$8 \times n = 72$

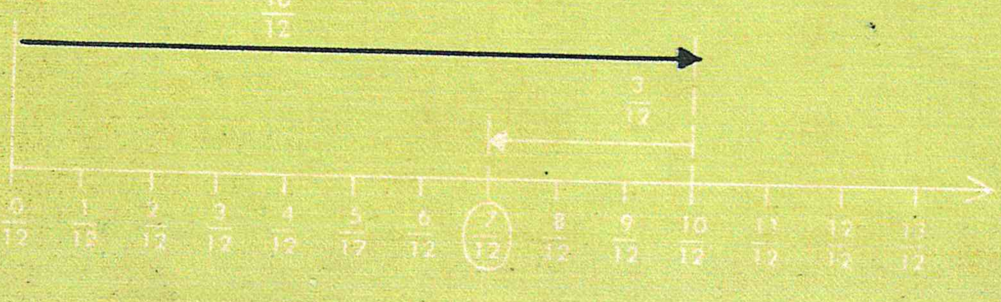
$n - 6 = 14$

ARITHMETIC

McSWAIN
BROWN
GUNDLACH
COOKE

5

$$\frac{10}{12} - \frac{3}{12} = n$$



ARITHMETIC

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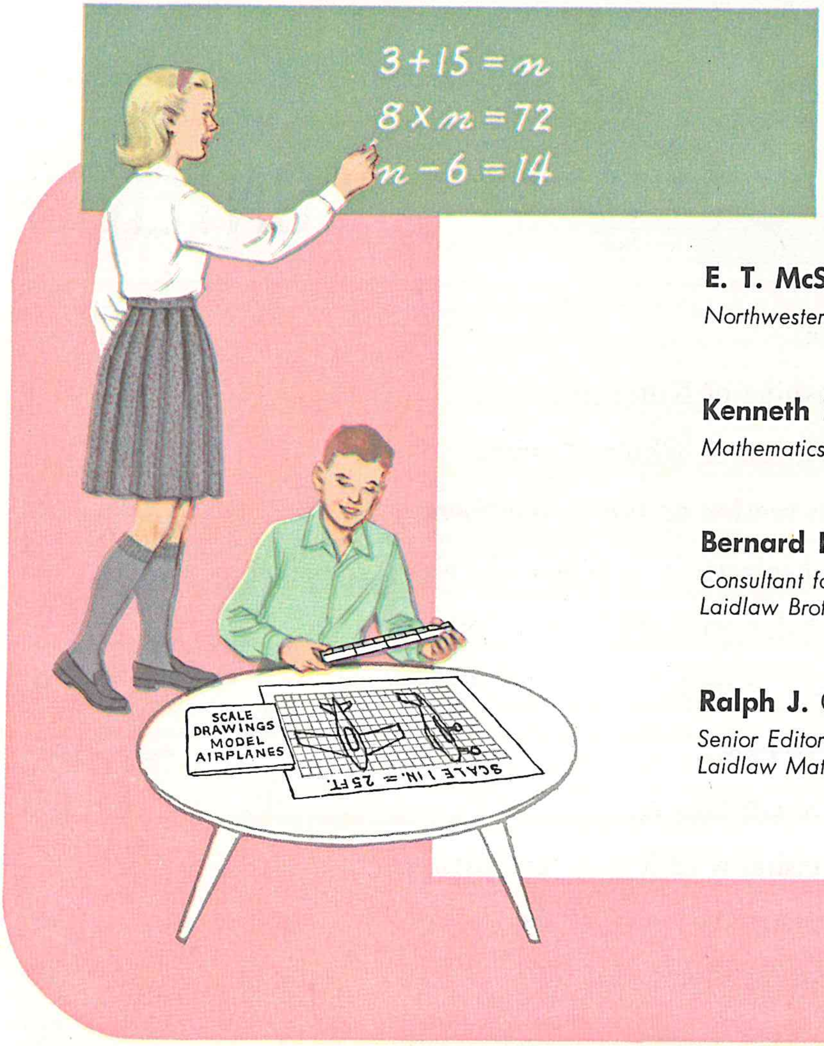

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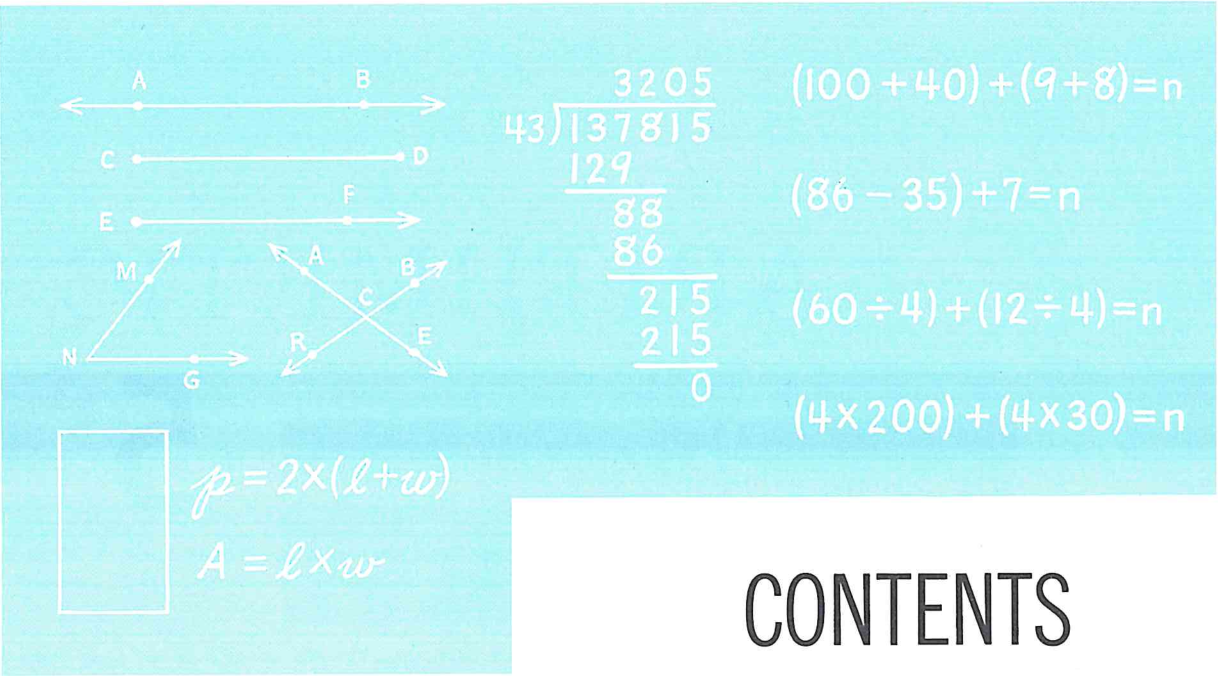

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$$\frac{2}{5} + 2\frac{3}{4} + \frac{1}{2} = n \qquad \frac{2}{5} = \frac{2\times4}{5\times4} = \frac{8}{20} \qquad \frac{3}{4} = \frac{3\times5}{4\times5} = \frac{15}{20} \qquad \frac{1}{2} = \frac{1\times10}{2\times20} = \frac{10}{20}$$

$$\frac{2}{5} + 2\frac{3}{4} + \frac{1}{2} = \frac{2}{5} + \left(2 + \frac{3}{4}\right) + \frac{1}{2} = 2 + \left(\frac{2}{5} + \frac{3}{4} + \frac{1}{2}\right) = 2 + \left(\frac{8}{20} + \frac{15}{20} + \frac{10}{20}\right) = 2 + \frac{33}{20} = 3\frac{13}{20}$$

$$\begin{array}{rcl} \frac{2}{5} & = & \frac{8}{20} \\ 2\frac{3}{4} & = & 2\frac{15}{20} \\ +\frac{1}{2} & = & \frac{10}{20} \\ \hline & & 2\frac{33}{20} = 2 + 1 + \frac{13}{20} = 3\frac{13}{20} \end{array} \qquad \begin{array}{rcl} \frac{2}{5} & = & .4 \\ 2\frac{3}{4} & = & 2.75 \\ +\frac{1}{2} & = & .50 \\ \hline & & 3.65 = 3\frac{65}{100} = 3\frac{13}{20} \end{array}$$

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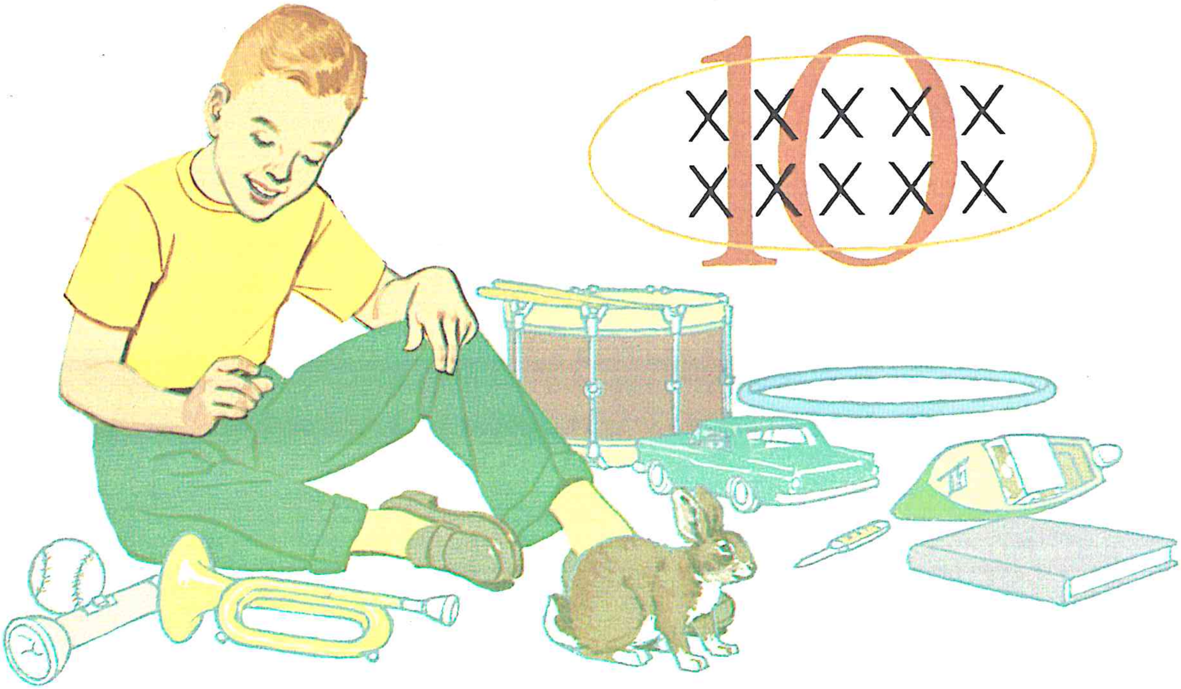
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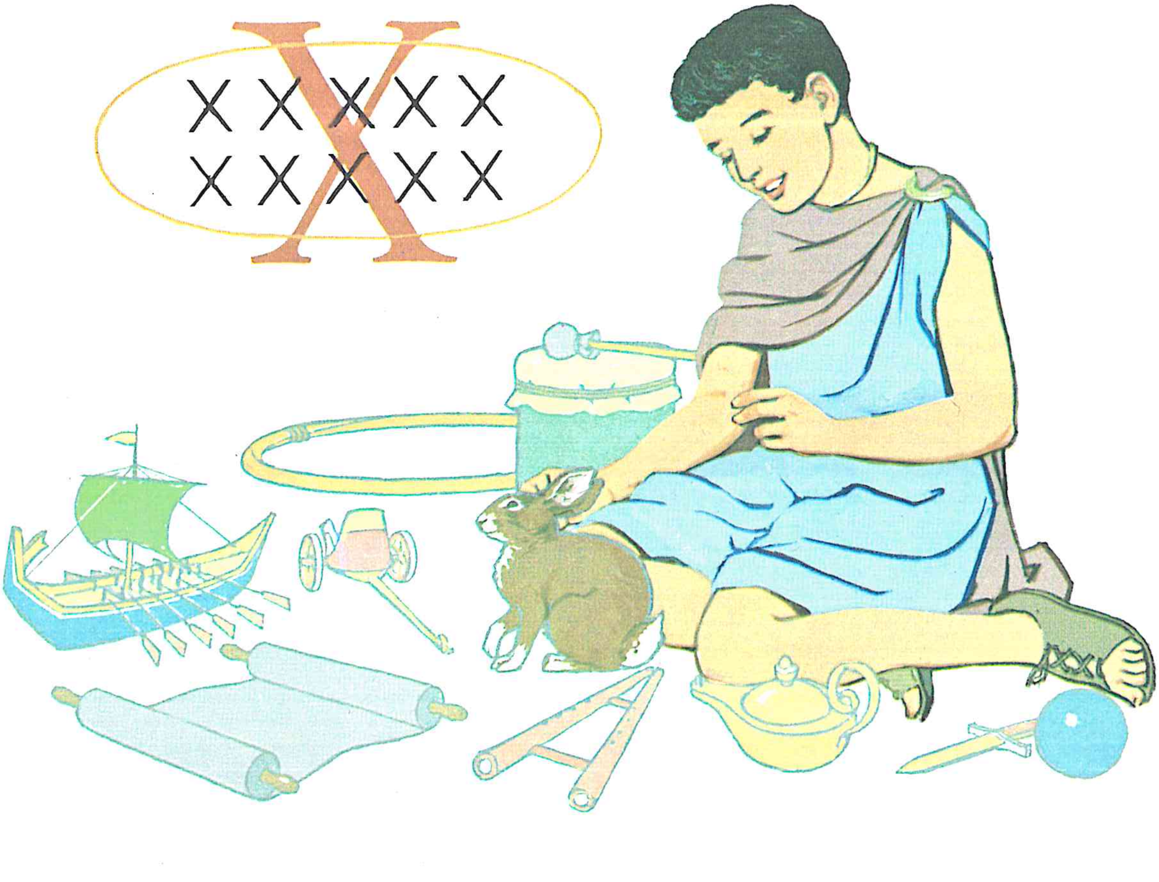
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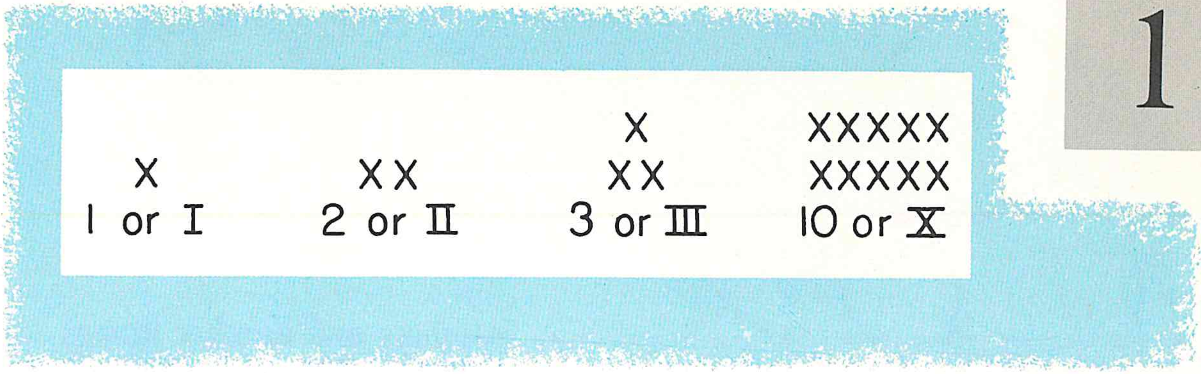


Finding How Many

The boys have counted their possessions. They will use different numerals to tell *how many*. Their numerals, which are Hindu-Arabic and Roman, are shown below the set patterns on this page.

In our Hindu-Arabic system only ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are needed to represent any number. We do this by using the same ten digits over and over again in a place-value system based on ten.

A kind of table called a **grid** will help you see that in our system the value of a digit depends upon its position, or place in the numeral. In the grid, Th stands for the set of thousands, H for the set of hundreds, T for the set of tens, and O for the set of ones. The number shown in the grid is two thousand, five hundred seventy-nine.

Th	H	T	O
2	5	7	9

Observe the numeral 5001 written in the grid. The 0 in tens place shows that there are *no* tens. The 0 in hundreds place shows that there are *no* hundreds. When there are no elements in a set, as in the tens set, the set is said to be empty. Do you see why zero is called the number of the empty set?

Th	H	T	O
5	0	0	1

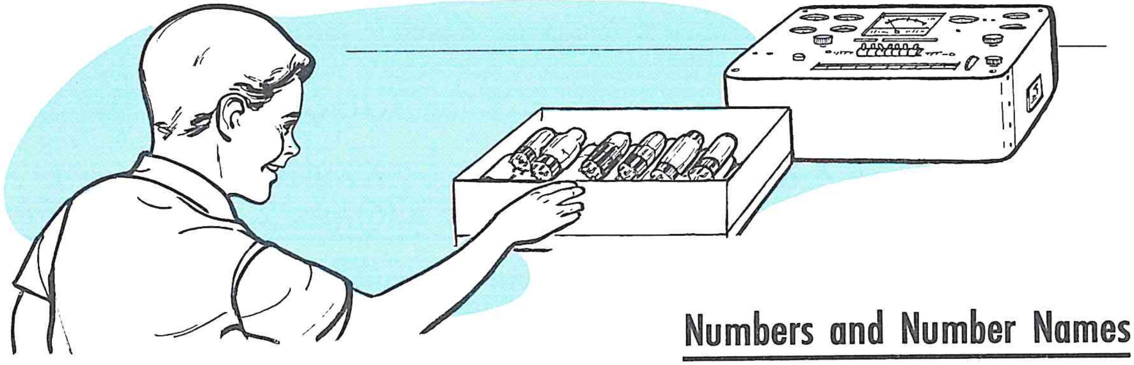
Oral Answer each question below.

1. How many objects did each boy on page 4 have?

2. What numeral is used in writing ten in the Hindu-Arabic system, and in writing ten in the Roman system?

3. In which system, Hindu-Arabic or Roman, are two digits used in writing ten?

4. What does the 0 in 10 tell you?



Numbers and Number Names

The boy in the picture has a name, Wally. Of course you know that "Wally" is not the boy but is only his name. His name and his picture are on this page, but the boy is not.

Wally has XXXXXX tubes in a group, or set. The set has a name, *six,* which when written as a numeral is **6**. Numerals are names for numbers, not the numbers themselves.

Most people can easily read numerals having only three digits. The number expressed by 324 is read *three hundred twenty-four*. If a numeral contains more than three digits, the digits can be grouped into sets of three digits each. Each set is given a name as shown.

Thousands	Ones
674	324

The digits in the thousands set are read *six hundred seventy-four* and are given the name *thousand.*

The digits in the ones set are read *three hundred twenty-four*.

For ease in reading, a comma is often used to separate the thousands set from the ones set. A comma is unnecessary in a four-digit numeral, although it may be used.

Oral Be ready to read each numeral expressed below.

	a	*b*	*c*
1.	7564	27576	112957
2.	3500	89230	560375

Written Write the number words you use in reading each numeral in rows 1 and 2 above. Number your answers 1 and 2.

Write in numeral form the numbers expressed by the following number words.

3. Three hundred

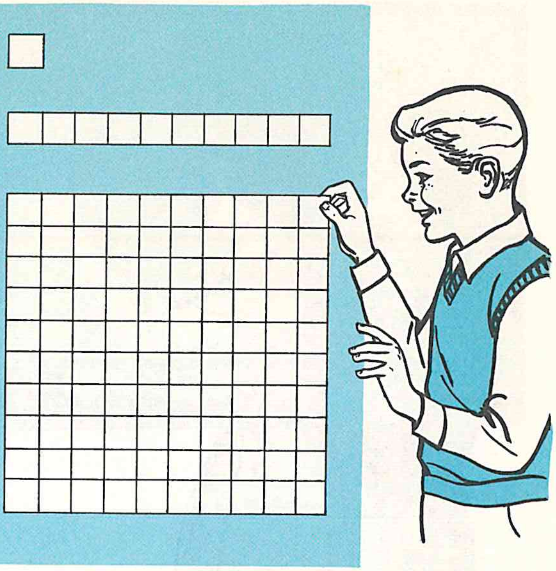
4. Forty thousand, nine

5. Thirty-five thousand, five hundred sixty-five

1 one

1 ten or 10 x 1

1 hundred or 10 x 10



The Decimal Numeration System

Our system of numeration is called a decimal system because of the importance of ten in the system. The word *decimal* comes from a Latin word meaning ten. The decimal grid and the numerals below will help you see the importance of ten.

Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Ones
1	1	1	1	1	1

$1 = 1 \times 1$
$10 = 10 \times 1$
$100 = 10 \times 10$
$1{,}000 = 10 \times 100$
$10{,}000 = 10 \times 1{,}000$
$100{,}000 = 10 \times 10{,}000$

The decimal value of each of the four digits in 5,647 is shown below.

5 means 5×1000 or 5000
6 means 6×100 or 600
4 means 4×10 or 40
7 means 7×1 or 7
The sum is 5647

Oral Tell the number of sheets of squared paper with one hundred squares on each that you would use in showing 500, 1,000, 10,000.

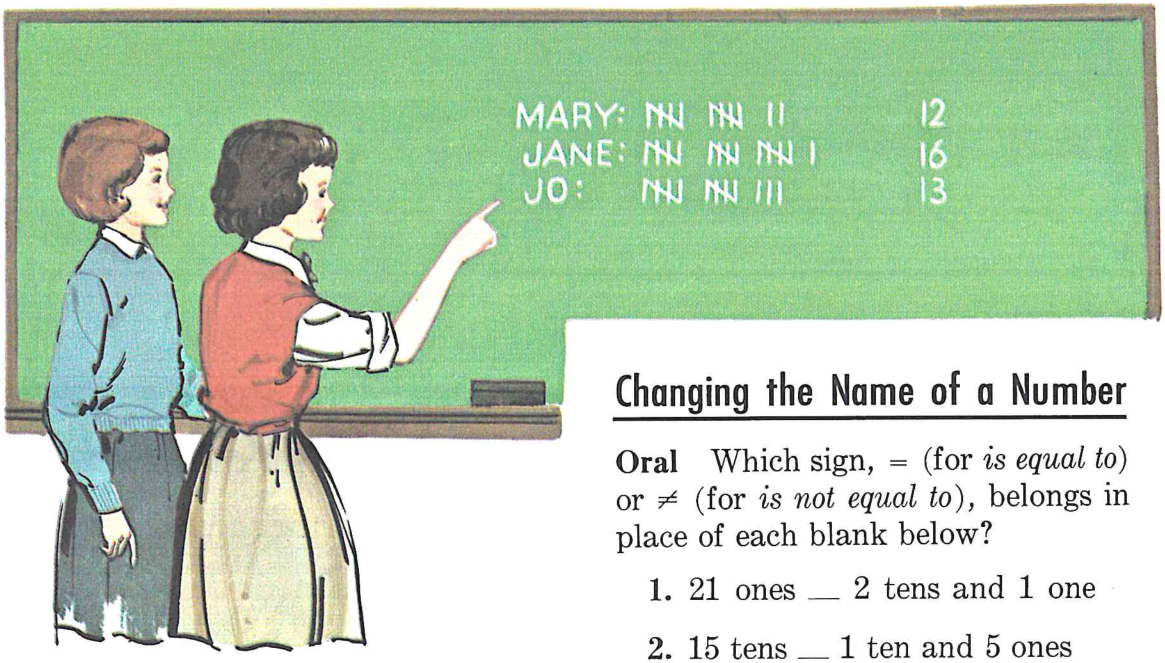
Written Write each statement below, using the correct numeral in each blank.

1. 70,836 = 7× __ or __
plus 0× __ or __
plus 8× __ or __
plus 3× __ or __
plus 6× __ or __
The sum is __.

2. 421,806 = 4× __ or __
plus 2× __ or __
plus 1× __ or __
plus 8× __ or __
plus 0× __ or __
plus 6× __ or __
The sum is __.

Show the meaning of each digit in the following.

	a	*b*	*c*
3.	1,625	20,412	214,576
4.	2,004	99,999	850,290



Changing the Name of a Number

Each tally mark can be thought of as a one. We have no single numeral for twelve ones. Twelve ones are named as 1 ten and 2 ones or 12. *Twelve ones* and *12* are two different names for the same number.

You must often change the name of a number when you add as in 8+9. The sum is seventeen ones, which is named as 1 ten and 7 ones or 17 in our decimal numeration system.

You also change the name of a number when you subtract as in 21−9. The 2 tens and 1 one may be thought of as 1 ten and 11 ones. You can subtract nine ones from eleven ones but not from one one.

MORE PRACTICE PAGE 295

Oral Which sign, = (for *is equal to*) or ≠ (for *is not equal to*), belongs in place of each blank below?

1. 21 ones __ 2 tens and 1 one
2. 15 tens __ 1 ten and 5 ones
3. 5 tens and 6 ones __ 56
4. 1330 __ 13 hundreds and 3 tens
5. 27 hundreds __ 2 thousands and 7 hundreds

How would you express the following in decimal form?

6. 21 ones
7. 15 tens and 5 ones
8. 32 hundreds, 8 tens, 3 ones
9. 85 hundreds, 11 tens, 0 ones
10. 96 tens and 15 ones

Written Using the decimal form, write the numerals for the numbers expressed in 6 through 10 above. Use a grid to help you.

Reading and Writing Larger Numerals

The decimal grid below has 9 place-value positions. In the grid, M stands for millions, TM stands for ten millions, and HM stands for hundred millions.

HM	TM	M	HTh	TTh	TH	H	T	O
5	2	8	4	7	9	5	6	6

The number shown in the grid has 9 digits. It is generally written as 528,479,566. Commas separate each set of three digits, making the numeral easier to read. The name given each group, or set, is shown.

Millions	Thousands	Ones
528	479	566

The digits in the millions set are read *five hundred twenty-eight,* and are given the name *millions*. The digits in the thousands set are read *four hundred seventy-nine thousand,* and the digits in the ones set are read *five hundred sixty-six.*

You read 528,479,566 like this: *five hundred twenty-eight million, four hundred seventy-nine thousand, five hundred sixty-six.*

Oral Tell where you would place a comma in each numeral stated below. Read each numeral.

	a	*b*
1.	6527818	600500049
2.	87900605	888000004
3.	196875200	720003000
4.	500700540	718562000

Read the census data which tells how many people lived in each major section in 1960.

Northeast—44,258,439

North Central—51,274,705

South—54,449,831

West—27,750,215

Written Follow these directions.

1. Make a list of the above regions according to population, putting the region with the smallest population first.

2. Express the counting numbers in natural order beginning with 999,995 and ending with 1,000,010.

3. Write in numeral form: one hundred twenty-seven million, six hundred nine thousand, fifty-one.

Using Numbers in the Billions

Do you know anyone who is 30 years old? That person has lived nearly a billion seconds.

Each of the numbers expressed below is 10 times the number expressed just above it. How many zeros follow the 1 in each numeral?

Ten	10
Hundred	100
Thousand	1,000
Ten thousand	10,000
Hundred thousand	100,000
Million	1,000,000
Ten million	10,000,000
Hundred million	100,000,000
Billion	1,000,000,000

How would you write ten billion? One hundred billion?

Astronomers sometimes use a different name for billion. The name they use is *one thousand million.*

To name numbers in the billions, you will need to remember that the digits can be arranged in sets of 3 digits. From left to right, the sets are named *billions, millions, thousands,* and *ones.* Read 476,532,156,785 like this: *four hundred seventy-six billion, five hundred thirty-two million, one hundred fifty-six thousand, seven hundred eighty-five.*

Oral Read each sentence below.

1. In 1961 the United States government spent \$80,410,000,000, which was about \$3,198,000,000 more than in 1960.

2. In a recent year the fire losses in our country amounted to approximately \$1,047,073,000 as compared with \$1,056,266,000 the year before.

Read the chart below.

U. S. Production of Petroleum

Year	Number of Barrels
1959	2,571,578,000
1958	2,448,987,000
1957	2,616,901,000
1956	2,617,283,000

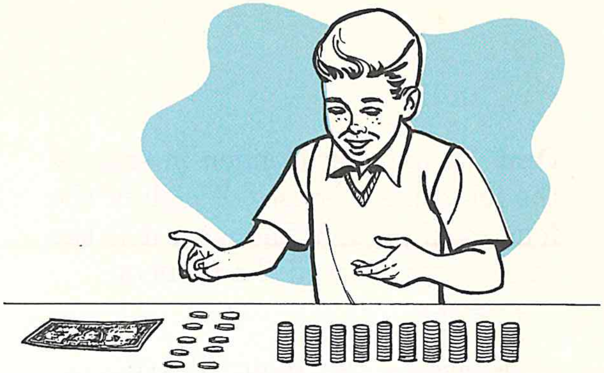
Written Write each of the numerals given below, placing commas as needed.

	a	*b*
1.	924319352	5802126
2.	878798325	6046691
3.	1599497005	1801287606
4.	4234568758	3756900095
5.	7000500000	9314601500

100 cents = 1 dollar

10 cents = $\frac{1}{10}$ dollar

1 cent = $\frac{1}{100}$ dollar



Expressing Money Value

The money in each of the three sets above has the value of a dollar. Each dime may be thought of as one of the 10 equal parts of the value of a dollar. Thus each dime has $\frac{1}{10}$ of the value of a dollar. Each penny may be thought of as one of the 100 equal parts of the value of a dollar.

In our money system, 10 has the same importance as in our number system. Because of the importance of 10, the money system and our number system are both called decimal systems. As a decimal numeral, $\frac{1}{100}$ is written .01 and $\frac{1}{10}$ is written .1. Using money symbols, .01 is written \$.01 and .1 is written \$.10.

$$\begin{array}{r} \$4000.00 \\ 300.00 \\ 50.00 \\ 2.00 \\ .40 \\ +.05 \\ \hline \$4352.45 \end{array}$$

At the right, \$4352.45 is expressed as a sum. Any decimal part of a dollar is written as hundredths. Notice that the points (.) lie along one vertical line.

Oral How would you write each of the following, using money symbols?

1. $\frac{3}{100}$ of a dollar
2. $\frac{15}{100}$ of a dollar
3. $\frac{23}{100}$ of a dollar
4. $\frac{1}{10}$ of a dollar
5. $\frac{3}{10}$ of a dollar
6. $\frac{9}{10}$ of a dollar

Written Write each of the following, using money symbols.

1. One dollar
2. Ten dollars and fifty cents
3. Eighty-five cents
4. Fifteen dollars and eighty cents
5. Two hundred dollars and eight cents

Write each of the following in the words that you would use to read each one.

	a	*b*	*c*
6.	\$25.15	\$175.50	\$550.50

Practice with Numerals

Oral A numeral is given in each of the following exercises. Which of the four word phrases following it is the correct way to read the numeral?

1. 6,250

six thousand, two hundred five
six hundred twenty-five
six thousand, one hundred fifty
six thousand, two hundred fifty

2. 18,002

eighteen thousand, two hundred
eighteen thousand, two
one hundred eighty thousand, two
eighteen thousand, twenty

3. 100,009

one hundred thousand, nine hundred
one hundred thousand, ninety
one hundred thousand, nine
one thousand, nine

4. 6,030

six thousand, three
six thousand, three hundred
six thousand, thirty
sixty thousand, three

5. 3,000,010

three million, one hundred
three million, one
three billion, ten
three million, ten

MORE PRACTICE PAGE 295

Written Write each of the following in the usual numeral form.

1. Eight hundred six
2. Six thousand, seven hundred
3. Five thousand, one
4. Sixty thousand, sixty
5. Four hundred fourteen
6. One hundred twenty-nine
7. One hundred nine
8. One hundred ninety
9. One hundred ninety-nine
10. Eighteen hundred sixty-two
11. Two hundred fifty thousand
12. Twenty-one thousand, eleven
13. Thirty-two hundred, fifteen
14. Five million, thirty thousand
15. Three thousand, eighty-seven

Express these numerals in words.

16. 5,063
17. 4,000,216
18. 72,409
19. 312,810
20. 6,666

Using Round Numbers

"How much do you think that you weigh, Peter?" asked Larry.

"Oh, in round numbers, about 70 pounds," said Peter.

Peter's weight was 68 pounds. The weight he expressed was rounded off to the nearer ten pounds.

In the statements below, 221,563 is rounded off first to the nearer ten, then to the nearer hundred, and then to the nearer thousand.

a. 221,560—to nearer ten, *6* not 7. The ones are dropped. 3 is nearer to 0 than to 10. No change is made in the tens.

b. 221,600—to nearer hundred, *6* not 5. The tens and ones are dropped. 63 is nearer to 1 hundred than to 0. 1 hundred is therefore added to the hundreds.

c. 222,000—to nearer thousand, *2*, not 1. The hundreds, tens, and ones are dropped. 563 is nearer to 1 thousand than to 0. 1 thousand is therefore added to the thousands.

Whenever an even digit precedes a 5 which is followed by zeros only, as the 8 in 850, the digit is not changed. If an odd digit precedes a 5 which is followed by zeros only, as the 7 in 750, it is increased by one.

750 is rounded to 800. 50 is no nearer to 0 than to 100, but 7 is odd, so 1 is added to hundreds.

850 is rounded to 800. Again, 50 is no nearer to 0 than to 100, but 8 is even. No change is made in hundreds.

Oral How could you round each number expressed below to the nearer ten? To the nearer hundred?

a	*b*	*c*	*d*
876	518	675	3,585

Written Round off to the nearer ten each number expressed below.

	a	*b*	*c*
1.	63	79	92
2.	17	36	88

Round off to the nearer hundred each number expressed below.

	a	*b*	*c*
3.	714	647	269
4.	436	573	328

Round off to the nearer thousand each number expressed below.

	a	*b*	*c*
5.	2,929	5,446	4,673
6.	7,818	3,737	6,364

MORE PRACTICE PAGE 295

Learning About Roman Numerals

The Roman numeration system involves the use of numerals that look like some of our letters as shown in the second column below.

One	I
Five	V
Ten	X
Fifty	L
One hundred	C
Five hundred	D
One thousand	M

The system follows several rules. One rule is that of *repetition.*

III—I repeated 3 times has a value of 3.

XXX—X repeated 3 times has a value of 30.

CC—C repeated twice has a value of 200.

A second rule is that of *addition.*

XII—X has a value of 10. II has a value of 2. 10+2=12.

XXXIII—30+3=33.

Another rule is that of *subtraction.*

IV—I on the left of V is subtracted as 5−1=4. The numeral IV has a value of 4.

IX—I on the left of X is subtracted as 10−1=9. The numeral IX has a value of 9.

XL—X on the left of L is subtracted as 50−10=40. The numeral XL has a value of 40.

Repetition may occur no more than 3 times. Subtraction may occur only once. 28 must be written XXVIII and not XXIIX.

Oral Answer each question below.

1. Use the rule of repetition. Tell the number words you would use in reading the following: CCC, MM, MMM.

2. Use the rule of addition. Tell how you would read each of the following: XXII, CCX, MMMCC.

3. Use the rule of subtraction. Tell how you would read each of the following: XC, CD, CM.

Written Write the following Roman numerals as they would be written in the Hindu-Arabic system.

	a	*b*	*c*
1.	IV	IX	XII
2.	VI	VII	VIII
3.	XVI	XVII	XVIII
4.	XIX	XXX	XL
5.	XXV	XLI	CCXII

Ones:	I	II	III	IV	V	VI	VII	VIII	IX
	1	2	3	4	5	6	7	8	9
Tens:	X	XX	XXX	XL	L	LX	LXX	LXXX	XC
	10	20	30	40	50	60	70	80	90
Hundreds:	C	CC	CCC	CD	D	DC	DCC	DCCC	CM
	100	200	300	400	500	600	700	800	900

Using Roman Numerals

In the above chart, you can see how to express certain numbers in Roman numerals and in Hindu-Arabic numerals.

Oral Answer each question below.

1. The rule of repetition is followed in writing II. In which other Roman numerals is the rule of repetition used?

2. Which Roman numerals on the chart illustrate the rule of addition?

3. Which Roman numerals on the chart illustrate the rule of subtraction?

4. How would you write as Roman numerals the Hindu-Arabic numerals 10 through 19? 40 through 49?

Written Follow these directions.

1. Write as Roman numerals the Hindu-Arabic numerals 40 through 49, and 60 through 69.

Write as Roman numerals each Hindu-Arabic numeral below.

	a	*b*	*c*	*d*
2.	3	6	15	17
3.	19	22	24	28
4.	32	35	39	51
5.	105	210	307	415

Write as Hindu-Arabic numerals each Roman numeral below.

	a	*b*	*c*	*d*
6.	II	VII	XIX	XXIV
7.	XXVII	XXI	XX	XXX
8.	LII	XLIV	LXII	LV
9.	LXX	XCV	LIX	LXXXI

Can you do this? Find a Roman numeral that is used as a date. Copy the numeral and explain its meaning to the class.

Egyptian Numerals

More than 5,000 years ago the Egyptians used picture writing to name some of the whole numbers, as shown below.

Our Numerals	Egyptian Numerals
1	|
10	∩
100	𓍢
1000	𓆼
10000	𓂭

Knowing only these symbols, the Egyptians could count by tens just as you do today. However, their system of writing numerals followed only these two simple rules: *repetition* and *addition.*

The way the Egyptians used these two rules in writing the numeral for 23 is shown below.

∩∩|||

10+10+1+1+1=23

In their system, the numeral for 241 would appear as follows.

𓍢𓍢∩∩∩∩|

100+100+10+10+10+10+1=241

Neither the Egyptians nor the Romans had a symbol for the number zero. There is no need for the symbol 0 unless you have place value.

Notice how many more Egyptian figures are necessary to write the name for 241 than in our system.

The Egyptian and Roman systems were fine for writing the names of the whole numbers, but were difficult to use in computation. For instance, you add 24 and 6 and get the sum of 30 very easily. The Romans might write XXIV+VI, and were almost forced to count to find the sum. Likewise, the Egyptians might have written it as ∩∩|||| +|||||| and would have had to count to find the sum.

Oral Answer the following.

1. What additional rule did the Romans use beyond what the Egyptians used in writing numerals?

2. In what ways is the decimal system better than the Egyptian system for doing arithmetic?

Written Write each of the following in Hindu-Arabic numerals.

1. 𓍢𓍢𓍢∩||||||
2. 𓆼𓍢𓍢∩∩∩||||

Write each of the following using Egyptian numerals.

3. 526

4. 3333

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Each time a count of 10, 20, 30, and so on, is reached, the digits are used over again. (5)

2. Zero is the number of the empty set. (5)

3. Numerals are names for numbers. (6)

4. The Hindu-Arabic numeration system is a decimal system because of the importance of ten in it. (7)

5. When a number is named in the decimal system, it can be thought of as the sum of ones, tens, hundreds, and so on. (8)

6. The Roman numeration system is based on three rules — the rules of repetition, addition, and subtraction. (14, 15)

Words to Know

1. Digits (5)

2. Grid (5)

3. Numerals and numbers (6)

4. Decimal numbers (7)

5. Billions (10)

6. Approximate (13)

Questions to Discuss

1. What is the meaning of each digit in 4,953? (7)

2. When might you want to change the name of a number? (8)

3. How does separating a numeral into sets of three digits each help you read the numeral? (9)

4. How do you round a number to the nearer ten? The nearer hundred? The nearer thousand? (13)

5. How do the Roman numeration and the Hindu-Arabic numeration systems differ from each other? (14)

Written Practice

1. Write the largest possible number using each of these digits only once: 1, 3, 5, 0. (7)

2. Name the counting numbers in natural order beginning with 999,999,990 and ending with the number 1,000,000,005. (9, 10)

3. Round to the nearer ten, the nearer hundred, and the nearer thousand each number expressed below.

5,283 12,898 25,682 (13)

4. Write the Roman numerals for 14, 49, and 90. (14, 15)

Self-Evaluation

Part 1 Write each of the following as a numeral.

1. Six hundred fifty-four thousand, two hundred twenty-four

2. Five million, ten thousand, sixty-five

3. Ten billion, five million, three hundred thousand, ten

4. Three hundred billion, five hundred million

5. Five hundred billion, eighty-five million, three hundred

6. Twenty-five billion, six hundred million, thirty thousand, one hundred twenty-five

Part 2 Copy each of the following numerals. Supply the missing commas. Then write number words to express each numeral in *a*.

	a	*b*
1.	89535	6300000
2.	456207	19001100
3.	1578000	347212350
4.	89435000	4756800000
5.	90005000	6423000000
6.	72114150	17500325000

Part 3 Make a chart like this one. Supply the missing numerals.

Exact Number	Rounded Numbers		
	Nearer ten	Nearer hundred	Nearer thousand
1. 4673			
2. 5739			
3. 8978			
4. 6762			
5. 9314			

Part 4 Make a chart like this one. Supply the missing numerals.

Number Word	Numeral	
	Hindu-Arabic	Roman
1. Ten		
2. One hundred		
3. One thousand		
4. Eighty-three		
5. Ninety-nine		

Part 5 Write each of the following, using the dollar sign ($) and the cents point (.).

1. $\frac{5}{100}$ of a dollar

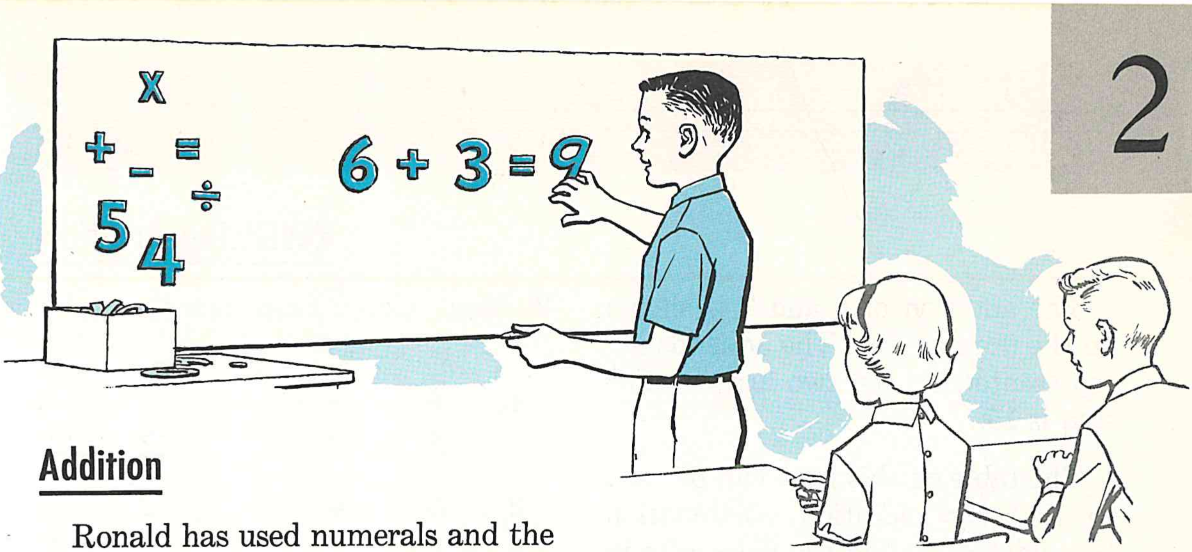
2. $\frac{17}{100}$ of a dollar

3. $\frac{5}{10}$ of a dollar

4. Eighty-nine cents

5. Ten dollars and five cents

2



Addition

Ronald has used numerals and the signs + and = to show an equation. He thought of 6 and 3 and got 9. This is an example of the operation called *addition*. In arithmetic an operation is a way of thinking about *two* numbers and getting *one* number.

In Ronald's equation the 6 and 3 are called **addends,** and the 9 is called the **sum.**

The sum of two whole numbers is always a whole number. This property is called the *closure property of addition of whole numbers.*

What numerals can you use for the n's to make the following true?

1. $9+7=n$ $7+9=n$

2. $4+8=n$ $8+4=n$

Do you see that two numbers may be added in either order to give the same sum? This property is called the *commutative property of addition.*

Now look at these equations.

3. $6+0=6$ $0+6=6$

4. $7+0=7$ $0+7=7$

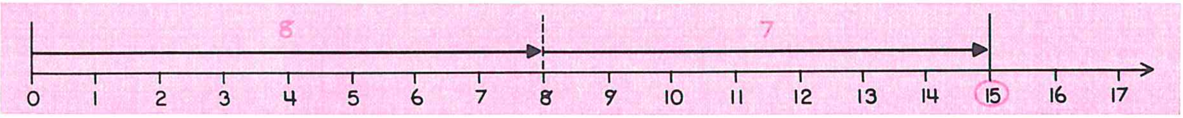
Does adding 0 to a number change the number? Does adding a number to 0 change the number?

As you can see, the number remains the same, or is identical, whether you add zero to the number, or add the number to zero. This is why zero is called the *identity number of addition.*

Oral Follow these directions.

1. Be ready to give equations to show that you can change the order of addition of two whole numbers without changing the sum.

2. Be ready to give equations to show that adding 0 to a number does not change the number.



Reviewing Addition

The addition of 8 and 7 is shown on the number line. The addends are represented by the two arrows. The sum is 15.

The table on this page can be used to find any addition combination through 18. To find the value of n in the equation 8+7=n, find 8 in the left hand column and 7 in the top row. The sum, 15, is in the square where the 8 row across and the 7 column down cross, or **intersect.** Use the table to solve 9+8=n.

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Oral Name each sum below.

a	*b*	*c*	*d*	*e*
7 +3	4 +8	3 +8	8 +6	7 +6

Written Copy. Find each sum.

	a	*b*	*c*	*d*	*e*
1.	6 +6	9 +5	8 +5	6 +9	5 +7
2.	5 +9	4 +7	7 +5	2 +8	6 +8
3.	6 +5	8 +4	7 +8	8 +9	5 +6
4.	3 +7	8 +7	9 +3	7 +4	7 +7
5.	9 +4	8 +3	6 +4	9 +6	4 +6
6.	9 +2	7 +9	4 +9	9 +1	9 +9
7.	9 +7	5 +5	8 +8	9 +8	5 +8

If you forget any sum, use an addition table or a number line.

Another way We just found the sum of 8 and 7 by reading the numeral in the 8 row and 7 column of the addition table. Now use the commutative property of addition to find the sum of 8 and 7 in this other way: Read the numeral in the 7 row and 8 column of the table.

Adding Numbers with One or Two Digits

Study the additions below.

$$\begin{array}{ccc} \textbf{A} & \textbf{B} & \textbf{C} \\ \begin{array}{c|c} T & O \\ 1 & 3 \\ +6 & \\ \hline 1 & 9 \end{array} & \begin{array}{c|c} T & O \\ 6 & 7 \\ + & 8 \\ \hline 7 & 5 \end{array} & \begin{array}{c|c|c} H & T & O \\ & 9 & 4 \\ & + & 7 \\ \hline 1 & 0 & 1 \end{array} \end{array}$$

In A, add the ones and leave the tens unchanged. 3 ones+6 ones=9 ones. Write 9 in ones place and 1 in tens place.

In B, add the ones. Change the form of 15 ones to 1 ten and 5 ones. Write 5 in ones place. Remember the 1 ten. Add the 1 ten remembered to 6 tens. Write 7 in tens place.

In C, add the ones. Think of 11 ones as 1 ten and 1 one. Add 1 ten to 9 tens. Think of 10 tens as 1 hundred and 0 tens.

Study the additions below. Are tens added in the same way as ones?

$$\begin{array}{ccc} \textbf{D} & \textbf{E} & \textbf{F} \\ \begin{array}{c|c} T & O \\ 1 & 3 \\ +2 & 6 \\ \hline 3 & 9 \end{array} & \begin{array}{c|c} T & O \\ 6 & 7 \\ +1 & 8 \\ \hline 8 & 5 \end{array} & \begin{array}{c|c|c} H & T & O \\ & 9 & 4 \\ + & 7 & 7 \\ \hline 1 & 7 & 1 \end{array} \end{array}$$

Oral Tell the steps to take in finding each sum below.

$$\begin{array}{rrrr} a & b & c & d \\ 76 & 95 & 54 & 78 \\ +7 & +8 & +65 & +86 \\ \hline \end{array}$$

Written Copy. Write each sum.

$$\begin{array}{lrrrr} & a & b & c & d \\ \textbf{1.} & 15 & 28 & 97 & 78 \\ & +4 & +5 & +86 & +89 \\ \hline \textbf{2.} & 17 & 19 & 94 & 91 \\ & +8 & +6 & +7 & +9 \\ \hline \textbf{3.} & 62 & 45 & 82 & 53 \\ & +17 & +23 & +94 & +95 \\ \hline \textbf{4.} & 86 & 65 & 49 & 57 \\ & +74 & +57 & +73 & +96 \\ \hline \textbf{5.} & 48 & 97 & 86 & 37 \\ & +37 & +54 & +68 & +87 \\ \hline \end{array}$$

Another way Regrouping addends may help you find sums quickly. 28+5 may be regrouped as 28+(2+3) because 2+3=5. The sum of (28+2)+3=30+3, or 33.

You can add 96 and 57 by thinking: How much of the 57 can I add to 96 to make 1 hundred? The answer is 4. Think: 57=4+53. Then 96+57=(96+4)+53, or 100+53, which is equal to 153.

MORE PRACTICE PAGE 296

Adding More Than Two Addends

Addition can be done with only two numbers at a time. When three numbers are to be added, either the first two or the last two may be grouped to be added first. See examples below. Notice that the sum is the same with both groupings. This property is called the *associative property of addition.*

$$(6+3)+4=9+4=13$$
$$6+(3+4)=6+7=13$$

Read this equation: $8+5+7+6=n$. One grouping for addition is shown.

$$\begin{array}{l} (8+5)+7+6=n \\ (13+7)+6=n \\ 20+6=n \\ 26=n \end{array}$$

Group the addends in another way and add again. Was the sum 26?

The equation $8+5+7+6=n$ is shown below in column and grid form.

$$\begin{array}{r} 8\\ 5\\ 7\\ +6\\ \hline n \end{array} \qquad \begin{array}{r|l} \text{T} & \text{O}\\ & 8\\ & 5\\ & 7\\ + & 6\\ \hline 2 & 6 \end{array} \qquad \begin{array}{l} \\ \\ (8+5=13)\\ (13+7=20)\\ (20+6=26)\\ \\ \end{array}$$

To check the addition, add the addends in another order. You should get the same sum.

Oral Tell the steps to take in finding each sum below.

a	*b*	*c*	*d*
4 8 7 +5	3 9 7 +6	7 7 8 +9	9 9 9 +7

Written Copy. Write each sum. Check by regrouping.

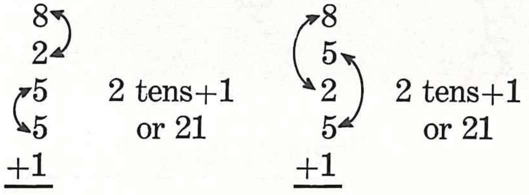
	a	*b*	*c*	*d*
1.	46 7 4 +5	3 8 7 +65	5 4 59 +7	4 37 9 +6
2.	89 72 56 +45	67 43 55 +39	53 80 88 +46	30 65 97 +54
3.	15 38 14 +23	18 17 26 +25	39 10 28 +14	15 29 26 +28

Copy the following equations. Show your groupings by writing in ( ). Find each sum.

	a	*b*
4.	$4+7+8=n$	$3+6+12=n$
5.	$9+13+11=n$	$8+7+4+5=n$
6.	$7+3+31=n$	$8+5+25=n$

Other Ways to Add

An excellent way to add numbers which are given in column form is to look for combinations of 10. Observe how combinations of 10 were used in adding the numbers indicated in the columns below.



In the equation 8+7+6=n, you do not *see* any combinations of 10. You should think of the many combinations of 10 you already know. In your mind you can change the way of naming a number and the order of the addends to enable you to use 10. Thus:

$$\begin{array}{r}8\\7\\\underline{+6}\end{array}\ \text{becomes}\ \begin{array}{r}8\\2\\5\\\underline{+6}\end{array}\ \text{or}\ \begin{array}{r}8\\2\\5\\5\\\underline{+1}\end{array}\ \begin{array}{l}\text{2 tens+1}\\\text{or 21}\end{array}$$

When adding two-digit numbers you can often save time by looking for combinations of 100. Thus: 44+63=n is solved as shown.

$$\begin{aligned}(40+4)+(60+3)&=n\\(40+60)+(4+3)&=n\\100\quad+\quad7\quad&=n\\107\quad&=n\end{aligned}$$

Such equations as 79+57=n may be easier to solve if you think of 57 as 1+56.

Then, 79+57=(79+1)+56=80+56.

Also, 80+56=(80+50)+6=130+6=136.

Hence, n=136.

Oral Tell different ways to find the sum in each addition below.

a	*b*	*c*	*d*
2	5	9	4
8	7	3	8
5	3	1	6
+5	+5	+7	+2

Written Copy. Find each sum. Check. (Reminder: Always try to find the easiest combinations and do as much as you can without writing.)

	a	*b*	*c*	*d*
1.	87 +6	97 +9	98 +7	76 +8
2.	83 +14	34 +45	41 +26	72 +23

Find a replacement for n which makes each of the equations true.

	a	*b*
3.	47+46=n	28+59=n
4.	67+75=n	95+29=n
5.	7+11+32=n	6+5+19=n

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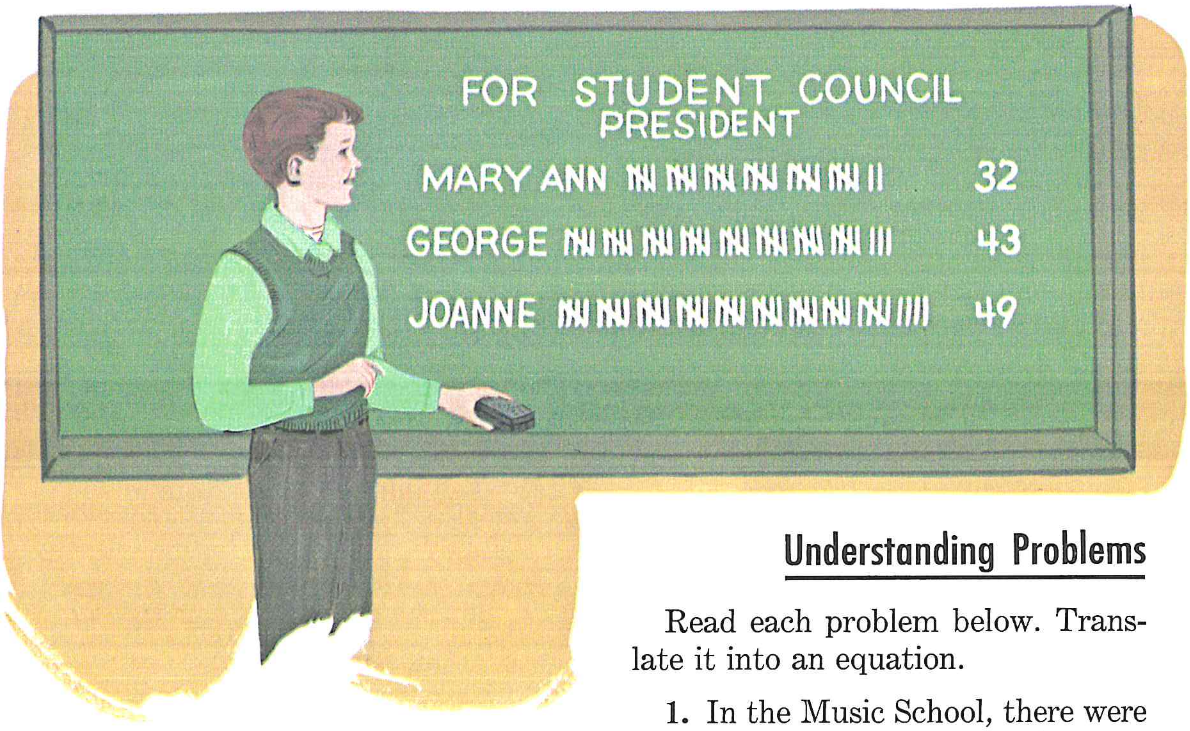


The teller wanted to know how many votes were cast in the election. What information does he have that he can use?

The teller has information telling how many votes were cast. He has a question about the votes which can be answered by joining the groups, or sets, of votes. Thus the teller has a problem. Any problem which can be solved by joining sets can also be solved by adding the numbers of the sets.

The problem above can be translated into an equation like this: $32+43+49=n$. The value of n is 124. The answer to the problem is a total of 124 votes.

MORE PRACTICE PAGE 296

Understanding Problems

Read each problem below. Translate it into an equation.

1. In the Music School, there were three fifth grades with 29 pupils in one grade, 33 pupils in another, and 31 in another. How many fifth graders attended the fifth grade in Music School?

2. The fifth graders sold 69 tickets for the first performance, 75 tickets for the second performance, and 92 tickets for the third performance of a circus. How many tickets did the pupils sell for all three performances?

3. Joe sold 72 papers on Monday, 87 on Tuesday, 77 on Wednesday, and 92 on Thursday. How many papers did he sell in the four days?

Oral Tell the equations you would use in solving problems 1, 2, and 3 above. Tell why you would use them.

Solving Problems

Each story problem on this page has a statement telling *how many* and a question related to the statement. The statement and question are related because they concern sets of objects that are alike. The following steps will help you solve each problem.

a. Read each problem carefully to determine how the statement and the question are related.

b. Decide what could be done with the sets to determine an answer. (If sets are to be joined, add the numbers of the sets.)

c. Write an equation, using n for the missing number.

d. Solve the equation to find a tentative answer.

e. Go over the problem again to see whether your tentative answer seems reasonable.

f. Write the answer, using the correct label.

Written Write and solve the equations for problems 1 through 5.

1. One Saturday John sold 78 papers, Donald sold 67 papers, and Michael sold 91 papers. How many papers did the boys sell altogether?

2. Don read 19 pages of his book in the morning and 27 pages in the afternoon. How many pages did he read during the day?

3. Janet received 32 votes, June received 39 votes, and Michael received 44 votes for secretary of the student council. How many votes were cast for secretary?

4. The three parking lots had spaces for cars as follows: Lot A, 32 spaces; Lot B, 60 spaces; Lot C, 48 spaces. How many spaces for cars were available on all three lots?

5. A room library had 28 books on geography, 38 books on history, and 29 books on science. How many books on all three subjects could be found in the library?

A quick review

1. $(20+40)+(6+8)=n$

2. $(30+20)+(9+5)=n$

3. $(50+60)+(8+7)=n$

4. $(40+80)+(6+7)=n$

5. $(90+60)+(5+7)=n$

6. $(30+42)+(7+3)=n$

7. $(100+40)+(9+8)=n$

MORE PRACTICE PAGE 296

Adding Numbers with Three Digits

Look at the equation 368+78=n. The addition is not difficult if you will do it in parts as shown in grid A.

A

H	T	O
3	6	8
+	7	8
4	4	6

The ones are added first. 8 ones+8 ones=16 ones. The form of 16 ones is changed to 1 ten and 6 ones. 6 ones is written in ones place. Next the 1 ten remembered is added to the tens. 1 ten+6 tens+7 tens=14 tens. The form of 14 tens is changed to 1 hundred and 4 tens, and 4 tens is written in tens place. Finally the hundreds are added. 1 hundred remembered+3 hundreds=4 hundreds. 4 hundreds is written in hundreds place. The total sum is 4 hundreds, 4 tens, and 6 ones, or 446.

Now look at the equation 436+385+574+957=n. It is written in column form in grid B. The only new step not used in grid A is that of changing the form of 23 hundreds to 2 thousands and 3 hundreds. Be ready to explain each step in the addition shown in grid B. How do you read the sum? Why does the sum contain four digits?

B

Th	H	T	O
	4	3	6
	3	8	5
	5	7	4
+	9	5	7
2	3	5	2

MORE PRACTICE PAGE 297

Oral Tell how you would solve each equation below. Always try to find the easiest combinations, and do as much as you can mentally.

	a	*b*
1.	379+21 = n	858+95 = n
2.	564+237 = n	379+465 = n

Written Copy. Find each sum. Add in a different order to check.

	a	*b*	*c*	*d*
1.	505 +699	479 +853	476 +589	999 +888
2.	23 42 +503	32 350 +74	55 880 +30	25 238 +33
3.	80 654 +67	61 84 +805	978 56 +25	92 49 +853
4.	106 328 132 +413	216 175 339 +146	400 139 607 +245	226 433 727 +540

Can you do this? What numerals must replace each dot below?

a	*b*
54●6 +3●7● ●752	4●72 ●85● +92●6 ●7775

Adding Numbers with Four Digits

A traffic count showed that on a Sunday 598 cars passed a certain school. On Monday, the count was 1,387 cars. What was the total count for the two days?

Because you are to think of a number that tells how many cars altogether, you add. The equation is 598+1387=n. The addition may be shown in column form as in A.

After adding the ones, you must change the form of 15 ones to 1 ten and 5 ones. After adding the tens, you must change the form of 18 tens to 1 hundred. and 8 tens. No changes in form are necessary after you add the hundreds. The sum is 1,985.

A

Th	H	T	O
	5	9	8
+1	3	8	7
1	9	8	5

Now look at the addition in column form in grid B below.

B

7206
6025
5704
+2043

TTh	Th	H	T	O
	7	2	0	6
	6	0	2	5
	5	7	0	4
+	2	0	4	3
2	0	9	7	8

In the addition shown in grid B, the sum in thousands place is 20 thousands. The form of 20 thousands must be changed to 2 ten-thousands (TTh) and 0 thousands. The sum is 20,978. Can you read this sum correctly?

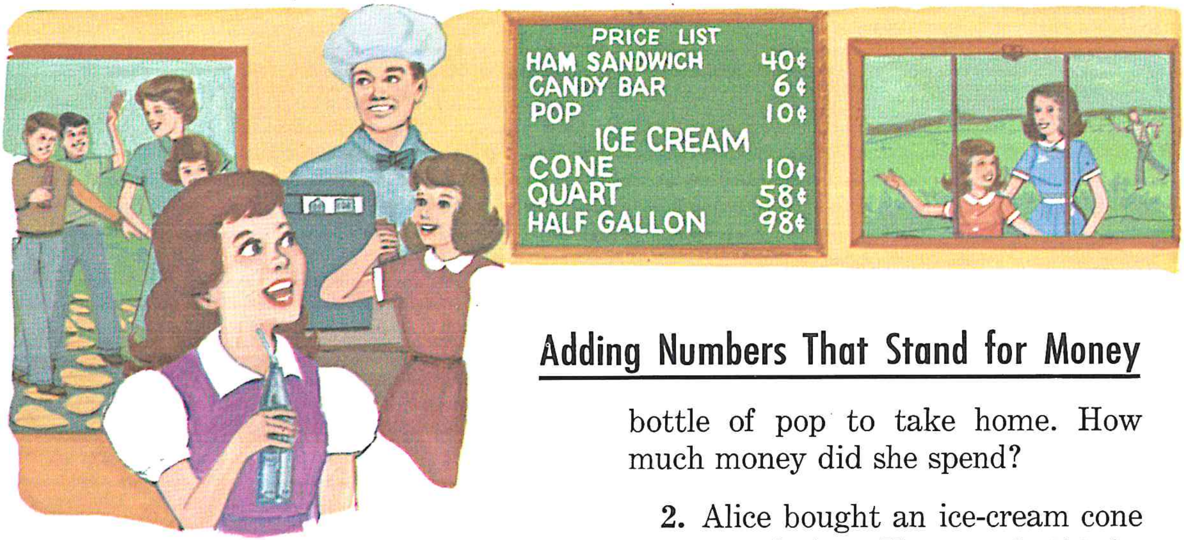
Oral Be ready to tell how you can find the sum for each addition in column form below.

	a	*b*	*c*
1.	8624 +634	4980 +3549	1324 +9715
2.	1032 3002 2460 +2503	2634 2000 1700 +2041	9354 3090 2270 +3103

Written Write each addition in rows 1 and 2 above with its sum. Number the rows 1 and 2. Then copy each addition below. Write each sum. Check by adding another way.

	a	*b*	*c*
3.	867 +4674	9556 +556	703 +4085
4.	7246 +5283	3134 +4906	5245 +2756
5.	600 4400 +300	280 350 +3650	3496 370 +870

MORE PRACTICE PAGE 297



Adding Numbers That Stand for Money

In column form, money numbers are written as shown. The dollar sign is used with the first addend and in the sum. The cents point is placed in each addend. The cents points are directly underneath each other. The numbers in each column are added in the same way as ones, tens, and hundreds.

$$\begin{array}{r} \$1.75 \\ .55 \\ .07 \\ +.25 \\ \hline \$2.62 \end{array}$$

Oral Why is a dollar sign unnecessary in each of the addends? Why is it necessary in the sum?

MORE PRACTICE PAGE 297

Written Using the price list above, write an equation for each of the following problems. (Reminder: The $ and . signs are not used in the equation, but are part of the answer.)

1. Larry's mother bought a quart of ice cream, a ham sandwich, and a bottle of pop to take home. How much money did she spend?

2. Alice bought an ice-cream cone and a candy bar. How much did she spend for the two items?

3. Nora bought a candy bar for herself, a bottle of pop for Sue, and a half gallon of ice cream to take home. How much money did she spend?

4. John's mother bought a bottle of pop for John, a candy bar for Roger, and an ice-cream cone for Roy. Then she bought a half gallon of ice cream. How much money did she spend?

Copy. Find each sum. Check by adding in a different order.

	a	*b*	*c*
5.	$\begin{array}{r} \$4.95 \\ .35 \\ +.27 \\ \hline \end{array}$	$\begin{array}{r} \$6.15 \\ 2.79 \\ +.37 \\ \hline \end{array}$	$\begin{array}{r} \$5.87 \\ 4.96 \\ +3.75 \\ \hline \end{array}$
6.	$\begin{array}{r} \$8.00 \\ 7.50 \\ +4.69 \\ \hline \end{array}$	$\begin{array}{r} \$3.57 \\ 4.78 \\ +7.69 \\ \hline \end{array}$	$\begin{array}{r} \$7.29 \\ 4.54 \\ +3.67 \\ \hline \end{array}$

Using Addition in Solving Problems

These instructions will help you do this page by yourself. Read each problem carefully to determine how the statement and question are related. Decide how the question could be answered by doing something with the sets. If you are to join them, add the numbers.

Write an equation for each problem. Find the value of n as a tentative answer. Check over the problem to see if the tentative answer is sensible. Write the answer. Reminder: The dollar sign ($) and the cent sign (.) are not part of an equation. In an equation, think of the prices as though they were all pennies, but use the dollar sign ($) and cent sign (.) in each answer.

1. James had $3.75. He earned $1.50 by running errands and $.65 by mowing his father's lawn. How much money did he then have?

2. The Mead family took a four-day trip. They drove 385 miles the first day, 417 the second day, 345 the third day, and 390 miles the fourth day. How many miles did they drive in four days?

3. Agnes read four books having 129 pages, 210 pages, 107 pages, and 263 pages. How many pages did she read altogether?

4. Mary's mother bought a dress for $29.95, a pair of shoes for $12.98, and a scarf for $1.50. How much did Mary's mother spend in all?

5. Jim's father sells groceries. The first day his total sales amounted to $879.25; the second day the total sales amounted to $750.95; the third day, $191.75; and the fourth day, $975.20. What were his total grocery sales in four days?

6. Mr. Johnson bought a coat for $59.50, a suit for $67.00, a pair of shoes for $16.98, and a hat for $7.50. How much money did he spend for these four items of clothing?

7. In one city there were three elementary schools. In one school 279 pupils were enrolled. In another school 1,095 pupils were enrolled, and in the third school 846 children were enrolled. How many pupils were enrolled in the three elementary schools of the city?

8. Mrs. Andersen bought a table for $129.75, a rug for $69.98, and bedroom furniture for $329.75. How much did she spend in all?

9. Mr. West keeps 43 cows, 7 horses, 2 dogs, and 215 chickens on his farm. What is the total number of animals he keeps?

Adding Numbers with Five Digits

If you can add numbers with four digits, you can probably add numbers with five digits. Ten thousands are added in exactly the same way as ones, tens, hundreds, or thousands. Look at the addition in grids A and B below.

A

41236
+3523

	TTh	Th	H	T	O
	4	1	2	3	6
+	3	5	2	3	
	4	4	7	5	9

B

91532
52274
+63378

	HTh	TTh	Th	H	T	O
		9	1	5	3	2
		5	2	2	7	4
	+	6	3	3	7	8
	2	0	7	1	8	4

In A there are no changes in form in any place-value position. What changes in form were made in B? Look at the digits in the ten-thousands place. Their sum is 20 ten-thousands, or 2 hundred-thousands and 0 ten-thousands. The sum in B is 207,184. Can you read it correctly?

MORE PRACTICE PAGE 297

Oral How would you solve the equation 54213+5314=n? The equation 42156+52449=n?

Written Copy. Write each sum. Add in a different order to check.

	a	*b*	*c*
1.	62134 +6319	76890 +5428	23769 +3807
2.	40765 +43189	75376 +34832	68753 +64729
3.	63694 2159 +4763	3797 58692 +1437	6543 7254 +99876
4.	97897 13687 +74532	58325 25890 +43760	63421 36925 +75689
5.	92367 32156 +46897	84752 43219 +29156	63214 74347 +46397

The additions below have more addends, but you do not need to know anything new in order to find each sum. Copy each addition below and write its sum.

	a	*b*
6.	\$754.67 426.74 795.27 +959.47	\$387.65 698.75 798.54 +645.36
7.	\$975.85 654.95 376.87 +49.50	\$325.75 469.89 729.75 +625.50

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. In the addition of whole numbers, the sum is again a whole number. (19)

2. Numbers may be added in any order. (19)

3. Zero is called the identity number of addition. (19)

4. Numbers may be grouped in any way for addition. (21, 22, 23)

5. You add ones to ones, tens to tens, hundreds to hundreds, thousands to thousands, and ten-thousands to ten-thousands. (21, 26, 27, 30)

Words to Know

1. Identity number of addition (19)

2. Operation, addend, sum (19)

Questions to Discuss

1. What are some of the ways you can use to find the answer to an addition fact if you forget it? (20)

2. In which ways can you find the sum of each addition below?

a	*b*	*c*	
28	96	79	
+5	+57	+54	(21)

3. In which ways can you find the sum of each addition below?

a	*b*	*c*	
8	8	8	
2	5	7	
5	5	1	
+5	+2	+6	(23)

4. When solving a problem how can you decide whether or not to add the numbers of the sets? (24, 25)

5. What steps should you use when solving a story problem? (25)

Written Practice

Copy. Write each sum.

	a	*b*	*c*	
1.	865	1325	4192	
	3742	4275	3875	
	651	6896	6742	
	+219	+3425	+2759	(27)

Copy. Write each sum.

	a	*b*	
2.	97857	52156	
	4653	34699	
	+27964	+78152	(30)
3.	92879	$975.50	
	74387	425.35	
	52745	275.50	
	+69318	+314.99	(30)

Self-Evaluation

Part 1 Copy. Write each sum.

	a	*b*	*c*	*d*
1.	63 +75	77 +19	98 +89	54 +87
2.	31 42 +37	54 36 +83	74 26 +35	68 72 +35
3.	157 68 +53	337 64 +73	719 91 +85	846 27 +58
4.	372 64 +453	395 236 +715	983 754 +127	681 429 +323
5.	634 235 875 +476	352 737 251 +879	485 313 697 +325	683 418 328 +796

Part 2 Copy. Write each sum.

	a	*b*	*c*
1.	1860 523 +417	3742 361 +518	4265 1434 +3575
2.	3453 7152 +2647	1214 9683 +8497	5461 2348 +5729
3.	5769 4331 5848 +6292	2457 7362 8648 +2753	9539 9480 8621 +2479
4.	69421 +4782	37495 +8978	26973 +17534
5.	53264 4146 +2328	53294 27103 +56817	76534 32016 +14572
6.	74326 16243 85764 +98887	94132 27879 23256 +86989	84765 32434 78340 +59175

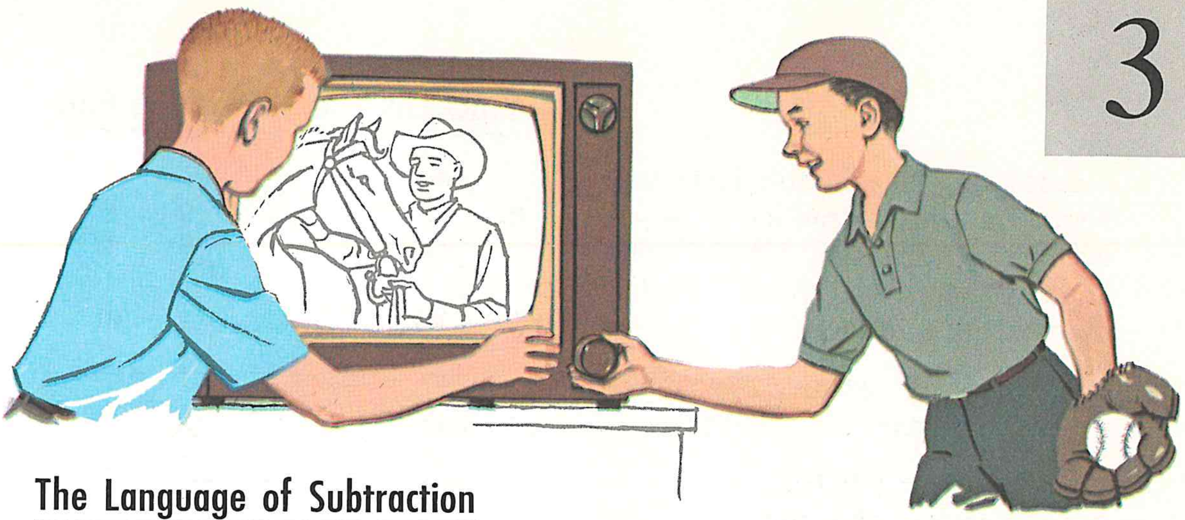
Part 3 Solve these problems.

1. Lincoln School had an enrollment of 1,045 pupils. In the same city, three other elementary schools had enrollments as follows: Roosevelt—735 pupils, Jefferson—498 pupils, Adams School—929 pupils. What was the total enrollment of the four schools?

2. Mr. Johnson paid $115 for rent. His car expenses were $31.85 and his bill for clothing was $112.79. What were his total expenses for the three items?

3. The Boy Scouts had a paper drive. Troop 13 collected 4,312 pounds of paper, Troop 21 collected 5,687 pounds, and Troop 74 collected 4,769 pounds. How many pounds of paper did the three troops collect altogether?

3



The Language of Subtraction

Arthur had just turned the television set on when Larry turned it off. Turning the set off *undoes* what turning it on *does*.

Many operations undo what others do. Untying a knot that someone has tied, opening a door that someone has closed, and unscrewing the top of a peanut-butter jar after someone has screwed the top on are examples of such operations. Can you think of other undoing operations?

An operation which undoes what another operation does is called the **inverse** of the original operation.

Look at the illustration below.

$$9+n=15 \longleftarrow \text{inverse} \longrightarrow 15-n=9$$

Adding 6 to 9 gives a sum of 15. Subtracting 6 from 15 gives a difference of 9. Does subtraction seem to undo what addition does? Try adding 6 to other numbers. Then subtract 6 from those sums. Why are subtraction and addition inverse operations?

From the following addition and subtraction, notice that each operation *undoes* what the other *does*.

$$\begin{array}{r} 9 \\ \underline{+6} \\ 15 \end{array} \longleftarrow \text{inverse} \longrightarrow \begin{array}{rl} 15 & \text{minuend} \\ \underline{-6} & \text{subtrahend} \\ 9 & \text{difference} \end{array}$$

Oral Answer these questions.

1. Which number in the addition above is the same as the minuend in the subtraction?

2. Which number in the addition above matches the subtrahend in the subtraction?

3. Do two other numbers match?

4. Why can you check subtraction by adding the subtrahend to the difference?

Practicing the Subtraction Facts

Knowing the addition facts well helps you with subtraction facts you may have forgotten. Thus if you know that $8+3=11$, you know that the value of n in $11-3=n$ is 8.

Oral Read each subtraction supplying each difference. Reminders:

a. Subtracting 0 from a number doesn't change the number. Thus, as a subtrahend, zero is called the *identity number of subtraction.*

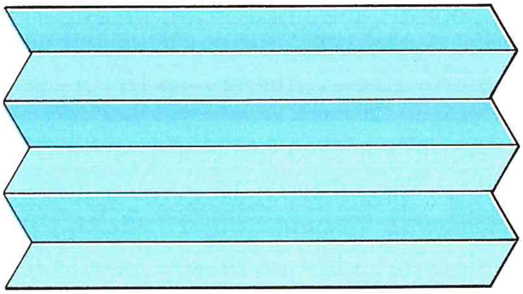
b. Subtracting a number from itself equals 0.

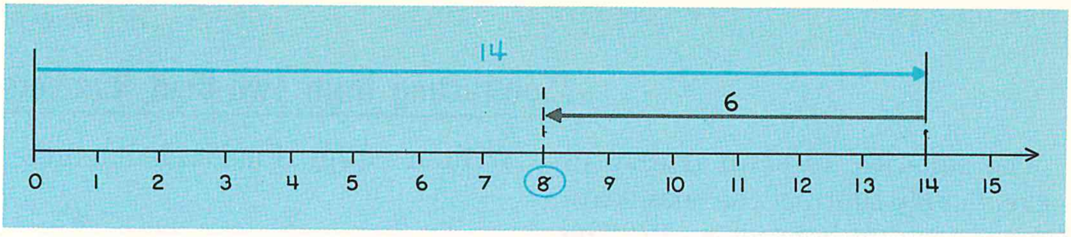
	a	*b*	*c*	*d*	*e*
1.	8 −4	7 −5	9 −8	7 −3	6 −4
2.	5 −4	9 −1	6 −2	5 −3	1 −1
3.	9 −0	8 −5	9 −3	8 −2	4 −1
4.	8 −7	3 −2	9 −4	6 −3	7 −4
5.	5 −5	8 −0	5 −2	7 −6	7 −2
6.	9 −6	8 −8	7 −0	4 −2	8 −3
7.	14 −6	17 −9	12 −6	16 −8	11 −3

MORE PRACTICE PAGE 298

	a	*b*	*c*	*d*	*e*
8.	15 −8	10 −2	12 −8	9 −9	5 −0
9.	10 −4	13 −4	11 −9	12 −3	10 −8
10.	11 −2	10 −7	10 −9	14 −5	7 −7
11.	4 −0	13 −6	15 −9	13 −5	12 −7
12.	12 −9	13 −9	14 −7	10 −5	18 −9
13.	3 −3	3 −0	11 −8	16 −7	10 −6
14.	15 −6	15 −7	11 −6	14 −8	11 −7

Written Fold a piece of paper as shown. Lay the top of your paper under row 1. Write each difference. Fold your answers under. Write row 2. Continue until you finish the page.

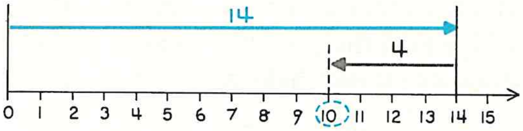




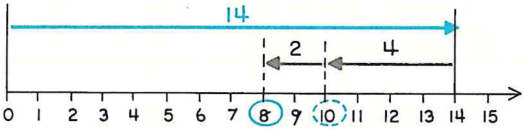
Using the Number Line in Subtraction

On the number line above the solution of $14-6=n$ is shown. The blue arrow shows the minuend. Notice that the gray arrow pointing to the left indicates the subtraction of 6. From the diagram you see that $14-6=8$.

You do not have to draw the *subtract 6* arrow in one piece. You may stop anywhere along the line. A convenient stop would be at 10.



You are at 10 because $14-4=10$. But $6=4+2$, so you must draw another 2-unit arrow from 10 to the left. So $14-6=8$.



In the form of an equation this is $14-6=14-(4+2)=(14-4)-2$, which becomes $10-2=8$.

Oral Study the solutions below. Be ready to explain each step.

$17-9=n$	$23-8=n$
$(17-7)-2=n$	$(23-3)-5=n$
$10-2=n$	$20-5=n$
$8=n$	$15=n$

Written Solve each equation below for n. Write the complete solution.

	a	*b*
1.	$16-9=n$	$17-8=n$
2.	$34-7=n$	$29-8=n$
3.	$26-8=n$	$31-5=n$
4.	$13-6=n$	$12-3=n$

A quick review Solve each equation for n. (Reminder: The ( ) means *do this operation first.*)

	a	*b*
1.	$(31-1)-9=n$	$(43-30)+6=n$
2.	$(89-40)+6=n$	$(75-20)+8=n$
3.	$(85-30)-6=n$	$(68-14)-9=n$
4.	$(91-20)+8=n$	$(86-35)+7=n$

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Subtracting from Two-Digit Numbers

The equation 19−7=n is easy to solve. Simply subtract the ones, leaving the tens unchanged. n=12. The equation 69−37=n is also easy to solve, for you subtract the ones and then the tens. n=32.

$$\begin{array}{c|c} \text{T} & \text{O} \\ 4 & 2 \\ - & 5 \\ \hline 3 & 7 \end{array}$$

To find n in the equation 42−5=n, you may think directly, 42−5=37. Or you might subtract as shown in the grid. The form of 4 tens and 2 ones was changed to 3 tens and 12 ones before subtracting. Why was the change necessary?

Why should you try to subtract, without using either a grid or a number line?

Oral Be ready to tell how you would find the difference for each subtraction below.

	a	*b*
1.	38−6=n	47−4=n
2.	76−35=n	89−30=n
3.	86−28=n	71−45=n
4.	64−12=n	94−76=n
5.	57−38=n	25−18=n
6.	81−56=n	76−49=n
7.	47−29=n	31−19=n

MORE PRACTICE PAGE 298

Written Copy. Find each difference.

	a	*b*	*c*	*d*
1.	79 −8	52 −2	67 −4	28 −7
2.	65 −8	73 −7	82 −8	95 −9
3.	80 −60	38 −15	49 −47	75 −45
4.	82 −47	71 −37	87 −68	93 −89

Another way The equation 92−37=n may be solved directly by thinking: 92−37=55. Another way to solve this is to think of 37 as the sum of 32 and 5 and do the subtraction in two steps as shown below.

$$\begin{aligned} 92 - 37 &= n \\ 92-(32+5) &= n \\ (92-32)-5 &= n \\ 60 - 5 &= n \\ 55 &= n \end{aligned}$$

Rewrite the subtrahend as an addition and solve for n.

	a	*b*
1.	37−19=n	26−16=n
2.	64−38=n	85−39=n
3.	93−47=n	78−45=n

Subtracting from Three-Digit Numbers

You can easily find the difference, n, in 573−21=n and 854−232=n, as shown in A and B below, because you subtract tens and hundreds as you subtract ones.

A

	H	T	O
	5	7	3
−		2	1
	5	5	2

B

	H	T	O
	8	5	4
−	2	3	2
	6	2	2

In each subtraction shown in C and D, do you see that 1 hundred was changed to 10 tens?

C

	H	T	O
	5	7	8
−		9	7
	4	8	1

D

	H	T	O
	8	2	9
−		7	5
	7	5	4

In grids E and F, 1 ten was changed to 10 ones.

E

	H	T	O
	7	8	0
−		3	5
	7	4	5

F

	H	T	O
	6	9	0
−	2	8	4
	4	0	6

In grids G and H, changes in form were made in the tens place and in the ones place.

G

	H	T	O
	7	1	6
−	5	6	7
	1	4	9

H

	H	T	O
	8	0	0
−	1	7	8
	6	2	2

Sometimes you need to change a hundred to 10 tens before you have any tens to change to ones.

Oral Be ready to tell how to find each difference in column *a*.

	a	*b*	*c*	*d*
1.	346 −34	498 −35	680 −320	879 −425
2.	548 −60	109 −39	820 −130	446 −255
3.	974 −19	142 −37	385 −118	878 −349
4.	273 −75	654 −98	731 −645	584 −397
5.	407 −98	702 −87	600 −149	800 −578

MORE PRACTICE PAGE 298

Written Copy the subtractions above. Write each difference.

Understanding Problems

Jack had 24 cards to use in taking notes. He spoiled 9 of the cards and threw them away. How many cards did he have left?

The problem has a statement telling about two groups, or sets. Then it asks a question that can be solved by using an arithmetic operation.

To solve the problem, try to picture the situation.

a. Were groups, or sets, combined?

b. Was a set separated?

The set of 24 was separated; so you subtract the numbers. In the language of arithmetic, the problem is written as $24-9=n$.

If necessary, use the grid form, a number line, or a picture of the sets to get the answer. Jack had 15 cards left.

As a next step, read the problem statement again to help you decide whether the answer seems reasonable. Check your subtraction by using the inverse operation, addition.

Oral Read each of the following problems. Refer to the previous suggestions. Decide which operation to use and be able to give an equation for each problem.

1. There are 35 pupils in Science Club. Of these 17 are girls. How many of the pupils are boys?

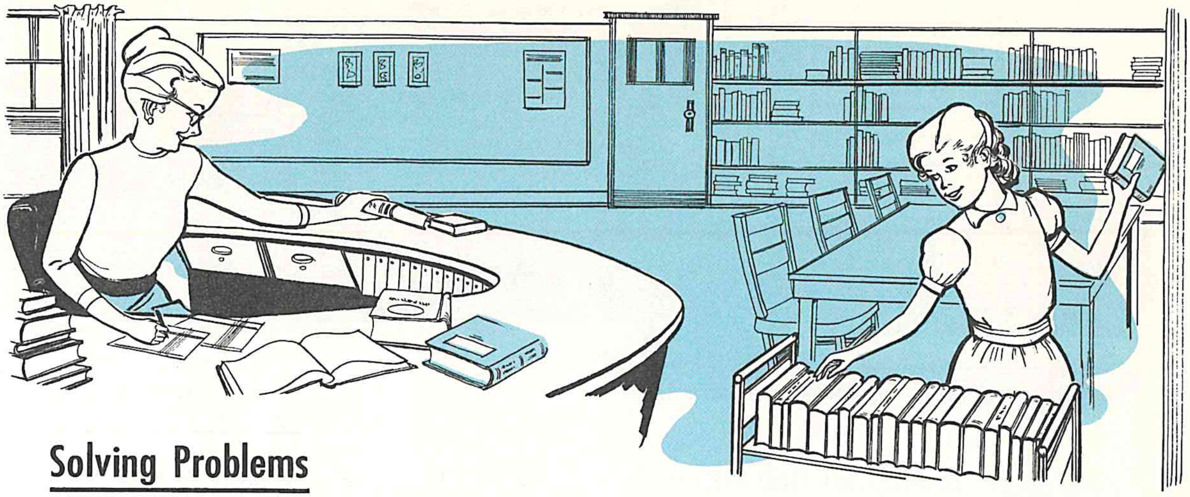
2. Mary knew that 41 books were on a shelf in the morning. At night only 19 books were on the shelf. How many books were not put back on the shelf?

3. Ed took 89 cents to the hobby store. He bought some glue and returned home with 54 cents. How much did he spend?

Written Write the equations and find the answers for problems 1 through 3 above.

Can you do this? Find the value of n which makes each equation true. (Reminder: *Addition and subtraction are inverse operations.*)

	a	*b*
1.	$43+n=52$	$23-n=15$
2.	$n+12=26$	$49+11=n$
3.	$23-9=n$	$31=n-7$
4.	$n-13=27$	$36=n+21$
5.	$n=52+16$	$72=50+n$
6.	$23=56-n$	$n=47-8$
7.	$98-n=80$	$121+n=136$



Solving Problems

Written Write an equation for each of the problems below. Solve each equation. Check the answer to decide if it is reasonable. Write the answer to each problem.

1. Bob had 74 pennies in a collection. He gave 29 of them to Andy. How many pennies did he have left?

2. Wally knew that he had 185 stamps in an album. Some came loose and fell out. When he looked in his album, he had only 119 stamps left. How many stamps had he lost?

3. The pupils wanted to make 100 programs for their play. At the end of an hour, they had made 53 programs. How many of the programs did they still have to make?

4. Bill counted 123 cars on the parking lot. Of these, 28 were foreign cars and the rest were made in America. How many of the cars were American-made cars?

5. We had 721 books in our school library. We bought 55 books for the library. Then how many books were in our school library?

6. A record of milk delivered at one school was as follows: Monday, 288 bottles; Tuesday, 312 bottles; Wednesday, 288 bottles; Thursday, 336 bottles; and Friday, 288 bottles. How many bottles of milk were delivered in the 5 days?

7. A milkman delivered 120 cartons of milk to a store. All of the cartons except 23 were sold. How many cartons of milk were sold?

8. A bridge has a load limit of 10,000 pounds. How much over the load limit is a truck which weighs 5,400 pounds and is carrying 4,750 pounds of freight?

Something to do Write 5 problems of your own. Write an equation to use in solving each of your problems.

Finding the Difference

A large parking lot had spaces for 125 cars. The smaller lot had spaces for 78 cars. How many more spaces were in the larger lot than in the smaller one?

The first step in solving the problem is to read it carefully. Then picture the situation in your mind, or on paper if you need to. The solution in grid form is shown at the right. The sets are to be compared to find by how many they differ in number. Thus you subtract the numbers of the two sets.

H	T	O
1	2	5
−	7	8
	4	7

Oral Read each problem below. Tell how you would decide upon the equation to use in finding each answer. Record the equations.

1. Bob's grandfather is 71 years old. His father is 36 years old. What is the difference between the ages of his father and grandfather?

2. Tom's big brother weighs 132 pounds. Tom weighs 79 pounds. How many pounds lighter is Tom than his big brother?

3. There are 398 boys and 429 girls enrolled in Roosevelt School. How many more girls than boys are enrolled in the school?

Written Solve the equations for problems 1 through 3 above. Write the answer to each problem. Number your answers 1, 2, and 3. Then solve the problems that follow. (Reminder: The signs $ and . are not to be used in the equations. They are written as part of the final answers.)

4. Alice bought a book for $1.75 and a notebook for $.29. How much more did she spend for the book than the notebook?

5. Andy spent $6.98 for shoes and $2.79 for a shirt. How much more than the shirt did the shoes cost?

6. Henry had $1.25 when he left for the store. He had only $.08 when he came home. How much of his money did he spend?

7. There were 528 pupils present in our school on Monday. The enrollment in our school is 570 pupils. How many were absent on Monday?

8. Ronald and his father are making a vacation trip of 800 miles. On the first day of their trip they drove 428 miles. How much farther did they have to drive?

9. Vicki has 76 postage stamps for 126 invitations she is writing. How many more stamps does she need to mail the invitations?

10. Bill can get a second-hand bicycle for $25.50. A new bicycle would cost $41.75. What is the difference between the prices of the two bicycles?

11. In January Dick's brother drove 1,729 miles on his sales route. He drove 1,455 miles in February. What was the difference in mileage for the two months?

12. A class needed 325 tickets for a play. Bob made 175 tickets. How many more tickets were needed?

13. A traffic count showed that 918 cars passed a school on Monday and that 1,215 cars passed the same school on Tuesday. How many more cars passed the school on Tuesday than on Monday?

14. Susan and her mother made 87 Red Cross bandages. They plan to make 175 bandages. How many more bandages should they make?

15. Andy used 1,250 feet of movie film. Three hundred feet of this was color film. How many feet of black-and-white film did Andy use?

Something to do Write 5 problems of your own. Write an equation to use in solving each of your problems.

Another way The large parking lot had spaces for 125 cars. The smaller lot had room for 78 cars. How many more cars could park in the larger lot than in the smaller one?

This problem could be solved by using an equation, which is given with its solution below.

$$\begin{aligned} 125 - 78 &= n \\ 125-(25+53) &= n \\ (125-25)-53 &= n \\ 100 - 53 &= n \\ (100-50) - 3 &= n \\ 50 - 3 &= n \\ 47 &= n \end{aligned}$$

There are 47 more spaces in the larger parking lot than there are in the smaller lot.

MORE PRACTICE PAGE 299

Subtracting from Four-Digit Numbers

In 10 years the population of a city grew from 7,150 to 9,560. How much did the population increase in this 10-year period?

A

Th	H	T	O
9	5	6	0
−7	1	5	0
2	4	1	0

You are to compare the numbers of two sets and determine the difference, so you subtract the numbers. This is done for you in grid A. You subtract the digits in each place-value position to find the difference.

Of course, you will sometimes have subtractions in which you must change the form of the minuend before you subtract. In grids B and C, 1 thousand was changed to 10 hundreds and added to the hundreds.

B

Th	H	T	O
9	4	8	7
−	8	6	2
8	6	2	5

C

Th	H	T	O
8	0	9	3
−4	7	2	1
3	3	7	2

MORE PRACTICE PAGE 299

The subtraction in grid D shows changes from thousands to hundreds, and from hundreds to tens. The subtraction in grid E shows changes in each place-value position.

D

Th	H	T	O
7	6	5	9
−	9	7	0
6	6	8	9

E

Th	H	T	O
6	0	0	0
−3	2	7	9
2	7	2	1

Oral What steps would you use to find each difference below?

	a	*b*	*c*
1.	8637 −524	9589 −3477	8738 −805
2.	8920 −7430	9138 −5646	7001 −4329

Written Follow these directions.

1. Copy the subtractions in row 1 above. Find each difference.

2. Copy the subtractions in row 2 above. Find each difference.

Copy the subtractions below. Find each difference.

	a	*b*	*c*
3.	4386 −709	5400 −600	8754 −693
4.	3579 −1189	8957 −2639	9576 −4368
5.	8439 −6987	7534 −6893	9818 −4579

Subtracting Four-Digit from Five-Digit Numbers

In a large city 13,585 pupils were enrolled in public schools and 8,250 pupils were enrolled in private schools. How many more pupils were enrolled in public schools than in private schools?

You are to find by how many the number of one set is greater than another, so you subtract the numbers. The equation is 13585−8250=n. The subtraction is done for you in grid A. There are more ones, tens, and hundreds in the minuend than in the subtrahend, so you subtract ones, tens, and hundreds. There are more thousands in the subtrahend than in the minuend, so you change the form of 1 ten-thousand to 10 thousands and add them to the thousands. Then you subtract in the thousands place. Hence n has a value of 5,335. There are 5,335 more pupils in public schools than in the private schools.

A

TTh	Th	H	T	O
1	3	5	8	5
−	8	2	5	0
	5	3	3	5

Study the completed subtractions in grids B through E. You need to change the form of each minuend in doing the subtractions. Changes are necessary in more than one place-value position in each subtraction.

B

TTh	Th	H	T	O
9	8	7	5	6
−	9	8	3	4
8	8	9	2	2

C

TTh	Th	H	T	O
7	4	3	2	5
−	7	6	5	1
6	6	6	7	4

D

TTh	Th	H	T	O
5	4	6	1	2
−	9	6	5	3
4	4	9	5	9

E

TTh	Th	H	T	O
7	0	0	0	4
−	8	5	3	7
6	1	4	6	7

Oral Explain the subtractions above. How would you check your work?

Written Copy. Find each difference. Check the accuracy of your work by using addition.

	a	*b*	*c*
1.	39623 −1265	72187 −4327	62398 −8543
2.	50876 −8532	96370 −2960	59765 −3857
3.	86093 −7185	73021 −1205	85837 −2461
4.	37043 −1565	40649 −3786	98416 −5667
5.	85432 −3745	76157 −7869	94001 −8954

Subtracting from Five-Digit Numbers

Look at the completed subtractions below.

A

	TTh	Th	H	T	O
	9	8	7	7	5
−	6	4	3	2	0
	3	4	4	5	5

B

	TTh	Th	H	T	O
	7	5	9	9	0
−	3	8	7	5	0
	3	7	2	4	0

Do you see that you subtract ten thousands in the same way that you subtract ones, tens, hundreds, or thousands?

Many subtractions can be done without using pencil and paper. In the equation 75475−50475=n, you would simply subtract 50 thousand from 75 thousand. Why?

In the equation 94385−9999=n, you would subtract 10,000 and add 1 to the difference. Why?

In any subtraction, whether you are working with large numbers, or small numbers, you can often save time by trying to subtract without using pencil and paper.

MORE PRACTICE PAGE 299

Oral Be ready to tell how to do each subtraction below.

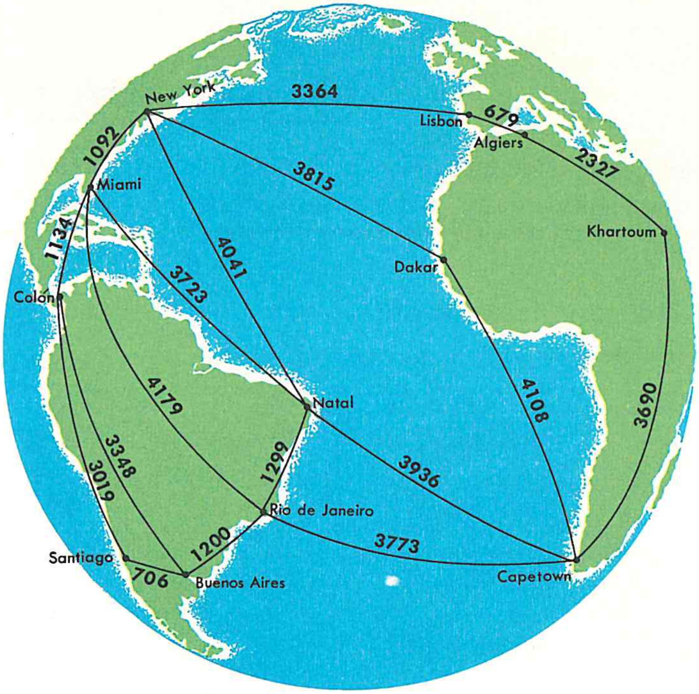
a	*b*	*c*
78960 −34650	54325 −29000	93175 −54175

Written Copy each subtraction below. Write each difference. Check.

	a	*b*	*c*
1.	86985 −34972	47980 −35269	86927 −51579
2.	98025 −43846	73659 −38234	60025 −34387
3.	61891 −43789	58765 −41987	29100 −19858
4.	43897 −14998	54276 −49395	78792 −28796
5.	67000 −29755	93005 −64321	80000 −43785
6.	75000 −49850	84005 −35985	62000 −46324
7.	87112 −71788	93421 −84679	19453 −15994
8.	32393 −18732	46269 −24637	58784 −53548
9.	95748 −47830	32005 −19575	67550 −29795

Can you do this? What numerals must replace each dot in the following subtraction?

$$\begin{array}{r} 8{\bullet}{\bullet}6 \\ -{\bullet}241 \\ \hline 319{\bullet} \end{array}$$



Using Addition and Subtraction in Problems

The picture above gives airline distances in miles between certain cities. Use the picture to answer the questions below.

1. What is the distance between New York and Capetown by way of Natal? By way of Dakar?

2. What is the distance between Miami and Capetown by way of Natal? By way of Rio de Janeiro?

3. What is the shortest distance between New York and Santiago?

4. What is the distance between New York and Capetown by way of Lisbon, Algiers, and Khartoum?

5. How much shorter is the trip from New York to Lisbon than the trip from New York to Natal?

6. How much closer is Miami to Natal than to Rio de Janeiro?

7. What is the distance from New York to Buenos Aires by way of Miami and Colon? By way of Natal and Rio de Janeiro? By way of Miami, Colon, and Santiago?

8. How much closer is Buenos Aires to Rio de Janeiro than Natal?

Something to do Make 4 problems of your own that can be solved by using the map on this page.

Reviewing Addition and Subtraction

On this page you will find out whether you need more help with addition and subtraction.

Part 1 Copy in column form. Write each sum or difference.

	a	*b*
1.	34+35=n	35+42=n
2.	36+28=n	56+19=n
3.	82−53=n	93−55=n
4.	66+98=n	87+66=n
5.	59−35=n	78−36=n

Part 2 Copy in column form. Write each sum or difference.

	a	*b*	*c*	*d*
1.	314 312 +341	225 220 +144	914 823 +175	760 335 +204
2.	114 129 232 +309	138 415 206 +129	150 263 141 +322	450 261 150 +150
3.	932 641 213 +312	571 652 750 +624	576 298 457 +863	898 750 895 +627
4.	828 −324	698 −275	684 −348	856 −549

Part 3 Copy. Write each sum or difference.

	a	*b*	*c*
1.	5790 3265 +4352	9389 4675 +2463	6324 3452 +1497
2.	4281 3564 +2714	3758 6429 +7384	7982 4654 +3793
3.	8879 −5867	9777 −5653	7594 −5416
4.	9758 −6549	6868 −4574	8802 −3652

Part 4 Copy. Write each sum or difference.

	a	*b*	*c*
1.	47564 +34751	38958 +49875	95246 +64752
2.	94325 34758 +85927	87416 29654 +96457	92100 18798 +97850
3.	94275 −1420	87156 −9750	72185 −8970
4.	87259 −37124	96150 −34100	75400 −29300
5.	64205 −29150	78915 −37985	92150 −24675

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Subtraction and addition are inverse operations. That is, each undoes what the other does. (33)

2. Subtraction can be checked by adding the difference to the subtrahend. The sum should always be equal to the minuend. (33)

3. Suggestions for solving a problem are: read it carefully, try to picture the situation, write the equation, and solve the equation. Check over the problem to see if the answer is reasonable. Then write the answer and check the accuracy of your work. (38, 40)

Words to Know

1. Inverse operation (33)

2. Minuend, subtrahend, and difference (33)

Questions to Discuss

1. What are some examples of inverse operations? (33)

2. What happens to a number when you subtract 0 from it? What is the difference when you subtract a number from itself? (34)

3. How can you use a number line to help you with subtraction? (35)

Written Practice

Copy in column form. Write each difference.

	a	*b*	
1.	$52 - 18 = n$	$95 - 37 = n$	(36)
2.	$674 - 32 = n$	$785 - 324 = n$	(37)
3.	$980 - 57 = n$	$870 - 86 = n$	(37)
4.	$515 - 338 = n$	$600 - 271 = n$	(37)

Copy. Write each difference. Check by using addition.

	a	*b*	*c*	
5.	$\begin{array}{r}8450\\-240\\\hline\end{array}$	$\begin{array}{r}9196\\-750\\\hline\end{array}$	$\begin{array}{r}7543\\-891\\\hline\end{array}$	(42)
6.	$\begin{array}{r}5792\\-4891\\\hline\end{array}$	$\begin{array}{r}7539\\-4876\\\hline\end{array}$	$\begin{array}{r}5201\\-3985\\\hline\end{array}$	(42)
7.	$\begin{array}{r}14750\\-3640\\\hline\end{array}$	$\begin{array}{r}37895\\-9765\\\hline\end{array}$	$\begin{array}{r}57150\\-3365\\\hline\end{array}$	(43)
8.	$\begin{array}{r}85790\\-34650\\\hline\end{array}$	$\begin{array}{r}70005\\-29590\\\hline\end{array}$	$\begin{array}{r}91625\\-85788\\\hline\end{array}$	(44)

Copy. Solve for n by using the equation method.

	a	*b*	
9.	$36 - 8 = n$	$23 - 5 = n$	(35)
10.	$72 - 37 = n$	$43 - 17 = n$	(36)

Self-Evaluation

Part 1 Copy. Write each difference.

	a	*b*	*c*	*d*
1.	689 −73	976 −35	792 −58	895 −89
2.	587 −324	975 −254	853 −391	642 −282
3.	952 −748	870 −657	755 −397	831 −452

Part 2 Copy. Write each difference. Check by adding.

	a	*b*	*c*
1.	8979 −535	8979 −639	$63.98 −2.98
2.	4198 −454	6389 −635	$78.67 −9.50
3.	9785 −591	8968 −385	$15.50 −4.99

Part 3 Copy. Write each difference. Check by adding.

	a	*b*	*c*
1.	15235 −4121	29460 −3320	54212 −4100
2.	37650 −8430	88798 −9435	92625 −8520
3.	45775 −24930	87520 −32710	63275 −42525

Part 4 Solve these problems after writing an equation for each one. (Reminder: Use the signs $ and . only in the answers, not in the equations, for problems 2 and 4.)

1. Doris and her mother started on a 2500-mile trip. At the end of 3 days they had driven 1135 miles. How many more miles did they still have to drive?

2. Laura needs $10.79 for a new dress. She has $6.98 and plans to earn the rest. How much money must she earn?

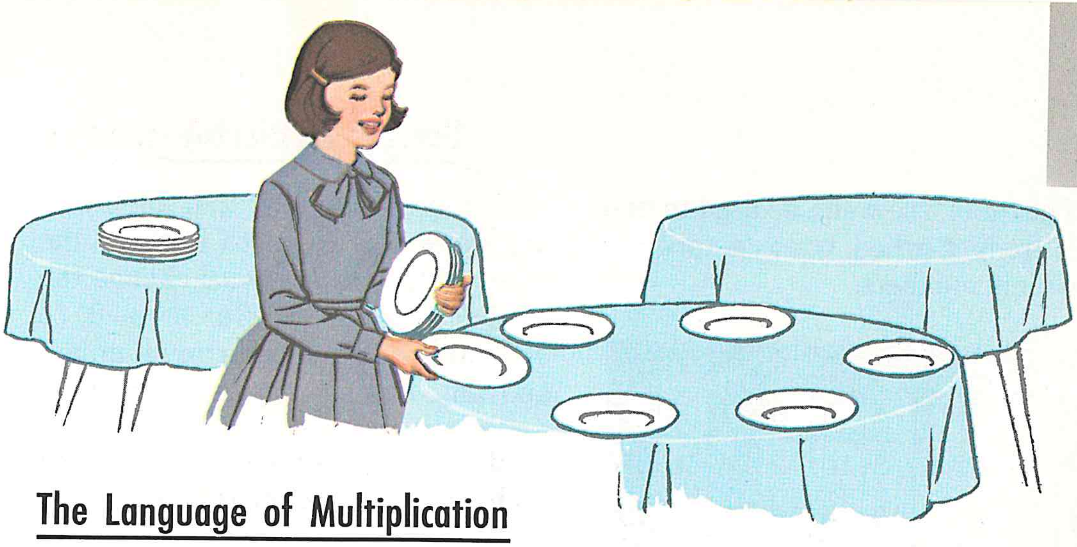
3. William's father drove 429 miles on Monday, 387 miles on Tuesday, and 196 miles on Wednesday. How many miles did he drive altogether during the three days?

4. John wants to buy a baseball glove that costs $6.50. He has $4.75. How much more money does he need to buy the baseball glove?

5. Tickets were sold for four home games at a football park as follows: 2485, 1918, 2005, and 1876. What was the total number of tickets sold for the four games?

6. Bill had some tropical fish. He sold nine of them and had twenty-two left. How many did he have before he sold the fish?

4

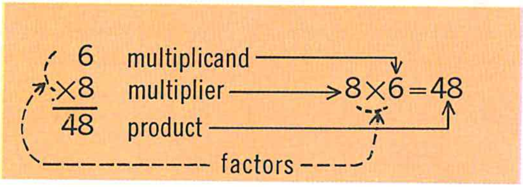


The Language of Multiplication

Betty Jo plans to set 8 tables with 6 plates for each table. How many plates does she need?

You may add 8 sixes to get the answer: $6+6+6+6+6+6+6+6=48$.

A fast way to find the answer is to multiply 6 by 8, as shown below in column form and equation form.



Betty Jo needed 48 plates.

Multiplication is an operation of arithmetic because you think of two numbers and obtain another number. The number to be multiplied is called the **multiplicand.** The number you multiply by is called the **multiplier.** The number obtained by multiplying is called the **product.**

When two or more numbers are to be multiplied, each is called a **factor.** For example, in the expression $5\times2=10$, the 5 and 2 are *factors.*

The product of two whole numbers is always a whole number. This is called the *closure property of multiplication of whole numbers.*

The multiplicand and the multiplier are actually names of factors.

Sometimes one of the factors is 0, as in $3\times0=n$. To find the value of n, add: $0+0+0=0$. Does $3\times0=0$? You will find that any number times 0 is equal to 0.

Oral Tell which numeral or numerals represent the multiplicand, the multiplier, the factors, and the product in each of the following.

	a	*b*	*c*
1.	$4\times9=36$	$7\times8=56$	$6\times0=0$
2.	$0\times8=0$	$9\times8=72$	$5\times5=25$

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Using a Multiplication Table

The table below can be used to find the product when both factors are one-digit numbers.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Notice that 9×7=63. Then solve 7×9=n. Does 7×9=9×7?

Do you see that the order of the two factors does not affect the product? This property is called the *commutative property of multiplication.*

Notice that 1 is a factor in each of the following multiplications:

1×3=3, 1×4=4, 1×7=7, and 1×8=8.

In each case the other factor and the product are identical, or the same. Check the table to find other products when 1 is a factor. Since multiplication by 1 leaves a number unchanged, 1 is called the *identity number of multiplication.*

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You may remember that any number times 0 is equal to 0. Since the order of factors does not change the product, 3×0=0×3. But 3×0=0, so 0×3=0. In fact, 0 times any number, and any number times 0, equals 0.

Oral Use the table to prove that each statement below is true.

	a	*b*
1.	7×8=8×7	7×6=6×7
2.	9×1=1×9	6×8=8×6
3.	6×9=9×6	9×8=8×9

Be ready to say the product for each multiplication below.

	a	*b*	*c*	*d*	*e*
4.	7 ×9	8 ×3	9 ×1	6 ×3	8 ×9
5.	6 ×2	9 ×7	8 ×8	7 ×1	9 ×4
6.	9 ×8	6 ×7	9 ×2	7 ×8	9 ×5
7.	5 ×1	3 ×8	4 ×6	2 ×3	3 ×0
8.	2 ×6	1 ×1	5 ×9	1 ×4	4 ×0
9.	8 ×1	7 ×3	8 ×6	6 ×5	6 ×0

Multiplying Tens and Ones by One-Digit Numbers

What happens when you multiply ones and tens by the same number?

$3 \times 1 = 3$ $3 \times 10 = 30$
$3 \times 2 = 6$ $3 \times 20 = 60$
$3 \times 3 = 9$ $3 \times 30 = 90$
$3 \times 4 = 12$ $3 \times 40 = 120$

Knowing how to multiply ones and tens will help you multiply 32 by 3, for 32 is 30+2. Study the multiplication below.

$$\begin{aligned} 3 \times 32 &= 3 \times (30+2) \\ &= (3 \times 30) + (3 \times 2) \\ &= 90+6 \\ &= 96 \end{aligned}$$

The same multiplication may be done on a grid as shown below.

	T	O	
	3	2	
	×	3	
3×2 ones = 6 ones		6	partial product
3×3 tens = 9 tens	9	0	partial product
Add	9	6	product

Both color blocks above show the *distributive property of multiplication over addition.* That is, to multiply the sum of two numbers, you may multiply each addend and add the two partial products.

Suppose the partial products are greater than 9. Then you will make changes in form as shown below.

A

H	T	O
	6	2
	×	4
		8
2	4	0
2	4	8

B

T	O
2	3
×	4
1	2
8	0
9	2

C

H	T	O
	3	8
	×	5
	4	0
1	5	0
1	9	0

In example A, how much is 4×6 tens? How was the form of 24 tens changed? In B, how was the form of 12 ones changed? What changes in form were made in C?

Oral Tell the steps in each of the multiplications below.

	a	*b*	*c*	*d*	*e*
1.	91 ×8	25 ×3	48 ×6	37 ×9	74 ×8
2.	56 ×6	83 ×9	75 ×5	71 ×9	43 ×7
3.	67 ×7	54 ×8	98 ×6	88 ×9	59 ×9

Written Find the products in the above multiplications.

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4×1=4	4×10=40	4×100=400	4×1000=4000
4×2=8	4×20=80	4×200=800	4×2000=8000
4×3=12	4×30=120	4×300=1200	4×3000=12000

Multiplying Three- or Four-Digit Numbers

Do you see that hundreds are multiplied in the same way as ones or tens? Are thousands also multiplied in the same way as hundreds, tens, and ones?

To multiply 132 by 3, think of 132 as 100+30+2. Then multiply using the distributive property of multiplication over addition.

	132
	×3
3×2=6	6
3×30=90	90
3×100=300	300
Add	396

Now suppose there are changes in form, as when you multiply 602 by 4. The multiplication is done for you below.

	602
	×4
4×2=8	8
4×0=0	00
4×600=2400	2400
Add	2408

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Do you see that the form of 24 hundreds is changed to 2 thousands and 4 hundreds?

To multiply 5245 by 6, first think of 5245 as 5000+200+40+5, or 5 Th+2 H+4 T+5 O. Then you can multiply as shown in the grid below.

	TTh	Th	H	T	O
		5	2	4	5
				×	6
6×5 O=30 O				3	0
6×4 T=24 T			2	4	0
6×2 H=12 H		1	2	0	0
6×5 Th=30 Th	3	0	0	0	0
Add	3	1	4	7	0

See how much time you save if you do not write the partial products. The digits in each place-value position are multiplied separately. Why? If you make mistakes using this short method, practice more cases where you write each of the partial products in doing the multiplication.

5245
×6
31470

Oral Study the multiplication of 5245 by 6. Then answer each question below.

1. What was done with 3 tens when the form of 30 was changed to 3 tens and 0 ones?

2. What is the sum of 24 tens and 3 tens?

3. How was the form of 27 tens changed?

Written Copy. Write each product.

	a	*b*	*c*	*d*
1.	440 ×2	233 ×3	110 ×8	314 ×2
2.	290 ×3	231 ×4	453 ×2	284 ×2
3.	227 ×3	108 ×9	219 ×4	326 ×3
4.	198 ×5	467 ×2	259 ×3	375 ×2
5.	445 ×5	789 ×2	298 ×9	934 ×3
6.	2861 ×2	7231 ×4	3262 ×2	8501 ×5
7.	1312 ×4	8234 ×2	7132 ×3	7523 ×3
8.	1001 ×7	4424 ×4	2618 ×3	5123 ×4

Another way To multiply, you do not always have to use the column form. For example, suppose you want to multiply to solve the equation $3\times 32=n$. Rename 32 as a sum of tens and ones, that is, as (3 tens + 2 ones) or $(30+2)$. Then $3\times 32=n$ becomes $3\times(30+2)=n$. Now use the distributive property of multiplication over addition to multiply each addend by the multiplier: $3\times(30+2)=n$ becomes $(3\times 30)+(3\times 2)=n$.

Study the complete solution which follows.

Equation	3×32	$=n$
Rename 32	$3\times(30+2)$	$=n$
Distributive prop.	$(3\times 30)+(3\times 2)$	$=n$
	$90 + 6$	$=n$
Product	96	$=n$

Can you do this? Use the above method in finding the value of n in each of the following equations.

	a	*b*
1.	$5\times 16=n$	$3\times 43=n$
2.	$6\times 24=n$	$8\times 17=n$
3.	$4\times 31=n$	$5\times 74=n$
4.	$2\times 97=n$	$7\times 42=n$
5.	$9\times 26=n$	$5\times 39=n$
6.	$5\times 76=n$	$8\times 33=n$
7.	$7\times 63=n$	$4\times 76=n$

To find this number:	Use this operation:
How many in all of the sets with unequal numbers?	Addition
How many in all of the sets with an equal number?	Multiplication
How many are left?	Subtraction
By how many one set differs from another?	Subtraction

Choosing the Correct Operation

Study each problem carefully by reading it several times. Using the suggestions at the top of this page, decide upon the operation to use in solving each problem. Then write the equation you would use in solving each problem. (Remember: The symbols for money are not part of an equation.)

1. Some of the boys helped set up chairs in the gym. They made 8 rows of chairs with 25 chairs in a row. How many chairs did they set up?

2. A truck carried 6500 pounds of wheat. How many pounds of wheat could it carry in 8 trips if each load weighed 6500 pounds?

3. Ann had $25.00 to spend for clothes. She spent $13.29. How much money did she have left?

4. Gene's scoutmaster says that if each of the 26 boys in the troop brings 9 boxes, they will be able to pack their equipment. How many boxes are needed?

5. Mr. Hauser earned $79.50 one week and $95.75 the next week. How much money did he earn in the 2 weeks?

6. In the morning Susan practiced her piano lesson. In the afternoon she practiced 40 minutes more. Then she said, "I practiced 85 minutes today." For how many minutes did she practice in the morning?

7. Ronald and his father drove from their home to Rockerville. After driving 73 miles they saw a sign which read, "Rockerville 58 miles." How far is it from their home to Rockerville?

Oral Tell the equation you would use in solving each problem above and why you would use it.

Written Solve each equation for problems 1 through 7. Read each problem again to decide whether your answer is reasonable. Write the answer to each problem by giving the kind of units or things asked for.

Problem Solving

Following the suggestions on page 54, solve each problem below.

1. A total of 2,765 tickets were sold for a high school football game. At this rate, how many tickets will be sold for 6 games?

2. Mr. Nelson earned $95.85. He had $27.98 taken out of his earnings to pay taxes and other expenses. How much money did he have left?

3. A truck loaded with steel weighed 15,750 pounds. The empty truck weighed 8,900 pounds. What was the weight of the steel?

4. A club treasurer collected $12.75 in January, $14.50 in February, and $9.85 in March. How much money did the treasurer collect in the three-month period?

5. The expenses for a party amounted to $11.29. These were paid from the $31.25 the club had in its treasury. How much money was left in the treasury after the expenses were paid?

6. Jack's father bought 3 shirts at $4.98 each. What was the total cost of the shirts?

7. Joyce is saving money for skates. One week she earned $2.41, the next week, $3.70. Then she spent $1.05. How much money does she still need if the skates cost $12.59?

8. The Grover Orchards shipped 125 boxes of apples to Mr. Ness. Each box of apples weighed 26 pounds. What was the weight of the shipment of apples sent to Mr. Ness?

A quick review Which operation do you think of when you hear each of the following words: sum, product, difference, minuend, multiplier, subtrahend, factor, multiplicand?

Can you do this? Which numeral must replace n if each of the following is to be true?

	a	*b*
1.	$(5\times3)+2=n$	$n=(4\times6)-8$
2.	$6\times(3+1)=n$	$n=12-(4\times3)$
3.	$8+(9-4)=n$	$n=(8-6)\times9$
4.	$4+(5\times2)=n$	$n=(3\times2)\times6$
5.	$6+(7+8)=n$	$n=20-(6\times3)$
6.	$(3\times0)+n=12$	$n=(7\times1)+0$
7.	$(12+0)\times5=n$	$n=(17\times0)+5$
8.	$(7-7)\times4=n$	$n=12-(7+2)$
9.	$(19-9)-8=n$	$n=42-(7\times6)$
10.	$(20+39)\times1=n$	$n=0\times(12-3)$

Multiplying by Tens

You know that $10 \times 1 = 10$ because 1 is the identity number of multiplication. What is the product of 10 and 2? Of 10 and 3?

Would you expect these statements to be true?

1 ten × 4 ones = 4 tens
1 ten × 8 ones = 8 tens
1 ten × 9 ones = 9 tens

Notice that *ten times ones* is equal to tens.

What product would you expect for the following: 20×2? 20×4? 30×4? 40×5? Do you see an easy way of obtaining the products?

If 10 sets, having 10 objects in each set, were joined, the sum would be 100 objects. This illustrates that $10 \times 10 = 100$. Then 10×20 would mean that 10 sets, having 20 objects in each set, were to be joined. The sum would be 200 objects. This illustrates that 10×20 is equal to 200.

10 times 10's is always equal to 100's. Try other multiplications to convince yourself that this is true.

This idea can be used to find the products of tens times 100's. That is, $10 \times 100 = 1000$ since the sum of ten 100's is 1000. What is the product of 10 and 300? Of 20 and 300?

Oral What numeral must replace n in each of the following?

	a	*b*
1.	$10 \times 1 = n$	$50 \times 1 = n$
2.	$10 \times 10 = n$	$20 \times 10 = n$
3.	$10 \times 100 = n$	$40 \times 100 = n$
4.	$10 \times 70 = n$	$9 \times 100 = n$
5.	$1 \times 300 = n$	$30 \times 100 = n$
6.	$10 \times 80 = n$	$70 \times 100 = n$

Written Write each equation below, replacing n by a numeral which makes the statement true.

	a	*b*
1.	$9 \times 40 = n$	$80 \times 6 = n$
2.	$6 \times 60 = n$	$70 \times 7 = n$
3.	$4 \times 80 = n$	$80 \times 8 = n$
4.	$5 \times 40 = n$	$20 \times 9 = n$
5.	$1 \times 800 = n$	$60 \times 90 = n$
6.	$20 \times 500 = n$	$50 \times 60 = n$
7.	$70 \times 40 = n$	$20 \times 80 = n$
8.	$30 \times 300 = n$	$3 \times 700 = n$
9.	$5 \times 200 = n$	$20 \times 40 = n$
10.	$40 \times 70 = n$	$20 \times 200 = n$

Multiplying by Tens and Ones

John had 23 egg cartons. He needed 12 eggs for each carton. How many eggs did he need to fill all of the cartons?

You are to think of 23 sets of 12 each as being joined, so you multiply the numbers. The equation is $23 \times 12 = n$.

The multiplication may be written in a grid as shown. When finding the partial products, think of the factors, 12 and 23, as follows:

$12 = 1$ ten $+ 2$ ones $= 10 + 2$
$23 = 2$ tens $+ 3$ ones $= 20 + 3$

The partial products 6 and 30 are found by multiplying 12 by 3. You have already learned how to multiply 12 by 3.

H	T	O
	1	2
×	2	3
		6
	3	0
	4	0
2	0	0
2	7	6

The partial product of 40 is obtained by multiplying 2 tens (in the multiplier) and 2 ones (in the multiplicand). 2 tens×2 ones=4 tens. The 4 tens are written as the partial product 40.

The partial product of 200 is obtained by multiplying 2 tens (in multiplier) and 1 ten (in multiplicand). 2 tens×1 ten=2 hundreds. The 2 hundreds are written as 200.

one×ones=ones	$1 \times 1 = 1$
one×tens=tens	$1 \times 10 = 10$
ten×ones=tens	$10 \times 1 = 10$
ten×tens=hundreds	$10 \times 10 = 100$

In shorter form, the multiplication may be done as follows.

12	multiplicand	factors
×23	multiplier	
36	partial product	add these
24	partial product	
276	product	

John needs 276 eggs to fill all of the cartons.

Oral Tell how to find each partial product and each product below.

a	*b*	*c*	*d*
24 ×12	31 ×32	14 ×12	45 ×11

Written Copy. Write each partial product and the product.

	a	*b*	*c*	*d*
1.	22 ×44	46 ×11	95 ×11	11 ×96
2.	24 ×21	21 ×43	23 ×32	12 ×42
3.	44 ×22	32 ×23	34 ×21	14 ×12
4.	12 ×32	43 ×21	11 ×57	33 ×32

Using a Two-Digit Multiplier

Ronald can put 18 plants in a row. He has enough room for 12 rows. How many plants does he need?

Do you see that Ronald must think of the 12 sets, each of 18 plants, as being joined? The equation is written: 12×18=n. The multiplication could be written in column form in a grid as shown.

H	T	O
	1	8
×	1	2
	3	6
1	8	
2	1	6

Multiply, first by the ones and then by the tens, as explained below.

Ones: 2 ones×8 ones=16 ones. Change the form of 16 ones to 1 ten and 6 ones. Write the 6 in ones place. Remember the 1 ten. 2 ones×1 ten=2 tens. 2 tens+1 ten remembered=3 tens. Write 3 in tens place.

Tens: 1 ten×8 ones=8 tens. Write 8 in tens place. 1 ten×1 ten=1 hundred. Write 1 in hundreds place.

The partial products are 36 and 180. Why? Do you see that the partial products were added? The product is 2 hundreds, 1 ten, and 6 ones or 216.

If both factors are two-digit numbers, can the product contain 2 digits? 4 digits? More than 4 digits?

Oral Study the completed multiplications below. Be ready to tell the changes in form that had to be made before each partial product could be found.

a	*b*	*c*
13	25	16
×14	×23	×64
52	75	64
13	50	96
182	575	1024

Written Copy. Find each of the following products.

	a	*b*	*c*	*d*
1.	13 ×60	24 ×40	18 ×50	37 ×20
2.	19 ×51	14 ×62	38 ×21	29 ×31
3.	13 ×14	12 ×15	12 ×16	15 ×12
4.	15 ×13	16 ×12	13 ×15	14 ×14
5.	17 ×52	15 ×65	19 ×42	18 ×43
6.	25 ×23	14 ×35	18 ×22	45 ×22
7.	13 ×57	48 ×22	28 ×23	14 ×45

More About Two-Digit Multipliers

In some multiplications you must change the form of a partial product before you write it. Study the multiplication in grid A. The first partial product, 104, is found by multiplying 26 by 4. Why is this partial product a three-digit number?

A

H	T	O
	2	6
×	1	4
1	0	4
2	6	
3	6	4

The second partial product is actually 260. Why? The product, 3 hundreds, 6 tens, and 4 ones, or 364, is obtained by adding the partial products.

Study the multiplication in B. Do you see that you must change the form of ones to tens and tens to hundreds to find the first partial product? Do you see why the product is a four-digit number?

B

Th	H	T	O
		1	5
	×	6	8
	1	2	0
	9	0	
1	0	2	0

Oral Be ready to tell how to find each partial product in the following multiplications.

	a	*b*	*c*	*d*
1.	42	22	28	39
	×33	×36	×35	×24
2.	41	23	19	36
	×26	×34	×22	×31
3.	72	15	34	31
	×17	×47	×25	×31

Written Copy. Find each of the following products.

	a	*b*	*c*	*d*
1.	20	31	21	50
	×48	×17	×46	×19
2.	61	54	41	21
	×15	×12	×24	×35
3.	23	34	68	23
	×39	×23	×13	×25
4.	64	33	12	22
	×14	×24	×49	×45
5.	19	14	27	17
	×46	×68	×29	×57
6.	24	25	37	12
	×39	×34	×23	×89
7.	58	26	14	16
	×12	×14	×78	×49
8.	31	83	71	32
	×26	×18	×17	×25
9.	34	46	38	92
	×29	×24	×19	×11

Bus fare was $.40 a day. How much money did Bill's brother spend on bus fare in the 22 days he rode the bus during May?

$$\begin{array}{r} \$\ .40 \\ \times 22 \\ \hline 80 \\ 8\ 0 \\ \hline \$8.80 \end{array}$$

You are to think of 22 sets, each of $.40, as being joined so you multiply the number of cents by 22 as shown. Do you see that the cents point in the product is directly under the cents point in the multiplicand? Why is the dollar sign needed in the product?

Oral Be ready to tell each answer and how you would write it, using a $ sign and cents point.

	a	*b*	*c*	*d*
1.	$.45 +.90	$7.50 +4.35	$19.50 −10.25	$.25 ×10
2.	$.36 +.79	$4.98 +5.35	$21.75 −11.98	$.35 ×12

Solving Problems About Money

Written Solve each problem below.

1. The children had 19 dozen cookies to sell. They planned to sell them for $.25 a dozen. How much money will they get if they sell all of the cookies?

2. Larry sold 60 notebooks at $.10 each. How much money did he get for the notebooks?

3. Mary has $1.15 in her pocket. When she entered the store she had $5.00. How much of the $5.00 had she spent?

4. The storekeeper at a school store collected $19.75 on Wednesday, $15.30 on Thursday, and $19.95 on Friday. How much money did he collect in the three days?

5. John's brother worked 5 days during one week. His pay was $17.50 a day. How much money had he earned during the week?

6. Roger saved his allowance of $.60 a week for 10 weeks. How much money did he save?

Changing the Form of the Partial Products

Study the completed multiplications A and B. Each of the second partial products actually is a four-digit number. Why? Check the addition of the partial products.

A	**B**
$\begin{array}{r} 61 \\ \times 31 \\ \hline 61 \\ 183 \\ \hline 1891 \end{array}$	$\begin{array}{r} 46 \\ \times 72 \\ \hline 92 \\ 322 \\ \hline 3312 \end{array}$

Now study the completed multiplications below. Is each partial product a three-digit number? Why? Notice the addition of partial products in both cases.

C	**D**
$\begin{array}{r} 51 \\ \times 78 \\ \hline 408 \\ 357 \\ \hline 3978 \end{array}$	$\begin{array}{r} 85 \\ \times 97 \\ \hline 595 \\ 765 \\ \hline 8245 \end{array}$

Oral Tell each partial product for the multiplications below.

a	*b*	*c*	*d*
82 ×30	57 ×31	99 ×72	64 ×28

Written Copy. Find each product.

	a	*b*	*c*	*d*
1.	21 ×61	31 ×61	41 ×51	21 ×71
2.	32 ×43	55 ×71	76 ×91	57 ×71
3.	85 ×60	68 ×41	75 ×81	13 ×92
4.	45 ×62	17 ×94	39 ×82	25 ×63
5.	59 ×51	36 ×32	39 ×62	28 ×73
6.	71 ×69	51 ×74	21 ×75	21 ×77
7.	92 ×64	54 ×72	32 ×74	72 ×83
8.	83 ×59	25 ×68	62 ×75	95 ×83
9.	63 ×38	72 ×44	83 ×63	62 ×98
10.	36 ×45	83 ×28	75 ×67	82 ×49
11.	54 ×73	24 ×76	29 ×46	68 ×47
12.	65 ×73	32 ×89	58 ×23	74 ×92

MORE PRACTICE PAGE 301

Multiplying by ones:	1×1=1	10×1=10
Multiplying by tens:	1×10=10	10×10=100
Multiplying by hundreds:	1×100=100	10×100=1000

Multiplying Three-Digit Numbers

You can add hundreds as you add tens or ones. You can subtract hundreds as you subtract tens or ones. Would you expect to be able to multiply hundreds as you multiply tens and ones?

Look at the completed multiplication. Do you see that hundreds are multiplied in the same way as ones and tens? Do you also see that 5 ones×3 hundreds equals 15 hundreds or 1 thousand and 5 hundreds. Notice that 2 tens×3 hundreds =6 thousands.

Th	H	T	O
	3	1	5
	×	2	5
1	5	7	5
6	3	0	
7	8	7	5

The product is 7 thousands, 8 hundreds, 7 tens, and 5 ones, which is written as 7875.

MORE PRACTICE PAGE 301

Oral Be ready to tell what was done to find each partial product in the multiplications below.

a	*b*	*c*
648 ×53 1944 3240 34344	400 ×32 800 1200 12800	896 ×75 4480 6272 67200

Written Copy. Find each product. Use the chart above if you wish.

	a	*b*	*c*	*d*
1.	731 ×69	528 ×32	814 ×76	648 ×53
2.	342 ×12	218 ×23	156 ×34	431 ×45
3.	304 ×28	705 ×89	200 ×43	400 ×32
4.	420 ×21	180 ×35	690 ×58	103 ×75
5.	985 ×59	594 ×46	572 ×73	896 ×75

Another way In such multiplications as 20×275, multiply by 2 tens. Write 0 in ones place in the product as shown. Why? Use this method in finding each of the following products.

275
×20
5500

	a	*b*	*c*	*d*
1.	890 ×70	610 ×40	380 ×20	230 ×30
2.	900 ×80	600 ×50	700 ×90	876 ×40

Multiplying Four-Digit Numbers

A theater had enough seats for 1250 people. Enough tickets were sold to fill the theater completely for 21 performances of a play. How many tickets were sold?

Think of 21 sets of 1250 tickets each as being joined. Multiply 1250 by 21. In column form, the multiplication can be done as shown.

TTh	Th	H	T	O
	1	2	5	0
		×	2	1
	1	2	5	0
2	5	0	0	
2	6	2	5	0

Do you see that 1 one×1 thousand=1 thousand? Do 2 tens×1 thousand=2 ten-thousands?

The product is 2 ten-thousands, 6 thousands, 2 hundreds, 5 tens, and 0 ones, or 26,250. A total of 26,250 tickets was sold.

Oral Be ready to tell what was done to find each product below.

a	*b*	*c*
4203 ×23 12609 8406 96669	1975 ×35 9875 5925 69125	1730 ×54 6920 8650 93420

Written Copy. Find each product. (Reminder: Each product contains 5 or more digits.)

	a	*b*	*c*
1.	1321 ×32	1327 ×32	2040 ×43
2.	1609 ×40	1937 ×50	4500 ×70
3.	1234 ×20	2750 ×30	1148 ×60
4.	8600 ×98	9876 ×32	6875 ×30
5.	5719 ×80	4324 ×20	3475 ×30
6.	2130 ×30	3208 ×20	1045 ×50
7.	3000 ×20	1200 ×40	4002 ×20
8.	7060 ×75	7146 ×83	4009 ×69
9.	6090 ×40	5320 ×36	4703 ×50
10.	3443 ×66	7061 ×88	9292 ×44
11.	4600 ×30	7100 ×70	5000 ×37

MORE PRACTICE PAGE 301

Using Three-Digit Multipliers

A storekeeper ordered 144 boxes of pencils. Each box held 144 pencils. How many pencils did the storekeeper expect to receive when the order was delivered?

You are to think of the 144 sets of 144 pencils each as being joined, so you multiply. The equation is 144×144=n. Study the multiplication that has been done for you in the following grid.

TTh	Th	H	T	O
		1	4	4
	×	1	4	4
		5	7	6
	5	7	6	
1	4	4		
2	0	7	3	6

Do you see that hundreds×ones = hundreds? That hundreds×tens = thousands? That hundreds×hundreds = ten thousands?

The storekeeper should have received 20,736 pencils.

In the multiplications A and B, there are no tens to multiply by in A, and no tens and no ones to multiply by in B. Do you see why only two partial products are needed in A and that no partial products are needed in B?

A

$$\begin{array}{r} 324 \\ \times 301 \\ \hline 324 \\ 9720 \\ \hline 97524 \end{array}$$

B

$$\begin{array}{r} 875 \\ \times 300 \\ \hline 262500 \end{array}$$

Oral Be ready to tell how to find each partial product in each multiplication below.

a	*b*	*c*
211 ×122	342 ×201	221 ×400

Written Copy. Find each product. (Reminder: Hundreds×ones = hundreds, hundreds×tens = thousands, hundreds×hundreds = ten thousands.)

	a	*b*	*c*	*d*
1.	309 ×635	700 ×879	564 ×543	724 ×413
2.	412 ×321	650 ×173	434 ×462	500 ×357
3.	333 ×323	987 ×111	402 ×123	310 ×382
4.	123 ×212	400 ×221	301 ×123	240 ×122
5.	300 ×204	765 ×302	607 ×809	970 ×708

Multiplying Larger Numbers

The builders of a bridge estimated that at least 3000 cars would use the bridge each day. If their estimate was correct, how many cars would use the bridge in 100 days?

To find the answer, you would solve the equation $100 \times 3000 = n$. Why? In column form, the multiplication may be done as shown.

HTh	TTh	Th	H	T	O
		3	0	0	0
		×	1	0	0
3	0	0	0	0	0

When one factor has four digits, you must multiply by thousands. The equations below give some products when one factor is 1000.

$$4 \times 1000 = 4000$$
$$40 \times 1000 = 40000$$
$$400 \times 1000 = 400000$$

In reading $400 \times 1000 = 400000$, note that hundreds times thousands is equal to hundred thousands.

Study the multiplications in A and B, paying special attention to the place-value of each digit in the partial products. How would you read each product? Would changing the order of the factors in A make the work any easier?

A	B
6000	3452
×245	×571
30000	3452
24000	24164
12000	17260
1470000	1971092

Oral Be able to explain how you would find each partial product in A and B.

Written Copy. Write each product.

	a	*b*	*c*
1.	4320 ×145	5155 ×360	3642 ×275
2.	8490 ×250	6425 ×475	7320 ×625
3.	1125 ×600	2465 ×205	4200 ×350
4.	9287 ×300	1432 ×960	5675 ×725
5.	7167 ×340	5430 ×950	9000 ×875
6.	4052 ×376	7603 ×249	8003 ×242
7.	3417 ×209	5621 ×407	8752 ×803

MORE PRACTICE PAGE 301

Other Ways to Multiply

In ordinary multiplication in a grid, you begin with the ones digit and work to the left. The multiplication shown in A is done in a different way, as follows: Step 1, 6×2 hundreds is 12 hundreds, or 1 thousand and 2 hundreds. Step 2, 6×4 tens equals 24 tens, or 2 hundreds and 4 tens. Step 3, 6×3 ones equals 18 ones, or 1 ten and 8 ones. The partial products, when added, give the product 1458. Do you understand why you can use this method of multiplication? Note that the partial products are written in reverse order from ordinary multiplication.

A

$$\begin{array}{r} 243 \\ \times 6 \\ \hline 12 \\ 24 \\ 18 \\ \hline 1458 \end{array}$$

In equation form the multiplication in A could be done as given below. Compare this solution with the one shown above.

$$\begin{array}{rcl} 6 \times 243 & = & n \\ 6\times(200 + 40 + 3) & = & n \\ (6\times200)+(6\times40)+(6\times3) & = & n \\ 1200 + 240 + 18 & = & n \\ 1458 & = & n \end{array}$$

As you have already seen, you often must *add* more than two numbers. In such cases, how do you find the *sum?* Now suppose the operation is not *addition* but *multiplication.* How would you expect to find the *product* of more than two numbers?

Multiplication can be done with only two numbers at a time. When three numbers are to be multiplied, either the first two or the last two may be grouped to be multiplied first. See examples below.

$$(3\times2)\times4=6\times4=24$$
$$3\times(2\times4)=3\times8=24$$

Notice that the product is the same with both groupings. That is, notice that $(3\times2)\times4=3\times(2\times4)$. This property is called the *associative property of multiplication.*

It is possible to make a multiplication easier by renaming one of the factors as a product. For instance, to find the value of n in $7\times24=n$, you could rename 24 as a product in several ways; such as 8×3, 6×4, 12×2. Since the multiples of 2 are easy to determine, replace 24 by (12×2). The equation could be solved as shown below.

$$\begin{array}{ll} \text{Equation} & 7 \times 24 = n \\ \text{Rename 24} & 7\times(12\times2)=n \\ \text{Assoc. prop.} & (7\times12)\times2=n \\ \text{Multiply} & 84 \times 2 = n \\ \text{Multiply} & 168 = n \end{array}$$

Since the order of factors leaves the product unchanged, you can check multiplication by interchanging the factors and multiplying. Study the multiplications given in B and C.

B	C
23	37
×37	×23
161	111
69	74
851	851

If the computation is correct, the product in each case should be the same, as in B and C.

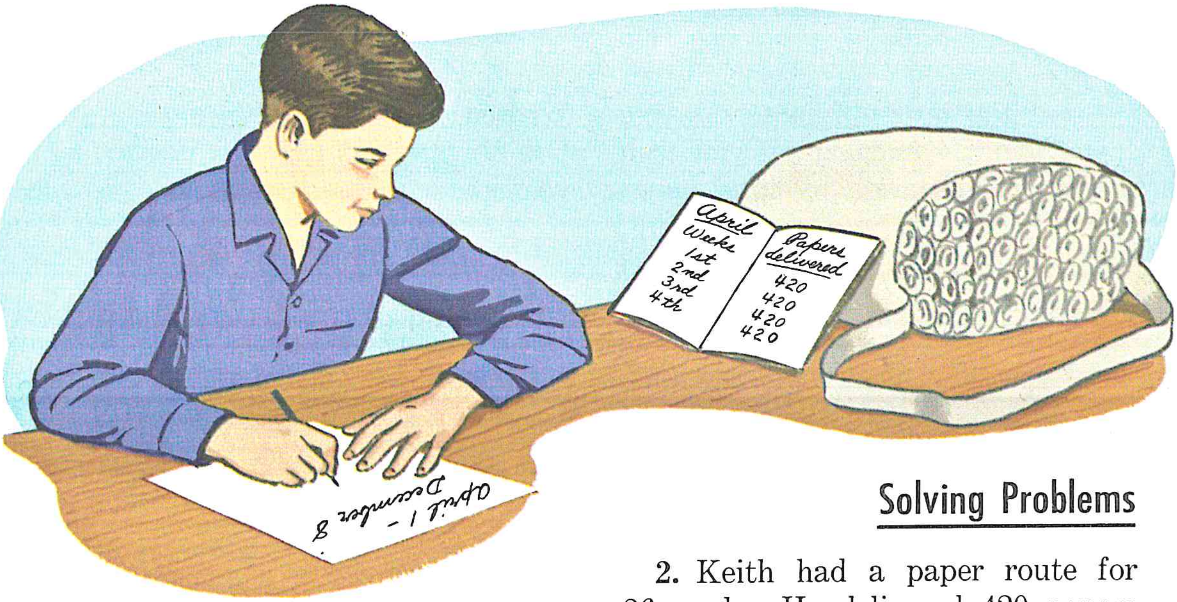
Which multiplication, B or C, would be easier for you to do? When would it be wise for you to interchange the factors?

Oral Tell which method you would use to find the following products. Also, be ready to explain why you chose each method.

	a	*b*	*c*
1.	24 ×6	20 ×53	549 ×3
2.	11 ×87	47 ×39	281 ×32
3.	12 ×59	76 ×31	176 ×54

Written Copy. Find each product. Use the method which is most convenient for you.

	a	*b*	*c*	*d*
1.	169 ×9	926 ×3	989 ×9	358 ×8
2.	421 ×2	300 ×5	372 ×2	921 ×5
3.	5079 ×6	2435 ×4	6589 ×9	9650 ×8
4.	2134 ×2	1903 ×3	7601 ×9	7605 ×7
5.	92 ×23	86 ×72	21 ×84	82 ×64
6.	10 ×29	20 ×48	28 ×36	73 ×93
7.	692 ×39	436 ×87	295 ×76	814 ×47
8.	426 ×32	723 ×42	295 ×65	832 ×73
9.	6291 ×43	3846 ×68	9217 ×73	7469 ×85
10.	683 ×286	639 ×365	483 ×622	3750 ×465
11.	382 ×213	246 ×435	493 ×619	725 ×361
12.	52 ×136	300 ×218	74 ×538	12 ×675



Solving Problems

You should be able to do the problems on these two pages without help. Read each problem several times, studying it carefully. Decide whether or not you are to join sets with unequal numbers or sets with an equal number. If so, you will either add or multiply the numbers. Instead, you may decide that you are to separate a set or are to find by how many two sets differ. Then you will subtract the numbers.

Write each equation and solve it. Finally, write the answer for the problem and check it.

1. On a Saturday the 31 members of a club took a bus trip. The bus fare was $.35 for the round trip. How much money should the bus driver have collected from all of the club members?

2. Keith had a paper route for 36 weeks. He delivered 420 papers each week. How many papers had he delivered in all?

3. Four persons agreed to pay the expenses for a party. The expenses were as follows: bus fare, $9.75; food, $10.85; other refreshments, $3.98. What were the total expenses?

4. Mr. Beyer spent $55.85 for motels and $24.75 for food on a trip. How much more did he spend for motels than for food?

5. At the end of a trip, a salesman had $14.50 of his expense money left. He had taken $75 with him to pay for his expenses. How much money had he spent?

6. The Boyer Creamery produces 375 gallons of ice cream each day. How many gallons of ice cream can they make in 12 days?

7. An airplane that was making a 3,250-mile flight was grounded 375 miles away from the airport where it planned to land. How many of the 3,250 miles had it already flown before being grounded?

8. The Andersen family drove 2,198 miles on their way to the coast and 2,429 miles on the return trip. How many miles did they drive in going to the coast and back again?

9. Baseball uniforms will cost the team members $12.75 each. How much money must the team of 9 boys have before they can purchase the baseball uniforms?

10. Sally's mother gave her 75 buttons to add to her button collection. Sally said, "Now I have 250 buttons." How many buttons did she have before receiving the buttons from her mother?

11. Mr. Naylor estimated that he had used 115 gallons of gasoline each month and that he could drive about 15 miles on each gallon. About how many miles had he driven during the month?

12. The seating units for a new school cost $31.50 each. What did it cost to buy 475 of the seating units for the school?

13. A used-car salesman bought a car for $1150. Later he sold it for $1325. How much less was the purchase price than the selling price?

14. Mr. Bloom estimated that his delivery truck carried 4750 pounds each day. How many pounds of cargo could he haul in 305 days?

A quick review What words come to mind when you think about addition? Subtraction? Multiplication? The inverse of addition? The inverse of subtraction? Arithmetic operation?

What numeral must replace n so that each of the following equations is true?

	a	*b*
1.	$10 \times 1 = n$	$30 \times 4 = n$
2.	$10 \times 100 = n$	$30 \times 200 = n$
3.	$14 \times 100 = n$	$23 \times 200 = n$
4.	$100 \times 100 = n$	$400 \times 600 = n$
5.	$160 \times 100 = n$	$150 \times 300 = n$
6.	$1 \times 1000 = n$	$7 \times 2000 = n$
7.	$10 \times 1000 = n$	$80 \times 3000 = n$
8.	$18 \times 1000 = n$	$21 \times 4000 = n$
9.	$100 \times 1000 = n$	$600 \times 7000 = n$
10.	$170 \times 1000 = n$	$310 \times 3000 = n$
11.	$152 \times 100 = n$	$234 \times 200 = n$
12.	$163 \times 1000 = n$	$413 \times 2000 = n$

Reviewing Addition and Subtraction

You should be able to find each sum or difference without help.

Part 1 Copy. Find each sum.

	a	*b*	*c*	*d*
1.	27	35	942	75
	19	57	50	45
	18	45	79	274
	+34	+96	+86	+86
2.	180	983	238	$3.00
	293	755	127	7.17
	251	700	315	8.06
	+170	+891	+206	+7.19
3.	2762	7986	9321	3157
	1906	6715	8679	1496
	2475	3240	4796	8975
	+3557	+4765	+5710	+9860
4.	9176	4324	1568	7472
	8534	9865	4789	3296
	3543	2916	1500	3750
	7650	4321	2905	7691
	+4030	+2900	+1576	+9875

Part 2 Copy. Find each sum.

	a	*b*	*c*
1.	17545	32976	47354
	36421	54876	35750
	+41375	+29650	+52400
2.	39875	89995	$750.45
	97650	75000	619.98
	47521	24500	725.50
	+69843	+36950	+450.35
3.	56755	75850	$867.25
	95298	62750	867.25
	45000	37900	900.00
	76275	94398	975.50
	+12471	+27564	+751.75

Part 3 Copy. Find the difference for each subtraction.

	a	*b*	*c*	*d*
1.	95	61	54	78
	−35	−46	−27	−39
2.	190	307	456	781
	−85	−29	−38	−64
3.	275	891	750	600
	−150	−387	−481	−342
4.	4653	9761	8050	6300
	−242	−861	−767	−925
5.	7375	6051	9003	5000
	−3259	−4875	−3658	−1917

Part 4 Copy. Find each difference.

	a	*b*	*c*
1.	37565	85231	$367.50
	−12690	−31485	−129.75
2.	74330	58750	$935.98
	−32450	−29898	−815.25
3.	61325	84005	$800.00
	−29950	−36850	−764.25

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Whenever 0 is a factor, the product is 0. (49, 50)

2. Whenever 1 is a factor, the other factor and the product are identical. (50)

3. Commutative property of multiplication: The order of the factors does not change the product. (50)

4. Associative property of multiplication: Factors may be grouped in any way for multiplication. (66)

Words to Know

1. Factor, product, multiplier, multiplicand (49)

2. Identity number of multiplication (50)

3. Partial product (51)

Questions to Discuss

1. Does 24×81=81×24? (50)

2. What is the least number of digits in: 1's×100's? 10's×10's? 10's×1000's? 100's×1000's? (52, 57, 58, 62, 65)

3. To solve a problem, how do you decide which operation to use? (54)

Written Practice

Copy. Find each of the following products.

	a	*b*	
1.	43 ×5	86 ×9	(51)
2.	645 ×8	7643 ×6	(53)
3.	37 ×20	27 ×13	(58)
4.	75 ×83	96 ×34	(61)
5.	785 ×73	496 ×46	(62)
6.	1708 ×52	5946 ×73	(63)
7.	2439 ×321	7682 ×608	(65)

Find a replacement for n which makes each of the following true.

	a	*b*	
8.	4×92=n	3×56=n	(53)
9.	60×80=n	40×600=n	(56)
10.	7×14=n	18×24=n	(66)

Self-Evaluation

Part 1 Copy. Solve each equation.

	a	*b*
1.	6×70 = n	n = 30×90
2.	34×12 = n	n = 9×16
3.	7×321 = n	n = 8×576
4.	4×1872 = n	n = 7×4583
5.	77×36 = n	n = 16×472

Part 2 Copy. Find each product.

	a	*b*	*c*
1.	53 ×29	70 ×36	68 ×40
2.	475 ×3	687 ×5	706 ×9
3.	246 ×21	385 ×42	783 ×67
4.	4321 ×2	8921 ×5	7604 ×8
5.	2121 ×23	3452 ×16	5684 ×93
6.	4008 ×7	3049 ×21	7006 ×59
7.	241 ×327	571 ×309	807 ×403
8.	577 ×88	984 ×475	713 ×808

Part 3 Copy. Find the sum, difference, or product as indicated by the operation sign.

	a	*b*	*c*
1.	789 −567	3467 +845	184 ×52
2.	486 +735	381 ×276	5628 −3849
3.	3002 −987	6798 +4736	5080 −3708
4.	676 ×99	3217 +8998	7878 +3636

Part 4 Solve each problem below.

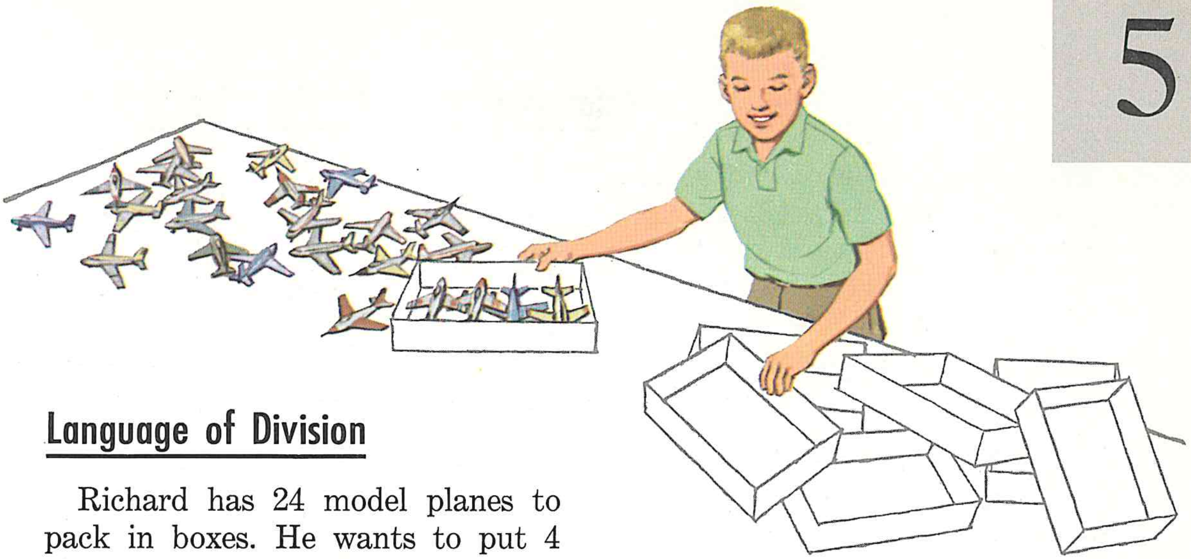
1. A creamery sold 73 cases of butter with 24 pounds in each case. How many pounds of butter were sold?

2. The fifth-grade library purchased 146 new books that cost $1.75 each. How much did all of the new library books cost?

3. Mrs. Snowden bought 4 yards of cloth at $2.98 a yard. How much change should she receive from $20.00?

4. Mr. Bowen bought 3 shirts that cost $4.50 each and 2 neckties at $1.75 each. How much did he spend for this purchase?

5



Language of Division

Richard has 24 model planes to pack in boxes. He wants to put 4 planes in each box. How many boxes will he need for all the planes?

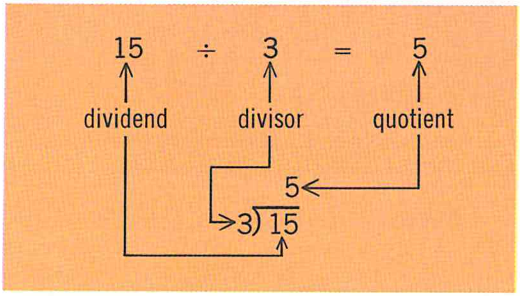
To solve the problem, ask yourself this question: If the group, or set, of 24 planes were separated into sets of 4 planes each, how many sets would there be?

If n represents the number of sets of 4 each, the equation to solve is $n \times 4 = 24$. This is easy to solve if you know your multiplication facts. $n = 6$. Richard would need 6 boxes.

However, in solving an equation like $n \times 3 = 84$, the value of n may not be so obvious. You would probably save time by using the arithmetic operation called *division.*

In division, the number being divided is called the **dividend;** the number by which you divide is called the **divisor;** and the number obtained as a result is called the **quotient.**

Division can be indicated in several ways. Two common ways of indicating the operation of division, with each part named, are shown below.



Oral Name the dividend, the divisor, and the quotient in each of the following divisions.

	a	*b*
1.	$32 \div 4 = 8$	$6 = 48 \div 8$
2.	$720 \div 5 = 144$	$50 = 350 \div 7$
3.	$\begin{array}{r} 307 \\ 3\overline{)921} \end{array}$	$\begin{array}{r} 204 \\ 5\overline{)1020} \end{array}$

Relation of Multiplication and Division

Think of a number, 12, for example. Multiply it by 2: 2×12=24. Now divide the product by 2: 24÷2=12. What is the result? If you thought of 12 in the beginning, you could write the solution as (2×12)÷2=12.

Think of another number. Divide it by 2. Now multiply the quotient by 2. What is the result? Had you thought of 8 in the beginning, the solution could be (8÷2)×2=8.

Is it possible that multiplication and division by the same number undo each other? Try some other numbers, as you did above, to verify that *multiplication and division are inverse operations.* Below are examples of this inverse relationship.

4×2=8 and 8÷2=4
5×3=15 and 15÷3=5
6×5=30 and 30÷5=6

You already used the idea of inverse operation when you checked division by using multiplication.

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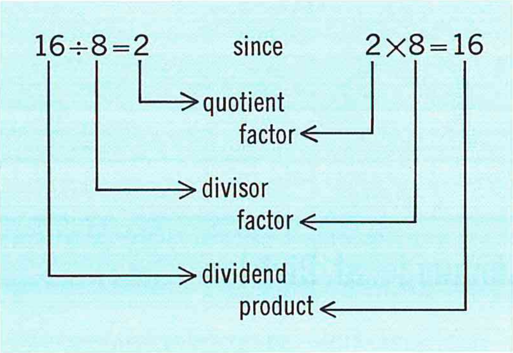
Study the following example of the inverse relationship between division and multiplication. Pay special attention to the name given each number under both operations. The quotient and the divisor correspond to the factors, and the dividend corresponds to the product.



Oral What is the value of n in each of the following?

	To solve:	*Think of:*
1.	12÷4=n	3×4=12
2.	18÷3=n	6×3=18
3.	32÷4=n	8×4=32
4.	48÷12=n	4×12=48
5.	150÷5=n	30×5=150
6.	555÷111=n	5×111=555

What is the inverse of each of the following? For example, for the inverse of *add 4,* say *subtract 4.*

	a	*b*
7.	Add 4.	Subtract 6.
8.	Divide by 5.	Multiply by 10.
9.	Subtract 8.	Divide by 12.
10.	Add 31.	Multiply by 9.

Zero in Division

What is the value of n in the equation $0 \div 5 = n$? Since you understand division by knowing how to multiply, $0 \div 5 = n$ becomes $5 \times n = 0$. Study this multiplication carefully. What value must n have in order that the product be 0? Would your answer be the same if you divided 0 by a number different from 5? What is the quotient when the dividend is 0 and the divisor is different from 0?

Suppose the divisor is 0, such as in $7 \div 0 = n$. In terms of multiplication, this becomes $0 \times n = 7$. Can you think of a number which when multiplied by 0 gives a product of 7? Try some. Of course you cannot find such a number. When 0 is a factor the product is always 0. Does it make sense to divide by 0? Hence, you will not be asked to divide by 0 because it has no meaning.

Two very important facts about 0 in division are as follows.

a. The quotient is 0 whenever the dividend is 0 and the divisor is not 0. That is, $0 \div n = 0$ for all values of n except $n = 0$.

b. Division by 0 is meaningless. To say you cannot divide by 0 really means that division by 0 is meaningless. That is, $n \div 0$ is meaningless for all values of n.

Oral Give the value of n in the following divisions and explain why your answer is correct.

	a	*b*
1.	$0 \div 7 = n$	$72 \div 2 = n$
2.	$\begin{array}{r} n \\ 21\overline{)0} \end{array}$	$\begin{array}{r} n \\ 5\overline{)125} \end{array}$

Written Write each quotient.

	a	*b*	*c*	*d*
1.	$3\overline{)24}$	$6\overline{)54}$	$3\overline{)21}$	$9\overline{)45}$
2.	$4\overline{)36}$	$9\overline{)72}$	$7\overline{)63}$	$5\overline{)30}$
3.	$6\overline{)0}$	$9\overline{)81}$	$8\overline{)48}$	$7\overline{)28}$
4.	$8\overline{)72}$	$5\overline{)40}$	$8\overline{)64}$	$6\overline{)36}$
5.	$3\overline{)27}$	$7\overline{)49}$	$9\overline{)63}$	$9\overline{)0}$

Find the value of n in each of the following equations.

	a	*b*
6.	$18 \div 2 = n$	$50 \div 5 = n$
7.	$27 \div 9 = n$	$25 \div 5 = n$
8.	$56 \div 7 = n$	$28 \div 4 = n$
9.	$12 \div 3 = n$	$48 \div 6 = n$
10.	$32 \div 4 = n$	$16 \div 2 = n$
11.	$18 \div 6 = n$	$0 \div 7 = n$

MORE PRACTICE PAGE 302

Questions Answered by Division

How many 4-cent stamps can James buy with 36 cents?

In this problem you are told how many cents in all, 36; how many cents in each set with an equal number, 4; and asked *how many sets of equal number* you can form. Division will answer the question.

That is, $36 \div 4 = 9$. James can buy 9 stamps.

Another question which is answered by division is illustrated by the following problem.

In the fifth-grade room there are 28 chairs placed in 4 rows. How many chairs are in each row?

You are told how many in all, 28; how many sets with an equal number there are, 4; and asked *how many in each of these sets*. To answer this question you could use division. You could write the division as $28 \div 4 = 7$. There are 7 chairs in each row.

Note the italicized words concerning the two previous problems. Division will aid you in answering the questions:

a. How many sets with an equal number are formed?

b. How many things are in each set with an equal number?

Oral Give the equation you would use to solve each of these problems.

1. June's father got 25 check blanks from the bank. If he uses 5 check blanks each week, how many weeks will this supply last?

2. Dennis bought 56 marbles. They were packed 8 per bag. How many bags of marbles did he buy?

3. Twenty-eight cookies were divided equally among the children at a birthday party. Each child received 4 cookies. How many children were at the birthday party?

4. The Willow Run School bought 70 new library books. If 7 of these books are placed in each room, how many rooms in the school will receive new library books?

5. Three girls bought a card of bobby pins for 30¢. What is each girl's fair share of the cost?

6. If 24 children are separated so as to form 4 teams of equal size, how many children are in each team?

Written Solve problems 1 through 6. Remember: After you have done the arithmetic, write the result so that it answers the question in the problem. That is, in problem 1, do not say 5; say 5 weeks.

Dividing Two-Digit Dividends

Sally had 60 photographs and put 6 of them on each page of her album. How many pages did she fill?

You are to find how many sets of 6 photographs each are contained in a set of 60 photographs. So you divide 60 by 6 as shown below.

$$\begin{array}{r} 10 \\ 6\overline{)60} \\ \underline{6} \\ 00 \end{array} \qquad\qquad \begin{array}{r} \text{Check} \\ 6 \\ \underline{\times 10} \\ 60 \end{array}$$

The division above was done as follows: *Divide* the tens. Since 6×1 T=6 T, then 6 T÷6=1 T. Write 1 in tens place in the quotient.

Multiply 1 ten by 6 to get 6 tens. *Subtract* in tens place, 6−6=0, leaving no tens. There are no ones in the dividend to divide, so write 0 in ones place in the quotient.

In dividing 84 by 3, you can proceed as indicated below.

$$\begin{array}{r} 28 \\ 3\overline{)84} \\ \underline{6} \\ 24 \\ \underline{24} \\ 0 \end{array} \qquad\qquad \begin{array}{r} \text{Check} \\ 3 \\ \underline{\times 28} \\ 24 \\ \underline{6} \\ 84 \end{array}$$

In the previous division, the tens digit, 8, cannot be exactly divided by 3. Since 2 T×3=6 T and 3 T×3=9 T, the largest number of tens you can use is 2. Write 2 in tens place in the quotient. Multiply: 2 T×3=6 T. Subtract: 8 T−6 T=2 T. Think of 2 tens as 20 ones. Add the 4 ones in the dividend, 20+4=24. 24 ones÷3=8 ones, so write 8 in ones place in the quotient.

To check, multiply the divisor by the quotient. The product should be equal to the dividend.

Oral How would you find the quotient in each division below?

	a	*b*	*c*	*d*
1.	$2\overline{)40}$	$3\overline{)45}$	$4\overline{)72}$	$6\overline{)72}$
2.	$8\overline{)96}$	$7\overline{)84}$	$7\overline{)70}$	$6\overline{)78}$
3.	$3\overline{)57}$	$4\overline{)68}$	$5\overline{)85}$	$2\overline{)68}$

Written Copy the above divisions and find each quotient.

Division with a Remainder Different from Zero

In some cases the divisor is not contained a whole number of times in the dividend. For example, consider the following problem.

Mr. Green packed 54 tomatoes into boxes. If each box held 4 tomatoes, how many boxes did he fill? How many tomatoes were left over?

You are asked to find how many sets of 4 each are contained in a set of 54, so you divide. Study the solution and the check given below.

$$\begin{array}{rcr} 13\text{ r}2 & & \text{Check} \\ 4\overline{)54} & & 13 \\ \underline{4} & & \underline{\times 4} \\ 14 & & 52 \\ \underline{12} & & \nearrow \underline{+2} \\ 2 & \longleftarrow \text{remainder} & 54 \end{array}$$

This example will help you remember about remainders in division. Mr. Green filled 13 boxes and had 2 tomatoes left over. The part of the dividend that is left over is called the **remainder.**

The divisor, 6, in the division $12 \div 6 = n$, is contained exactly twice in the dividend, 12. Note the 0 in the remainder position in the division. Is it correct to say the remainder is 0? Does every division have a remainder?

$$\begin{array}{r} 2 \\ 6\overline{)12} \\ \underline{12} \\ 0 \end{array}$$

Every division has a remainder. However, if the remainder is 0, you usually give only the quotient as your answer. To say there is no remainder really means that the remainder is 0.

Oral Name the quotient and the remainder in each of the following divisions.

	a	*b*	*c*	*d*
1.	$2\overline{)3}$	$5\overline{)9}$	$3\overline{)8}$	$4\overline{)12}$
2.	$4\overline{)9}$	$5\overline{)15}$	$9\overline{)20}$	$6\overline{)31}$

Written Copy. Find the quotient and the remainder in each of the following divisions.

	a	*b*	*c*	*d*
1.	$4\overline{)39}$	$9\overline{)55}$	$6\overline{)49}$	$7\overline{)44}$
2.	$8\overline{)70}$	$7\overline{)68}$	$6\overline{)52}$	$5\overline{)47}$
3.	$7\overline{)29}$	$5\overline{)19}$	$8\overline{)65}$	$2\overline{)15}$
4.	$9\overline{)37}$	$4\overline{)11}$	$3\overline{)19}$	$3\overline{)28}$
5.	$2\overline{)13}$	$4\overline{)32}$	$7\overline{)0}$	$5\overline{)26}$
6.	$5\overline{)63}$	$6\overline{)79}$	$7\overline{)25}$	$9\overline{)39}$

When the Remainder Is Zero

Study the following divisions and note each quotient.

A	B	C
$\begin{array}{r} 3 \\ 2\overline{)6} \\ \underline{6} \\ 0 \end{array}$	$\begin{array}{r} 3 \\ 5\overline{)15} \\ \underline{15} \\ 0 \end{array}$	$\begin{array}{r} 3 \\ 9\overline{)27} \\ \underline{27} \\ 0 \end{array}$

In the divisions A, B, and C each dividend is different, each divisor is different, but the quotients are all the same. Also, notice that each remainder is 0.

Can you give three other divisions in which the quotient is 3 and the remainder is 0? Are there more than three possible answers?

Whenever the remainder is 0, the dividend is said to be *a multiple of* the divisor. For instance, $21 \div 7 = 3$ since $3 \times 7 = 21$, and 21 is a multiple of 7. Is 21 a multiple of 3?

Furthermore, if 21 is a multiple of 7, then 21 is said to be *divisible by* 7. In other words, the dividend is divisible by the divisor if the remainder is 0.

Divide 2, 4, 6, 8, and 10 by 2. What were the remainders? Are each of these numbers divisible by 2?

Numbers that are divisible by 2 are called *even numbers.* All other whole numbers are called *odd numbers.*

Oral What are the only possible remainders when you divide by 2? By 3? By 4?

How many different remainders are possible when the divisor is 4? 5? 8? n?

How many even numbers can you name? How many odd numbers can you name?

Written Write the answers for each of the following.

1. Write the first twelve multiples of 2.

2. Write any 8 multiples of 4.

3. Write all the multiples of 3 which are between 20 and 40.

4. Write five two-digit numbers which are divisible by 7.

5. Which of the following numbers are divisible by both 3 and 4?

6, 8, 12, 20, 24, 32, 36

6. If a number is divisible by both 3 and 4, is it divisible by 12, because $3 \times 4 = 12$?

7. If a number is divisible by 10, is it divisible by 2? By 5?

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Using Division in Solving Problems

Carol bought 35 roses from the florist. How many bouquets of 7 roses each could she make?

You are asked to find how many sets of 7 each are contained in one set of 35. Hence, you divide 35 by 7. The equation to solve is $35 \div 7 = n$. Since $5 \times 7 = 35$, then $35 \div 7 = 5$. Carol could make 5 bouquets.

In solving each problem below, read the problem carefully to determine which of the numbers given is the dividend, and which is the divisor. Write an equation, solve it, and check by using multiplication.

1. On school visitation day the boys set up 96 chairs for guests. They arranged them in 8 equal rows. How many chairs were in each row?

2. The 32 children in Miss Poe's room formed 6 teams for a game. The same number of pupils were on each team. The remaining pupils were to keep score. How many were on each team? How many pupils kept score?

3. At her birthday party, Joan gave each child 3 balloons. She handed out 51 balloons. How many children were at the party?

4. Mrs. Bauer paid \$.94 for 2 pounds of candy. What was the cost for a pound of candy?

5. How many 5-cent candy bars can Dick buy with \$.83? Will he have any money left? If so, how much will he have left?

6. The science class wanted to plant 56 bean seeds. They were told to plant 4 seeds per box. How many boxes were needed?

7. Ann has \$.60 to spend on treats for herself and 3 friends. If each of them shares alike, how much can she spend for each person? How much money would she have left?

Practicing Division

Copy. Find the quotient and the remainder for each division. Write the multiplications which check each division.

	a	*b*	*c*
1.	$5\overline{)60}$	$4\overline{)85}$	$6\overline{)84}$
2.	$8\overline{)96}$	$3\overline{)84}$	$8\overline{)70}$
3.	$4\overline{)62}$	$7\overline{)49}$	$6\overline{)52}$
4.	$5\overline{)42}$	$9\overline{)60}$	$5\overline{)46}$
5.	$8\overline{)51}$	$4\overline{)59}$	$3\overline{)93}$
6.	$6\overline{)30}$	$5\overline{)72}$	$7\overline{)78}$
7.	$3\overline{)66}$	$4\overline{)68}$	$5\overline{)68}$
8.	$4\overline{)55}$	$4\overline{)99}$	$7\overline{)66}$

Solve each of the following equations for n.

	a	*b*
9.	$72 \div 8 = n$	$70 \div 7 = n$
10.	$57 \div 4 = n$	$92 \div 5 = n$
11.	$47 \div 8 = n$	$87 \div 6 = n$
12.	$84 \div 9 = n$	$93 \div 4 = n$
13.	$96 \div 6 = n$	$72 \div 7 = n$
14.	$84 \div 8 = n$	$97 \div 4 = n$

Can you do this? An easy way to determine whether or not a number is divisible by 3 is explained below. Is 81 divisible by 3? First, add the digits in 81. $8+1=9$. Since 9 is divisible by 3, so is 81.

Try again. Is 213 divisible by 3? $2+1+3=6$. Six is divisible by 3. So 213 is divisible by 3. You can check the answer by actually dividing 213 by 3. The quotient would be 71.

Which of the following are divisible by 3?

51, 123, 241, 318, 1152, 954

Can you tell an easy way of knowing whether or not a number is divisible by 2?

To tell whether or not a number is divisible by 4 is almost as easy as divisibility by 2 or 3.

Is 216 divisible by 4? Certainly 200 is divisible by 4, so merely ask: Is 16 (the last two digits of 216) divisible by 4? If the answer is yes, then 216 is divisible by 4.

Try again. Does 4 divide 756 exactly? $56 \div 4 = 14$, so 756 is divisible by 4.

Which of the following numbers are divisible by 4?

48, 512, 340, 721, 636, 1016

Two-Digit Quotients

Study the completed divisions shown below.

$$\begin{array}{ccc} \text{A} & \text{B} & \text{C} \\ \begin{array}{r} 50 \\ 6\overline{)300} \\ \underline{30} \\ 0 \end{array} & \begin{array}{r} 50 \text{ r4} \\ 6\overline{)304}\phantom{\text{ r4}} \\ \underline{30}\phantom{\text{ r4}} \\ 4\phantom{\text{ r4}} \end{array} & \begin{array}{r} 52 \text{ r2} \\ 6\overline{)314}\phantom{\text{ r2}} \\ \underline{30}\phantom{\text{ r2}} \\ 14\phantom{\text{ r2}} \\ \underline{12}\phantom{\text{ r2}} \\ 2\phantom{\text{ r2}} \end{array} \end{array}$$

In example A, you cannot divide 3, the hundreds digit, by 6, the divisor. So think of 3 hundreds as 30 tens. 30 T÷6=5 T, since 5 T×6=30 T. That is why 5 is written in tens place in the quotient.

In example B, 4 ones remain from the dividend after the subtraction of the 30 tens. Certainly 6 ones are not contained in 4 ones because $4<6$ (read < as *is less than*). Therefore 0 appears in ones place in the quotient, and 4 is the remainder.

In example C, 1 ten remains after the subtraction of tens. Think of it as 10 ones. Add the 4 ones of the dividend, 10 ones+4 ones=14 ones. Hence you are to divide 14 by 6.

Why does each quotient in examples A, B, and C contain two digits?

Oral What is the first digit in the quotient of each division below? Does this first digit represent hundreds, tens, or ones?

	a	*b*	*c*
1.	$3\overline{)372}$	$5\overline{)268}$	$2\overline{)172}$
2.	$9\overline{)216}$	$7\overline{)413}$	$2\overline{)101}$
3.	$4\overline{)390}$	$6\overline{)572}$	$8\overline{)328}$

Written Copy. Find the quotient and the remainder in each division.

	a	*b*	*c*
1.	$4\overline{)382}$	$8\overline{)499}$	$4\overline{)320}$
2.	$5\overline{)206}$	$6\overline{)540}$	$5\overline{)354}$
3.	$6\overline{)547}$	$4\overline{)329}$	$7\overline{)288}$
4.	$9\overline{)659}$	$2\overline{)168}$	$3\overline{)217}$
5.	$8\overline{)520}$	$7\overline{)651}$	$3\overline{)236}$
6.	$2\overline{)156}$	$9\overline{)342}$	$5\overline{)474}$
7.	$8\overline{)648}$	$5\overline{)447}$	$9\overline{)135}$
8.	$3\overline{)202}$	$2\overline{)111}$	$7\overline{)376}$
9.	$9\overline{)874}$	$4\overline{)296}$	$8\overline{)555}$
10.	$5\overline{)225}$	$7\overline{)456}$	$3\overline{)241}$

Three-Digit Quotients

In the following divisions, the divisor is less than the hundreds digit of the dividend. In A, $2<6$. In B, $4<9$.

A	B
$\begin{array}{r} 314 \\ 2\overline{)628} \\ \underline{6} \\ 2 \\ \underline{2} \\ 8 \\ \underline{8} \\ 0 \end{array}$	$\begin{array}{r} 243\text{ r3} \\ 4\overline{)975}\phantom{\text{ r3}} \\ \underline{8}\phantom{75\text{ r3}} \\ 17\phantom{5\text{ r3}} \\ \underline{16}\phantom{5\text{ r3}} \\ 15\phantom{\text{ r3}} \\ \underline{12}\phantom{\text{ r3}} \\ 3\phantom{\text{ r3}} \end{array}$

In example A, you get the first digit of the quotient by dividing the hundreds digit, 6, by the divisor, 2. $6\div2=3$. Why is 3 written in hundreds place in the quotient?

After the subtraction of 6 hundreds, there remains a two-digit division, $28\div2$. You should be able to solve this division.

In example B, find the first digit of the quotient as in example A, $9\div4=2$ r1. Use 2 as the hundreds digit in the quotient. 4×2 H $=8$ H, and after the subtraction of 8 hundreds from 9 hundreds, there remains 1 hundred. The rest of this division, 17 T$\div4$ and 15 ones$\div4$, should cause you no trouble.

Oral Why are both of the quotients in the preceding examples three-digit numbers?

Tell whether each of the following divisions has a two-digit or a three-digit quotient.

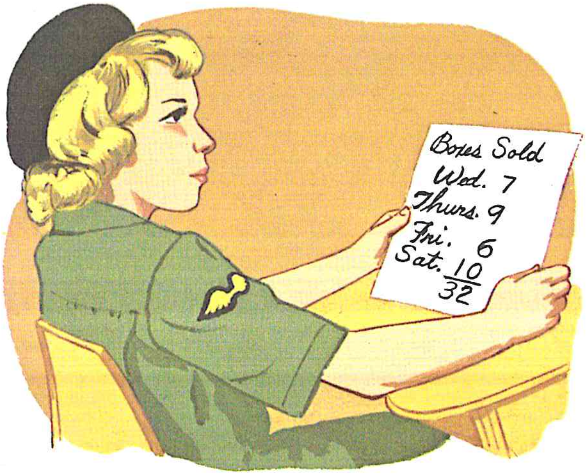
	a	*b*	*c*
1.	$4\overline{)216}$	$5\overline{)525}$	$8\overline{)324}$
2.	$4\overline{)829}$	$3\overline{)129}$	$6\overline{)972}$
3.	$9\overline{)495}$	$8\overline{)684}$	$7\overline{)735}$

Give the first digit of the quotient in each of the above divisions.

Written Copy. Find the quotient and the remainder in each division.

	a	*b*	*c*
1.	$5\overline{)850}$	$4\overline{)760}$	$8\overline{)960}$
2.	$2\overline{)597}$	$4\overline{)610}$	$3\overline{)791}$
3.	$5\overline{)670}$	$7\overline{)994}$	$3\overline{)768}$
4.	$2\overline{)967}$	$3\overline{)548}$	$4\overline{)608}$
5.	$3\overline{)489}$	$2\overline{)566}$	$6\overline{)846}$
6.	$7\overline{)987}$	$5\overline{)805}$	$8\overline{)968}$
7.	$5\overline{)759}$	$6\overline{)908}$	$7\overline{)919}$

Finding Averages



Sandra kept a record of her sales of Girl Scout cookies for the first 4 days. Her record is shown above. She said, "I sold an average of 8 boxes a day."

Do you know what Sandra meant by the word **average?**

Sandra sold 32 boxes of cookies, but did not sell the same amount each day. Her division, $32 \div 4 = 8$, tells us that she would have sold the same total amount if she had sold 8 boxes each day for the 4 days. In this case, 8 is called the average.

Notice how Sandra found the average. She found the total sales for the 4 days. Next the sum, 32, was divided by 4, the number of addends. The quotient, 8, is the average.

In some cases, the sum and the number of addends are given. Then you divide the sum by the number of addends to find the average.

Study the following example of finding 16 as the average of 12, 16, 13, 17, and 22.

$$\begin{array}{r} 12\\ 16\\ 13\\ 17\\ +22\\ \hline 80 \end{array} \qquad \begin{array}{r} 16\\ 5\overline{)80}\\ 5\\ \hline 30\\ 30\\ \hline 0 \end{array}$$

Note that some of the numbers are larger than the average while others are smaller.

Oral Estimate the average of each set of numbers below.

1. 24, 32, 28, 25, 26
2. 7, 14, 9, 5, 8, 13
3. 11, 19, 38, 25, 33, 38

Written Find the average of the numbers in each exercise above.

Solve the following problems.

4. The fifth grade sold tickets for the school play. On Monday they sold 23; on Tuesday, 19; on Wednesday, 28; and on Thursday, 14. What was the average daily sale?

5. James delivered 98 grocery orders in 7 days. What was the average number of deliveries per day?

Review of Addition, Subtraction, and Multiplication

You should be able to find each sum, difference, or product and solve each problem without help.

Part 1 Copy. Find each sum.

	a	b	c	d
1.	22 38 +92	46 9 +82	35 9 +6	78 21 +85
2.	208 182 46 +59	76 431 57 +634	214 852 541 +476	$1.35 5.17 4.61 +.56
3.	5641 3492 8426 +2733	3281 762 49 +5131	7246 385 9542 +761	4765 8172 6396 +1237
4.	3742 645 2481 1957 +823	5781 36 581 4627 +436	472 3582 7602 1093 +867	9321 2468 7531 4357 +6160

Part 2 Copy. Find each difference.

	a	b	c	d
1.	39 −26	74 −48	37 −9	57 −48
2.	542 −31	463 −76	871 −424	643 −502
3.	7349 −238	4581 −3656	4008 −592	7007 −4329

Part 3 Copy. Find each product.

	a	b	c	d
1.	453 ×4	671 ×5	493 ×32	132 ×68
2.	1762 ×6	341 ×576	505 ×72	407 ×306
3.	2605 ×7	572 ×300	920 ×70	3008 ×703

Part 4 Copy. Carry out the indicated operation.

	a	b	c
1.	58210 +4821	76824 −6936	7205 ×481
2.	4763 −4762	4879 ×16	52718 +47292
3.	3056 ×99	8765 +1234	50004 −7538

Part 5 Copy. Find a replacement for n which makes each of the following statements true.

	a	b
1.	12×11 = n	34+16 = n
2.	126−87 = n	73×24 = n
3.	n−12 = 41	n+36 = 127
4.	n = 406×27	47+1287 = n

To answer:	Use:
How many in all? (sets with unequal numbers)	Addition
How many in all? (sets with an equal number)	Multiplication
How many left?	Subtraction
By how many do 2 sets differ?	
How many sets with an equal number?	Division
How many in each set with an equal number?	

Choosing the Correct Operation

It is wise to read a story problem several times before doing any writing. The suggestions at the top of this page should help you decide which operation to use.

It may be necessary to use more than one operation in solving a problem, such as in finding the average.

Oral What operation would you use to solve each of the following problems? Give the equation you would use for each problem.

1. John has 27 crayons in his box. The box will hold 36 crayons. How many more crayons does he need to fill the box?

2. A jet plane flew at a speed of 525 miles an hour. Flying at this speed, how many miles would it travel in 7 hours?

3. Esther practiced the flute 25 minutes on Monday, 35 minutes on Tuesday, and 45 minutes on Wednesday. How many minutes did she practice on the three days?

4. Mr. Goodman feeds each of the 6 bears at the city zoo the same amount of food. He uses 84 pounds a day. How many pounds of food does each bear get in a day?

5. Robert pitched 9 innings in the baseball game and threw 18 strikes. What was the average number of strikes per inning?

Written Solve problems 1 through 5 above. Read each problem again to be sure your answer is reasonable.

Something to do Make 5 problems of your own for the class to solve. Write an equation for each of your problems.

Dividing Four-Digit Dividends

In many problems you may want to solve, the dividend will have more than three digits. For example, study division A.

A

$$\begin{array}{r} 310 \\ 4\overline{)1240} \\ \underline{12} \\ 4 \\ \underline{4} \\ 0 \end{array}$$

Check

$$\begin{array}{r} 4 \\ \underline{\times 310} \\ 1240 \end{array} \quad \text{or} \quad \begin{array}{r} 310 \\ \underline{\times 4} \\ 1240 \end{array}$$

The divisor, 4, will not divide the thousands digit, 1, of the dividend. So think of 1 thousand as 10 hundreds, and add the 2 hundreds of the dividend. 10 H+2 H=12 H. Since 12 H÷4=3 H, write 3 in the hundreds place in the quotient. Multiply, 3 H×4=12 H. Subtract 12 hundreds from the dividend, leaving 40 ones.

Now proceed with the remaining two-digit dividend of 40 as you have previously done.

Study the divisions B and C. Notice that in each case the divisor will divide the thousands digit of the dividend, but there will be some thousands remaining. Think of the remaining thousands as hundreds and proceed as you did in example A at the top of this page.

B

$$\begin{array}{r} 2487 \\ 3\overline{)7461} \\ \underline{6} \\ 14 \\ \underline{12} \\ 26 \\ \underline{24} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

C

$$\begin{array}{rl} 2413 & \text{r1} \\ 4\overline{)9653} & \\ \underline{8} & \\ 16 & \\ \underline{16} & \\ 5 & \\ \underline{4} & \\ 13 & \\ \underline{12} & \\ 1 & \end{array}$$

Oral Explain divisions B and C.

Written Find the quotient for each of the divisions below. Check each division by using multiplication.

	a	*b*	*c*
1.	$4\overline{)2000}$	$5\overline{)3050}$	$3\overline{)2106}$
2.	$9\overline{)6375}$	$8\overline{)5601}$	$6\overline{)5498}$
3.	$3\overline{)2926}$	$7\overline{)6069}$	$4\overline{)3945}$
4.	$5\overline{)2392}$	$2\overline{)1864}$	$8\overline{)6904}$
5.	$3\overline{)9000}$	$3\overline{)6900}$	$4\overline{)8002}$
6.	$5\overline{)5007}$	$7\overline{)7706}$	$3\overline{)9857}$
7.	$2\overline{)8746}$	$6\overline{)6489}$	$5\overline{)5545}$
8.	$5\overline{)7895}$	$8\overline{)8860}$	$4\overline{)8047}$
9.	$7\overline{)9784}$	$5\overline{)9402}$	$4\overline{)8364}$

MORE PRACTICE PAGE 302

Basic Operations of Arithmetic

Addition, subtraction, multiplication, and division are called the *basic operations* of arithmetic.

You have learned that each basic operation is an operation on *two* numbers. That is, you can add, subtract, multiply, or divide only two numbers at a time. If an addition has more than two addends, or a multiplication more than two factors, you make repeated additions or repeated multiplications.

The following summary and examples will help you understand the relationships among the operations.

ADDITION

1. Addition is derived from the joining of two sets. When a set containing 10 objects is joined with a set containing 15 objects to form a set containing 25 objects, you merely write $10+15=25$.

2. *Commutative property* Two numbers may be added in either order.

$4+5=9$ and $5+4=9$, so $4+5=5+4$

3. *Associative property* Regrouping does not change the sum.

$6+4+5=n$	$6+4+5=n$
$(6+4)+5=n$	$6+(4+5)=n$
$10+5=n$	$6+9=n$
$15=n$	$15=n$

4. You may change both the order and the grouping in an addition.

$7+6+3+4=n$

$7+(6+3)+4=n$ Why?

$7+(3+6)+4=n$ Why?

$(7+3)+(6+4)=n$ Why?

$10+10=n$

$20=n$

SUBTRACTION

1. Subtraction is the inverse operation of addition.

$11+5=16$ and $16-5=11$

2. Subtraction must be done in a specified order.

3. Subtraction is checked by using addition.

$27-14=13$ and $13+14=27$

MULTIPLICATION

1. Multiplication may be based on repeatedly adding the same number.

Instead of: $4+4+4+4+4=20$
You can think: $5\times4=20$
So, 5×4 means five 4's added.

2. *Commutative property* Two numbers may be multiplied in either order.

$3\times7=21$ and $7\times3=21$

$3\times7=7\times3$

3. *Associative property* Regrouping does not change the product.

$$\begin{array}{rr} 2\times4\times5=n & 2\times4\times5=n \\ (2\times4)\times5=n & 2\times(4\times5)=n \\ 8\times5=n & 2\times20=n \\ 40=n & 40=n \end{array}$$

4. You may change both the order and the grouping of factors in a multiplication.

$$\begin{array}{rl} 2\times4\times6=n & \\ (2\times4)\times6=n & \text{Why?} \\ (4\times2)\times6=n & \text{Why?} \\ 4\times(2\times6)=n & \text{Why?} \\ 4\times12=n & \\ 48=n & \end{array}$$

DIVISION

1. Division is the inverse operation of multiplication.

$$35\div7=5 \text{ and } 35\div5=7$$
$$5\times7=35$$

2. Division may be based on repeatedly subtracting the same number.

$$\begin{array}{r} 15 \\ -5 \\ \hline 10 \\ -5 \\ \hline 5 \\ -5 \\ \hline 0 \end{array} \qquad \begin{array}{r} 5 \\ 3\overline{)15} \\ 15 \\ \hline 0 \end{array}$$

3. Division must be done in a specified order.

4. The remainder in a division may be 0 or any whole number less than the divisor.

5. Division is checked by using multiplication.

$$\begin{array}{r} 12 \\ 3\overline{)36} \\ 36 \\ \hline 0 \end{array} \qquad \text{Check} \quad \begin{array}{r} 3 \\ \times12 \\ \hline 36 \end{array} \quad \text{or} \quad \begin{array}{r} 12 \\ \times3 \\ \hline 36 \end{array}$$

Oral Answer the following questions about the basic operations.

1. Which one of the basic operations do you consider most important? Give reasons for your answer.

2. Which of the operations is most like counting?

Written Copy. Find the value of n in each equation, and tell which operation you used.

	a	*b*
1.	$7-4=n$	$n\times3=18$
2.	$n=8\div4$	$n+2=6$
3.	$n\div2=7$	$15\div n=5$
4.	$n=5+17$	$4\times n=24$
5.	$2\times5=n$	$13-n=5$
6.	$n-8=11$	$12+n=21$
7.	$28+n=42$	$n-13=122$
8.	$72\div n=18$	$n\div5=14$

Practicing Problem Solving

You should be able to solve all the problems on this page without help. Use the suggestions on page 86 if necessary.

1. At summer scout camp the boys caught 4 fish one afternoon. These fish were 9 inches, 11 inches, 8 inches, and 12 inches long. What was the average length of the fish?

2. Melvin earned \$1.50 by raking leaves from the neighbor's lawn. Now he has enough money to buy a new football which costs \$4.25. How much money did he have before he earned the \$1.50?

3. The science club made 36 cardboard signs for the science fair. They made 3 signs from each sheet of cardboard. How many sheets of cardboard did they use?

4. Jeff sold 224 papers last week, including Saturday and Sunday. What was his average sale per day?

5. A swimming pool was 92 feet long and 45 feet wide. Susan walked all the way around the edge of the pool. How far did she walk?

6. On his vacation trip, Paul's father drove 215 miles on Monday, 178 miles on Tuesday, and 387 miles on Wednesday. How many miles did he travel in the three days?

7. Mrs. Hawkins bought 13 cases of canned food for the school cafeteria. Each case contained 24 cans. How many cans of food did she buy?

8. In science class Bill learned that the planet Jupiter has about 4,380 days in one of its years. Our year has 365 days. How many days longer is Jupiter's year than ours?

9. Mr. Swanson pays \$150.00 a month rent for a house. How much rent does he pay in a year?

Can you do this? Find a replacement for n which makes each of the following sentences true.

	a	*b*
1.	$(6\times3)+13=n$	$n=75+(45-27)$
2.	$4+(7+8)=n$	$n=86-(5\times9)$
3.	$(28\div4)-3=n$	$n=18+(96\div6)$
4.	$(8\times9)\div4=n$	$n=7\times(42\times3)$
5.	$(12\div3)\div2=n$	$n=(5\times6)+71$
6.	$78-(49-17)=n$	$n=(21\div3)\times127$
7.	$15+(11-3)=n$	$n=(18\div2)+56$
8.	$(49-14)\div n=5$	$(12+8)\div5=n$
9.	$(6\times3)\div n=9$	$(80\div10)\div n=2$
10.	$(56-16)\div n=4$	$(32+12)\div n=11$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Division can be indicated in different ways. (73)

2. Multiplication and division are inverse operations. (74)

3. When zero is divided by a number different from zero, the quotient is zero. (75)

4. Addition, subtraction, multiplication, and division are the basic operations of arithmetic. (88, 89)

Words to Know

1. Dividend, divisor, quotient (73)

2. Remainder (78)

3. Multiple, divisible (79)

4. Is less than, < (82)

5. Average (84)

Questions to Discuss

1. Is division by 0 used? (75)

2. What questions are answered by using division? (76)

3. How do you check your work in division? (77, 80)

4. What is meant by saying there is no remainder? (78)

5. How do you find the average of 4 numbers? Of 5 numbers? (84)

Written Practice

Copy. Find each answer.

	a	*b*	
1.	$7\overline{)63}$	$5\overline{)35}$	(77)
2.	$6\overline{)71}$	$8\overline{)43}$	(78)
3.	$5\overline{)215}$	$9\overline{)351}$	(82)
4.	$6\overline{)767}$	$3\overline{)764}$	(83)
5.	$5\overline{)3465}$	$6\overline{)1956}$	(87)
6.	$7\overline{)1763}$	$2\overline{)5697}$	(87)

Find the average of each set of numbers.

7. 12, 16, 19, 9, 14 (84)

8. 96, 112, 101, 105, 111 (84)

Use the set of numbers 12, 17, 21, 26, 30, 39 to answer the questions below.

9. Which of the numbers are multiples of 2? (79)

10. Which of the numbers are divisible by 3? (79)

Self-Evaluation

Part 1 Copy. Solve each equation.

	a	*b*
1.	$64 \div 4 = n$	$72 \div 6 = n$
2.	$600 \div 6 = n$	$328 \div 8 = n$
3.	$848 \div 4 = n$	$286 \div 2 = n$
4.	$180 \div 9 = n$	$550 \div 5 = n$
5.	$4000 \div 5 = n$	$1250 \div 2 = n$
6.	$2610 \div 3 = n$	$3645 \div 9 = n$
7.	$2768 \div 8 = n$	$7824 \div 6 = n$

Part 2 Copy. Find the quotient and the remainder for each division.

	a	*b*	*c*
1.	$2\overline{)136}$	$5\overline{)357}$	$6\overline{)469}$
2.	$4\overline{)239}$	$9\overline{)463}$	$3\overline{)153}$
3.	$7\overline{)412}$	$8\overline{)912}$	$6\overline{)181}$
4.	$8\overline{)350}$	$4\overline{)724}$	$3\overline{)526}$
5.	$9\overline{)992}$	$5\overline{)510}$	$3\overline{)333}$
6.	$7\overline{)705}$	$8\overline{)994}$	$3\overline{)546}$
7.	$2\overline{)2008}$	$4\overline{)8080}$	$4\overline{)8364}$
8.	$5\overline{)4060}$	$2\overline{)6469}$	$3\overline{)8952}$
9.	$7\overline{)7700}$	$4\overline{)9669}$	$2\overline{)6008}$
10.	$8\overline{)8860}$	$4\overline{)9875}$	$3\overline{)9857}$

Part 3 Copy. Find each answer.

	a	*b*	*c*
1.	$\begin{array}{r}3475\\-852\\\hline\end{array}$	$\begin{array}{r}4050\\-3862\\\hline\end{array}$	$\begin{array}{r}8152\\-8096\\\hline\end{array}$
2.	$\begin{array}{r}351\\\times 9\\\hline\end{array}$	$\begin{array}{r}712\\\times 18\\\hline\end{array}$	$\begin{array}{r}509\\\times 32\\\hline\end{array}$
3.	$\begin{array}{r}5172\\\times 6\\\hline\end{array}$	$\begin{array}{r}7109\\\times 21\\\hline\end{array}$	$\begin{array}{r}8176\\\times 249\\\hline\end{array}$

Part 4 Solve the following problems.

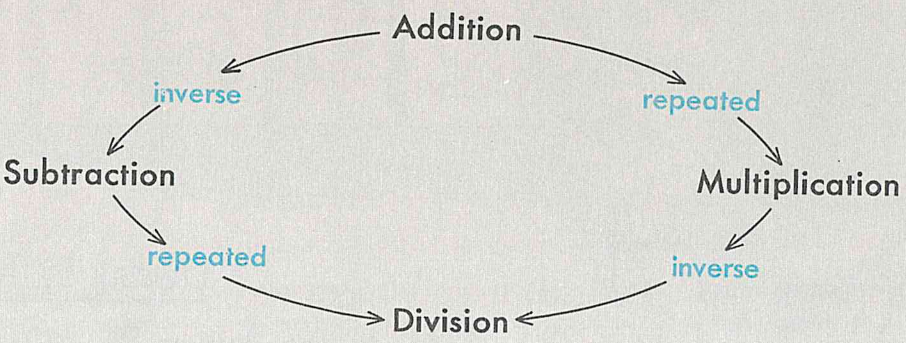
1. Mr. Lind drove his car 1448 miles in 4 days. What was the average number of miles driven each day?

2. The children placed 23 books on each shelf in the room. They filled 16 shelves and had 9 books left over. How many books were there?

3. In scouting, Bill earned 11 merit badges, Jim earned 8, Bob earned 12, Richard earned 6, Norman earned 13, and Ed earned 10. What was the average number of merit badges earned by each boy?

4. Peter and James had 72 cookies to divide equally among the children at the picnic. The two boys ate 12 cookies on the way to the picnic. When the food was distributed, each child received 4 cookies. How many children attended the picnic?

6



Using Two-Digit Divisors

Westside School used 15 classrooms for the 435 pupils who enrolled. What was the average number of pupils per room?

To answer the question, think: 435 pupils were assigned equally to 15 rooms, so divide 435 by 15. The equation is $435 \div 15 = 29$. There was an average of 29 pupils per room.

Many problems will have two-digit divisors. Before using two-digit divisors it is a good plan to review the basic operations.

The basic operations are so related that you can expect to use more than one of them in any given exercise of arithmetic. Refer to the diagram at the top of this page for the relationships among the operations.

When doing a division problem, you multiply the divisor by the quotient and subtract this product from the dividend. To check a division, you multiply the divisor by the quotient and add the remainder. You can see that it is important to know the facts for each of the basic operations.

Oral Give the answer to each of the following exercises by performing the indicated operation.

	a	*b*	*c*	*d*
1.	$5\overline{)45}$	$8\overline{)64}$	$7\overline{)42}$	$6\overline{)0}$
2.	$4\overline{)120}$	$5\overline{)250}$	$3\overline{)396}$	$7\overline{)490}$
3.	8 $+9$	12 -5	9 $\times 6$	12 $+0$
4.	18 -0	7 $\times 4$	15 -6	23 $+14$
5.	7 $\times 8$	52 -37	32 $\times 4$	15 $\times 0$

Dividing by Tens

The art class selected 30 of its drawings for display. There is room for 10 drawings in each row on the bulletin board. How many rows of artwork will there be?

You must find how many sets of 10 there are in a set of 30. So you divide 30 by 10, as shown below.

$$\begin{array}{r} 3 \\ 10\overline{)30} \\ 30 \\ \hline 0 \end{array} \qquad \begin{array}{r} \text{Check} \\ 10 \\ \times 3 \\ \hline 30 \end{array}$$

Why is the quotient of 3 written in the ones place?

Study the divisions in A and B.

$$\begin{array}{r} \textbf{A} \\ 4 \\ 20\overline{)80} \\ 80 \\ \hline 0 \end{array} \qquad \begin{array}{r} \textbf{B} \\ 4\text{ r}6 \\ 20\overline{)86} \\ 80 \\ \hline 6 \end{array}$$

In A, the divisor, 20, will not divide the tens digit, 8, so think: 8 tens is 80, and 80÷20=4. So 4 must be the quotient.

In B, the divisor, 20, will not divide the tens digit, 8; so think: 8 tens is 80 ones, and 80 ones plus 6 ones is 86 ones. Now you can think of this division as 86 ones divided by 20. Since 20×4 ones=80 ones, use 4 ones as the quotient. What is the remainder?

Oral Tell the steps to use when dividing 60 by 20; when dividing 63 by 20.

Written Copy. Divide and check.

	a	*b*	*c*
1.	$10\overline{)40}$	$20\overline{)60}$	$40\overline{)80}$
2.	$30\overline{)60}$	$30\overline{)90}$	$20\overline{)40}$
3.	$20\overline{)43}$	$30\overline{)62}$	$40\overline{)89}$
4.	$30\overline{)95}$	$40\overline{)47}$	$20\overline{)65}$
5.	$70\overline{)78}$	$30\overline{)33}$	$30\overline{)62}$

Dividing by a Two-Digit Divisor

Martha has saved 69 stamps to place in her album. She can put 23 stamps on a page. How many pages will she use for the 69 stamps?

To find the answer, divide 69 by 23, as given below. 23 will not divide 6, the tens digit of the dividend. So think: 6 tens equal 60 ones, 60 ones+9 ones equal 69 ones, and 69 ones ÷ 23 = 3 ones. Write 3 in ones place in the quotient. Multiply: $3\times23=69$. The product is 69 and the dividend is 69. What is the remainder? Why do you know that 3 is the correct quotient? What does the quotient tell you?

$$\begin{array}{r} 3 \\ 23\overline{)69} \\ 69 \\ \hline 0 \end{array}$$

You can proceed in the same way when the division has a remainder different from 0. Study the following examples.

$$\begin{array}{r} 2\text{ r}3 \\ 21\overline{)45\phantom{\text{ r}3}} \\ 42\phantom{\text{ r}3} \\ \hline 3\phantom{\text{ r}3} \end{array} \qquad\qquad \begin{array}{r} 2\text{ r}1 \\ 34\overline{)69\phantom{\text{ r}1}} \\ 68\phantom{\text{ r}1} \\ \hline 1\phantom{\text{ r}1} \end{array}$$

What is the largest possible remainder if the divisor is 20? How many possible remainders are there when the divisor is 20?

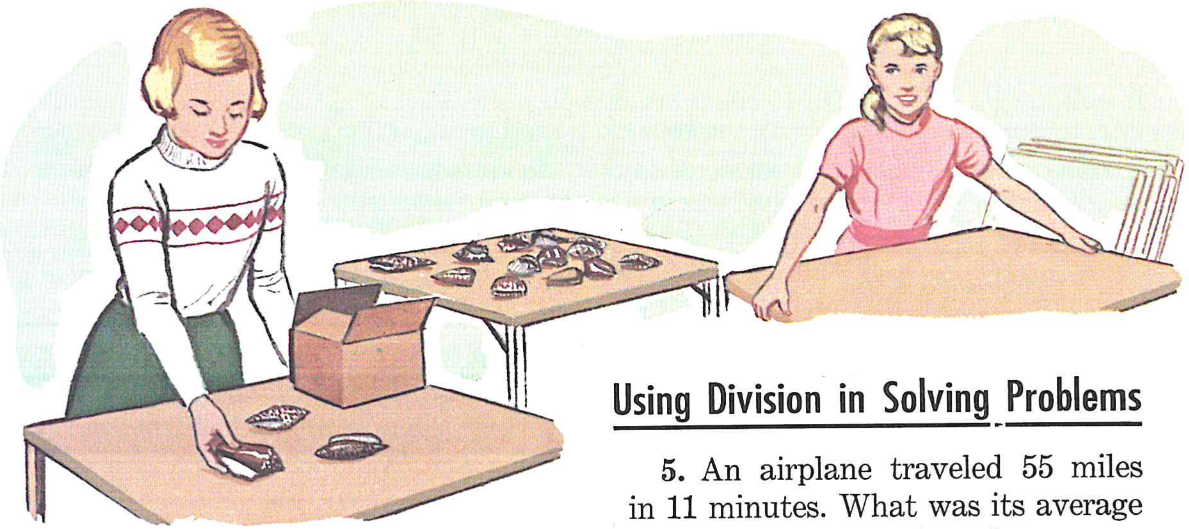
In doing a division, you must be able to multiply. The better you know multiplication, the faster you can carry out the steps of division.

Oral Read. Supply each product. (Reminder: Multiply the tens and multiply the ones by the first factor: $3\times12=(3\times10)+(3\times2)=36$.)

	a	*b*
1.	$3\times13=n$	$4\times11=n$
2.	$4\times22=n$	$6\times18=n$
3.	$7\times12=n$	$9\times12=n$
4.	$3\times24=n$	$4\times23=n$

Written Copy. Find each quotient and remainder. Check your work.

	a	*b*	*c*
1.	$32\overline{)64}$	$11\overline{)88}$	$22\overline{)66}$
2.	$12\overline{)48}$	$91\overline{)93}$	$24\overline{)48}$
3.	$20\overline{)61}$	$51\overline{)57}$	$22\overline{)89}$
4.	$23\overline{)49}$	$21\overline{)63}$	$33\overline{)68}$
5.	$33\overline{)67}$	$12\overline{)27}$	$31\overline{)93}$
6.	$41\overline{)89}$	$17\overline{)19}$	$13\overline{)20}$
7.	$13\overline{)26}$	$11\overline{)68}$	$22\overline{)47}$



Using Division in Solving Problems

Read each of the following problems carefully. Decide which number is the dividend and which number is the divisor. Write the equation, solve it, and give the answer for each problem.

1. Becky and Linda had a collection of 48 sea shells. For the school science exhibit they placed 12 shells on each table. How many tables did the girls need to display their 48 sea shells?

2. The 21 girls in Troop 54 sold 84 boxes of Girl Scout candy. What was the average number of boxes sold by each girl?

3. Robert's father pays 32¢ for a gallon of gasoline. How many gallons can he buy for 96¢?

4. The 14 cantaloupes for the picnic weighed 70 pounds. What was the average weight?

5. An airplane traveled 55 miles in 11 minutes. What was its average speed in miles per minute?

6. A man had 49 eggs to place in cartons. Each carton held 1 dozen eggs. How many cartons did he fill, and how many eggs were left over?

7. Philip had 46¢ and spent 27¢ for treats. How many 15¢ tickets for rides at the carnival could he buy with the remaining money?

8. Bobby saved 98 old and foreign coins. His coin collection book allows him to put 32 coins on each page. How many pages will he need? How many coins will be left over?

9. The 96 boys in Little League summer baseball were asked to form teams of 12 boys each. How many teams were formed?

10. Mr. Knutson took 64 pictures on his vacation trip. He got 16 pictures from each roll of film. How many rolls of film did he use?

Another way To find the value of n in the equation $46 \div 2 = n$, you could rename 46 as a sum. A good choice would be $46 = 40 + 6$. A solution of the equation $46 \div 2 = n$ is given below. Study each step carefully.

$$\begin{aligned} 46 \div 2 &= n \\ (40+6) \div 2 &= n \\ (40 \div 2) + (6 \div 2) &= n \\ 20 + 3 &= n \\ 23 &= n \end{aligned}$$

In the above solution, 46 was renamed as $(40+6)$. Next each addend of $(40+6)$ was divided by 2, and the partial quotients were added. This method of dividing uses the *distributive property of division over addition.* That is, to divide the sum of two numbers by a third number, you may divide each addend and add the two partial quotients.

Another solution of the equation, $46 \div 2 = n$, is given below. Notice that 46 was again named so that each addend was divisible by 2, the divisor.

$$\begin{aligned} 46 \div 2 &= n \\ (30+16) \div 2 &= n \\ (30 \div 2) + (16 \div 2) &= n \\ 15 + 8 &= n \\ 23 &= n \end{aligned}$$

Why would $(25+21)$ be a poor way of renaming 46 in the above solution? Solve $46 \div 2 = n$ by using $(25+21)$ for 46 and see what happens.

In solving $72 \div 4 = n$, rename 72 as a sum in which each addend is divisible by 4. Study the solution given below.

$$\begin{aligned} 72 \div 4 &= n \\ (60+12) \div 4 &= n \\ (60 \div 4) + (12 \div 4) &= n \\ 15 + 3 &= n \\ 18 &= n \end{aligned}$$

Why would $(70+2)$ be a poor way of renaming 72 in the solution above? Can you suggest another way of writing 72 as a sum which would simplify the division of 72 by 4?

Written Use the method described above to find each of the following quotients.

	a	*b*
1.	$68 \div 4 = n$	$32 \div 4 = n$
2.	$88 \div 8 = n$	$94 \div 2 = n$
3.	$81 \div 3 = n$	$75 \div 5 = n$
4.	$84 \div 4 = n$	$65 \div 5 = n$
5.	$96 \div 4 = n$	$72 \div 3 = n$
6.	$72 \div 6 = n$	$98 \div 7 = n$
7.	$54 \div 9 = n$	$15 \div 5 = n$
8.	$82 \div 7 = n$	$35 \div 5 = n$
9.	$64 \div 4 = n$	$93 \div 3 = n$
10.	$46 \div 2 = n$	$87 \div 3 = n$

Trial Quotients for Two-Digit Dividends

In the divisions you have been working up to this time, the quotient could be obtained by dividing the tens digit of the dividend by the tens digit of the divisor. In many cases, however, the number obtained by dividing the tens digits is not the correct quotient, so call it a **trial quotient.**

Study the division and the trial quotient equation below.

Trial Quotient

6 T÷3 T=2

$$\begin{array}{r} 2 \\ 34\overline{)68} \\ \underline{68} \\ 0 \end{array}$$

In the above example, the trial quotient of 2 was tried as the quotient. $2\times34=68$, and $68-68=0$. Hence the trial quotient was actually the quotient you wanted.

In example A, begin by solving the equation 7 T÷3 T=n. Since n is more than 2, but less than 3, try 2 as the quotient. $2\times35=70$, and the product 70 can be subtracted from the dividend. What is the remainder? Again, the trial quotient was the correct quotient for the division.

A

$$\begin{array}{r} 2 \\ 35\overline{)70} \\ \underline{70} \\ 0 \end{array}$$

Study the divisions in B and C below. Pay special attention to the use of trial quotients.

B

$$\begin{array}{r} 4 \\ 27\overline{)81} \\ \underline{108} \\ ? \end{array}$$

C

$$\begin{array}{r} 3 \\ 27\overline{)81} \\ \underline{81} \\ 0 \end{array}$$

In B the trial quotient is 4 since 8 T÷2 T=4. When 4 is used as the quotient, notice that $27\times4=108$. But 108 is larger than the dividend. Is 4 the quotient? Is 4 too large to be the quotient?

Since 4 is too large to be the quotient, what would be your next trial quotient? Try the next smaller whole number, 3. Illustration C shows that 3 is the quotient.

To solve $81\div27=n$, you can think: 27 is nearly 30, so the quotient of $81\div27$ should be nearly the same as the quotient of $81\div30$. For $81\div30=n$, $n>2$ but $n<3$ (read > as *is greater than* and < as *is less than*). Thus, use 2 or 3 as a trial quotient.

What would you do if the product of the trial quotient and the divisor were less than the dividend? Be careful in answering this question.

Examples D, E, and F show that you may have to use more than two trial quotients in order to complete the division.

D	**E**	**F**
$\begin{array}{r} 5 \\ 17\overline{)51} \\ \underline{85} \\ ? \end{array}$	$\begin{array}{r} 4 \\ 17\overline{)51} \\ \underline{68} \\ ? \end{array}$	$\begin{array}{r} 3 \\ 17\overline{)51} \\ \underline{51} \\ 0 \end{array}$

Begin the division of 51 by 17 by finding a trial quotient. Use the equation, $5\text{ T} \div 1\text{ T} = 5$. Example D shows that $5 \times 17 = 85$, which is larger than the dividend. So 5 is too large to be the quotient.

In E, 4 is used as a trial quotient, $4 \times 17 = 68$. 68 is also larger than the dividend, so 4 is too large.

In F, 3 is used as the trial quotient. This example shows that 3 is the quotient you want.

Another way to think of division F would be to round off the divisor. Think: 17 is close to 20, and for $51 \div 20 = n$, $n > 2$ but $n < 3$. So you could use either 2 or 3 as a trial quotient.

Using 2 as a trial quotient for $51 \div 17$ leaves a remainder which is equal to the divisor since $2 \times 17 = 34$ and $51 - 34 = 17$. Hence you should use the next larger whole number as a trial quotient.

When using trial quotients, do as much of the multiplication mentally as possible. This will avoid your having to erase or rewrite your work, and you will find that you can do division faster.

Oral What would you use as a trial quotient in each of the following divisions? Check each answer.

	a	*b*	*c*
1.	$18\overline{)54}$	$24\overline{)72}$	$12\overline{)60}$
2.	$13\overline{)78}$	$32\overline{)96}$	$14\overline{)42}$
3.	$15\overline{)75}$	$47\overline{)94}$	$37\overline{)74}$
4.	$41\overline{)77}$	$29\overline{)58}$	$19\overline{)95}$

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$12\overline{)72}$	$14\overline{)56}$	$18\overline{)36}$
2.	$15\overline{)45}$	$18\overline{)72}$	$13\overline{)52}$
3.	$29\overline{)87}$	$38\overline{)76}$	$24\overline{)48}$
4.	$23\overline{)92}$	$29\overline{)58}$	$24\overline{)96}$
5.	$46\overline{)92}$	$27\overline{)54}$	$32\overline{)64}$
6.	$37\overline{)74}$	$28\overline{)84}$	$24\overline{)72}$

MORE PRACTICE PAGE 303

Remainders in Division

Floyd bought 85 stamps for his collection. Each page of his stamp book holds 24 stamps. How many pages can he fill with the 85 stamps? How many stamps will be left over?

To answer the questions asked in this problem, you divide 85 by 24, as shown below.

$$\begin{array}{r} 3\text{ r}13 \\ 24\overline{)85} \\ \underline{72} \\ 13 \end{array} \qquad \begin{array}{r} \text{Check} \\ 24 \\ \underline{\times 3} \\ 72 \\ \underline{+13} \\ 85 \end{array}$$

To find a trial quotient, round off 24 to 20 and think: $85 \div 20 = 4$ with r5. Use 4 as a trial quotient. Multiply mentally, and you find that $4 \times 24 = 96$ which is greater than the dividend. So 4 is too large for the quotient. Notice that $3 \times 24 = 72$. When 72 is subtracted from the dividend, 85, the remainder is 13.

Floyd can fill 3 pages in his book with 13 stamps left over.

To check a division, multiply the divisor by the quotient. Then add the remainder to this product.

Oral Any of the following multiplications could occur while doing a division. Do each one mentally and give the product.

	a	*b*
1.	$5 \times 12 = n$	$3 \times 19 = n$
2.	$2 \times 32 = n$	$7 \times 21 = n$
3.	$6 \times 20 = n$	$4 \times 32 = n$
4.	$8 \times 41 = n$	$9 \times 22 = n$
5.	$2 \times 72 = n$	$6 \times 31 = n$
6.	$7 \times 23 = n$	$4 \times 14 = n$
7.	$8 \times 12 = n$	$3 \times 92 = n$

Written Copy. Find the quotient and the remainder in each of the following divisions.

	a	*b*	*c*
1.	$11\overline{)24}$	$50\overline{)55}$	$27\overline{)58}$
2.	$37\overline{)79}$	$42\overline{)87}$	$31\overline{)99}$
3.	$49\overline{)99}$	$26\overline{)58}$	$19\overline{)39}$
4.	$15\overline{)35}$	$18\overline{)38}$	$44\overline{)89}$
5.	$25\overline{)57}$	$10\overline{)65}$	$34\overline{)69}$
6.	$23\overline{)82}$	$15\overline{)67}$	$38\overline{)77}$
7.	$13\overline{)75}$	$22\overline{)80}$	$33\overline{)91}$
8.	$14\overline{)27}$	$42\overline{)71}$	$15\overline{)58}$

Using Division in Solving Problems

The operation of division is used in solving many problems. Before you begin writing a solution, read the problem carefully to determine which number is the dividend and which number is the divisor.

Solve the following problems. When you write your answer, label it with the kind of units or objects that were asked for in the question in each problem.

1. An airplane traveled 95 miles in 19 minutes. What was its average speed per minute?

2. Jane found that by working steadily she could type 13 business letters in an hour. How long would it take her to type 65 letters?

3. Jack put 75 golf balls into boxes. Each box held 24 golf balls. How many boxes did he fill? How many balls were left over?

4. Cheryl's mother made 95 pieces of chocolate candy. She kept 27 pieces for the family. She divided the rest equally among the 17 children at Cheryl's party. How many pieces did each child receive?

5. Charles took 95¢ to the hobby store. He bought 23 large balloons and received 3¢ change. What was the cost of each balloon?

6. David rode his bicycle 84 miles in 6 days on his paper route. How many miles did he average per day?

7. A grocer sold 26 pounds of potatoes for 78¢. What was the price per pound?

8. The hobby store sells 15 pictures of airplanes for 75¢. What is the cost of one picture?

9. The 78 boys at day camp decided to take turns at putting away equipment and cleaning up the grounds. Day camp lasted 13 days, and the same number of boys helped each day. How many boys helped put away equipment and clean up each day?

10. It cost Mr. Brown 87¢ to mail 29 advertising folders. What was the cost of mailing each folder?

Trial Quotients for Three-Digit Dividends

Sandy's TV Center sold 160 TV sets in the last 40 days. What was the average daily sales?

To find the answer you divide 160 by 40, as shown below. You can show that 4 is the quotient because 40×4=160. But how is the quotient determined when the dividend has three digits?

$$\begin{array}{r} 4 \\ 40\overline{)160} \\ 160 \\ \hline 0 \end{array}$$

The corresponding divisions in column A and column B give the same quotient.

A	B
$160 \div 80 = 2$	$16 \div 8 = 2$
$240 \div 40 = 6$	$24 \div 4 = 6$
$420 \div 60 = 7$	$42 \div 6 = 7$
$320 \div 80 = 4$	$32 \div 8 = 4$
$400 \div 50 = 8$	$40 \div 5 = 8$

Notice in the first row above that $160 \div 80 = 2$ and also $16 \div 8 = 2$. This means that there are as many groups of 80 in 160 as there are groups of 8 in 16. Think of 160 as 16 tens, and 80 as 8 tens. Then $16 \div 8 = 2$ is merely dividing *the number of tens in the dividend* by *the number of tens in the divisor*. This is how you could obtain a trial quotient in such divisions. The equation, $16\text{ T} \div 8\text{ T} = 2$, is a short way of denoting the division of tens.

Study the rest of the rows illustrated in A and B. How is each entry in column A obtained from the corresponding entry in column B?

To solve the division, $160 \div 40$, you can get a trial quotient mentally. Think: There are as many groups of 40 in 160 as there are groups of 4 in 16. Since $16 \div 4 = 4$, try 4 as the quotient. Why is the 4 written in ones place in the quotient?

Oral Use the above method to determine a trial quotient for each division below. Check the trial quotient mentally. Be ready to give the quotient for each division.

	a	*b*	*c*
1.	$60\overline{)120}$	$90\overline{)180}$	$70\overline{)490}$
2.	$70\overline{)280}$	$20\overline{)140}$	$30\overline{)210}$
3.	$50\overline{)350}$	$60\overline{)360}$	$90\overline{)810}$
4.	$40\overline{)200}$	$80\overline{)480}$	$90\overline{)540}$
5.	$50\overline{)500}$	$60\overline{)180}$	$30\overline{)270}$
6.	$40\overline{)120}$	$70\overline{)350}$	$80\overline{)240}$
7.	$20\overline{)160}$	$90\overline{)360}$	$60\overline{)540}$

Dividing Three-Digit Dividends by Two-Digit Divisors

Study the divisions in A and B below. Notice that neither divisor is a multiple of 10.

A

$$\begin{array}{r} 6 \\ 31\overline{)186} \\ 186 \\ \hline 0 \end{array}$$

B

$$\begin{array}{r} 4\text{ r}2 \\ 62\overline{)250} \\ 248 \\ \hline 2 \end{array}$$

Use the division, 18 T÷3 T=6, to find a trial quotient in A. Why is 6 written in ones place in the quotient? 6×31=186 and 186−186=0. Therefore, 6 is the quotient and 0 is the remainder.

To determine the trial quotient in division B, divide 25 T by 6 T. The result will be more than 4 but less than 5. Since 5×62=310, which is more than the dividend, you use 4 as a trial quotient.

Oral How would you find a trial quotient in each of the following divisions?

	a	*b*	*c*
1.	$21\overline{)42}$	$51\overline{)408}$	$41\overline{)205}$
2.	$22\overline{)66}$	$72\overline{)288}$	$52\overline{)156}$
3.	$37\overline{)74}$	$62\overline{)310}$	$47\overline{)376}$

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$73\overline{)219}$	$84\overline{)168}$	$23\overline{)115}$
2.	$63\overline{)252}$	$74\overline{)444}$	$94\overline{)188}$
3.	$21\overline{)105}$	$36\overline{)108}$	$83\overline{)581}$
4.	$37\overline{)222}$	$21\overline{)147}$	$30\overline{)150}$
5.	$54\overline{)324}$	$73\overline{)365}$	$46\overline{)368}$
6.	$87\overline{)522}$	$57\overline{)342}$	$63\overline{)504}$

Copy. Find the quotient and the remainder for each division.

	a	*b*	*c*
7.	$39\overline{)219}$	$47\overline{)430}$	$38\overline{)162}$
8.	$63\overline{)128}$	$72\overline{)217}$	$31\overline{)219}$
9.	$40\overline{)205}$	$59\overline{)330}$	$27\overline{)199}$
10.	$43\overline{)368}$	$27\overline{)180}$	$35\overline{)109}$
11.	$92\overline{)465}$	$25\overline{)165}$	$23\overline{)117}$
12.	$62\overline{)438}$	$31\overline{)249}$	$28\overline{)150}$
13.	$54\overline{)219}$	$92\overline{)276}$	$61\overline{)366}$
14.	$23\overline{)176}$	$37\overline{)185}$	$83\overline{)332}$
15.	$45\overline{)363}$	$88\overline{)594}$	$91\overline{)755}$

Division Practice

You may be able to determine many quotients mentally because of your knowledge of multiplication. If the quotient is not obvious, use a trial quotient.

Part 1 Copy. Find each quotient. If you use a trial quotient, check it mentally before writing it as the quotient.

	a	*b*	*c*
1.	$10\overline{)70}$	$20\overline{)120}$	$50\overline{)250}$
2.	$14\overline{)56}$	$53\overline{)77}$	$23\overline{)83}$
3.	$61\overline{)98}$	$27\overline{)59}$	$36\overline{)75}$
4.	$70\overline{)150}$	$80\overline{)250}$	$30\overline{)150}$
5.	$36\overline{)210}$	$47\overline{)315}$	$58\overline{)277}$
6.	$48\overline{)200}$	$57\overline{)405}$	$82\overline{)290}$
7.	$40\overline{)126}$	$70\overline{)500}$	$63\overline{)218}$
8.	$13\overline{)113}$	$92\overline{)888}$	$47\overline{)303}$

Part 2 Write an equation for each of the following problems. Solve each equation.

1. Mark delivered 216 newspapers on 24 blocks of his route. What was the average number of papers delivered per block?

2. If 259 sheets of art paper are shared equally by 37 pupils, how many sheets will each child receive?

3. When milk sells for 80¢ a gallon, what is the cost of 1 quart?

Part 3 Solve the equations below.

	a	*b*
1.	$56 \div 4 = n$	$208 \div 26 = n$
2.	$315 \div 7 = n$	$738 \div 82 = n$
3.	$4520 \div 8 = n$	$5719 \div 7 = n$
4.	$332 \div 83 = n$	$539 \div 77 = n$
5.	$2274 \div 3 = n$	$4496 \div 8 = n$

Part 4 The divisions below are for review and practice. Copy each exercise, and find the quotient and remainder.

	a	*b*	*c*
1.	$6\overline{)84}$	$8\overline{)96}$	$5\overline{)72}$
2.	$4\overline{)93}$	$3\overline{)64}$	$3\overline{)20}$
3.	$6\overline{)509}$	$4\overline{)319}$	$8\overline{)700}$
4.	$5\overline{)393}$	$7\overline{)79}$	$3\overline{)185}$
5.	$3\overline{)1242}$	$6\overline{)2046}$	$5\overline{)1585}$
6.	$7\overline{)2571}$	$9\overline{)5463}$	$3\overline{)1264}$

More Three-Digit Dividends and Two-Digit Divisors

All the divisions thus far, where two-digit divisors were used, had one-digit quotients. Suppose you were to solve 114÷19=n.

Study the divisions below and the explanations that follow.

A	B
$\begin{array}{r} 9 \\ 19\overline{)114} \\ \underline{171} \\ ? \end{array}$	$\begin{array}{r} 6 \\ 19\overline{)114} \\ \underline{114} \\ 0 \end{array}$

In example A, a trial quotient appears to be 11 since 11 T÷1 T=11. But you know that 19×10=190, and 19×11 would be still larger. When a trial quotient appears to be greater than 9, try 9 as the quotient. If 9 is too large, as it is in A, try the next smaller whole number. If that is still too large, try the next smaller. Example B shows that 6 is the correct quotient.

You would have saved time by thinking of 19 as being near 20, and using 114÷20 or 11÷2 to get a trial quotient.

How can you tell if a trial quotient is too small? Use 5 as a trial quotient in 119÷17. Note the size of the remainder.

The following divisions have two-digit quotients.

$$\begin{array}{r} 20\text{ r}6 \\ 40\overline{)806} \\ \underline{80} \\ 6 \end{array} \qquad \begin{array}{r} 30\text{ r}20 \\ 25\overline{)770} \\ \underline{75} \\ 20 \end{array}$$

Oral Is the first digit in each of the above quotients the same number you would have used as a trial quotient? Why are these two-digit quotients? Why are 0's in the ones place of the quotient?

Written Copy. Find each quotient and remainder.

	a	*b*	*c*
1.	$30\overline{)600}$	$22\overline{)880}$	$13\overline{)390}$
2.	$21\overline{)650}$	$31\overline{)950}$	$12\overline{)480}$
3.	$14\overline{)840}$	$16\overline{)800}$	$18\overline{)720}$
4.	$32\overline{)992}$	$41\overline{)861}$	$21\overline{)672}$
5.	$36\overline{)756}$	$45\overline{)990}$	$16\overline{)496}$
6.	$20\overline{)684}$	$33\overline{)697}$	$42\overline{)885}$
7.	$47\overline{)988}$	$14\overline{)996}$	$24\overline{)986}$
8.	$57\overline{)692}$	$35\overline{)738}$	$42\overline{)506}$

MORE PRACTICE PAGE 303

Which Operation Should You Use?

When you solve written problems, take time to read the problem several times. Decide which operation to use. Express the problem as an equation. Solve the equation and use the result to answer the question asked in the problem. Always read the problem again, to be sure your answer is reasonable.

Use the above suggestions in solving the following problems.

1. Mr. Benson put 92 pairs of socks in a special sale box in his store. That night there were only 27 pairs left. How many pairs of these socks were sold during the day?

2. Mark had delivered 87 papers by the time he got to Robert's house. He still had 49 papers to deliver. How many papers did he have to deliver altogether?

3. The Coffee Shop cuts each pie into 6 pieces. How many people are served dessert from 17 pies?

4. Mr. Johnson sold 3,248 quarts of milk in 7 days. What was his average daily sale of milk?

5. The 59 pupils in two classes made 472 programs. If each child made his fair share, how many programs did each pupil make?

6. Harry was reading a 215-page book about animals. He had read 87 pages. How many pages did he still have to read to complete the book?

7. Jan can buy 3 records with the money she earned last month. Each record costs $1.98. How much did she earn last month?

8. Mary read two books. One book had 178 pages and the other had 97 pages. How many pages did she read in both books?

Can you do this? Study the following multiplications in which 5 is a factor.

$1 \times 5 = 5$ $4 \times 5 = 20$ $7 \times 5 = 35$

$2 \times 5 = 10$ $5 \times 5 = 25$ $8 \times 5 = 40$

$3 \times 5 = 15$ $6 \times 5 = 30$ $9 \times 5 = 45$

What is the ones digit in each of these products? Find other products where one of the factors is 5. Do these products end in 0 or 5 also?

How can you tell whether or not a number is divisible by 5? If a number is divisible by 5, the ones digit of the number is either 0 or 5.

Which of the following numbers are divisible by 5?

13, 20, 56, 75, 151, 1450, 2155

More About Three-Digit Numbers in Multiplication and Division

Suppose you want to solve the equation $4 \times 237 = n$. You could proceed to do the work in column form, as shown at the right. What is the product? Which numbers are called the factors? Partial products?

$$\begin{array}{r} 237 \\ \times 4 \\ \hline 28 \\ 120 \\ 800 \\ \hline 948 \end{array}$$

The work could just as well be shown in equation form, and would probably be easier for you to do. The factor 237 could be expressed as the sum: $200+30+7$. Study carefully the following solution.

$$\begin{aligned} 4 \times 237 &= n \\ 4 \times (200+30+7) &= n \\ (4 \times 200)+(4 \times 30)+(4 \times 7) &= n \\ 800+120+28 &= n \\ 948 &= n \end{aligned}$$

The partial products 800, 120, and 28 appear in both methods. Most of the work in the latter solution could be done mentally.

This approach can also be used in many divisions if you rename the dividend as a sum.

If the dividend contains three digits, as in $375 \div 5 = n$, you could rename 375 as $300+70+5$. Each of the above addends is divisible by 5, and the quotient could be found as shown below.

$$\begin{aligned} 375 \div 5 &= n \\ (300+70+5) \div 5 &= n \\ (300 \div 5)+(70 \div 5)+(5 \div 5) &= n \\ 60+14+1 &= n \\ 75 &= n \end{aligned}$$

Notice that each addend, 300, 70, and 5, was divided by 5. Then these separate quotients were added to get the quotient of $375 \div 5$.

In the equation, $354 \div 3 = n$, rename 354 as a sum such that each addend is divisible by 3. One choice would be $354 = 300+30+24$. Why are these addends a good choice? Then proceed as shown above.

Oral Which number would you rename, and how would you rename it, in each of the following equations?

a	*b*
$4 \times 631 = n$	$372 \div 6 = n$

Written Copy. Solve for n.

	a	*b*
1.	$2 \times 246 = n$	$5 \times 322 = n$
2.	$365 \div 5 = n$	$378 \div 3 = n$
3.	$3 \times 875 = n$	$7 \times 147 = n$
4.	$279 \div 9 = n$	$322 \div 7 = n$

Review of Addition, Subtraction, and Multiplication

You should be able to find each sum, difference, or product correctly without help.

Part 1 Copy. Find each sum.

	a	*b*	*c*	*d*
1.	31 27 +52	7 49 +92	18 7 +63	91 47 +56
2.	132 51 215 +76	572 89 316 +51	772 581 15 +634	921 567 404 +920
3.	854 472 391 507 +804	127 483 546 724 +82	433 755 616 812 +376	320 516 754 483 +197
4.	8172 5436 7820 4117 +9504	1723 1009 2107 5072 +3622	4826 3755 2817 7115 +6827	7472 6412 3050 7601 +8200

Part 2 Copy. Find each sum.

	a	*b*	*c*
1.	23421 45617 70210 +13589	10517 22762 48513 +12987	35912 18472 10008 +27051
2.	$417.52 47.81 +122.46	$876.53 112.48 +74.62	$125.50 237.25 +472.64

Part 3 Copy. Find each difference.

	a	*b*	*c*	*d*
1.	87 −47	53 −27	40 −32	71 −58
2.	212 −81	507 −47	750 −582	476 −87
3.	4345 −434	8472 −856	2105 −609	4000 −491

Part 4 Copy. Find each product.

	a	*b*	*c*
1.	56 ×9	31 ×15	76 ×23
2.	217 ×8	315 ×31	476 ×57
3.	593 ×68	376 ×250	2059 ×8
4.	2107 ×34	4724 ×98	5436 ×143
5.	5406 ×500	6725 ×346	8704 ×807

Part 5 Answer these questions.

1. Name the sum in 15+6=21.

2. Name the dividend in 63÷7=9. What is the quotient?

3. Name the minuend in 72−37=35. Name the subtrahend.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Divide the number of tens in the dividend by the number of tens in the divisor to find a trial quotient. (98, 102, 103)

2. A trial quotient may be obtained by rounding the divisor to the nearest multiple of 10. (98, 99, 105)

3. When the trial quotient appears to be 10 or more, use 9. (105)

4. A sum may be divided by using the distributive property of division over addition. (97)

Words to Know

Trial quotient, is greater than, > (98)

Questions to Discuss

1. If the trial quotient you choose is too large, how do you proceed? (98, 99, 105)

2. How can you tell the place value for the trial quotient? (94, 98, 102, 103)

3. Why is it a good idea to check a trial quotient mentally? (99)

4. What is a good way to check a division when the remainder is different from 0? (100)

5. How do you find a trial quotient for $375 \div 62$? For $375 \div 69$? (103)

6. How can you tell whether or not the trial quotient is actually the quotient you want? (98)

7. What steps can you give for working written problems? (101, 106)

Written Practice

Copy. Find each quotient and remainder.

	a	*b*	*c*	
1.	$30\overline{)85}$	$50\overline{)97}$	$20\overline{)64}$	(94)
2.	$14\overline{)29}$	$53\overline{)57}$	$32\overline{)97}$	(95)
3.	$26\overline{)52}$	$37\overline{)74}$	$47\overline{)94}$	(99)
4.	$60\overline{)240}$	$70\overline{)420}$	$80\overline{)560}$	(102)
5.	$53\overline{)427}$	$26\overline{)173}$	$73\overline{)532}$	(103)
6.	$47\overline{)546}$	$37\overline{)921}$	$24\overline{)648}$	(105)

Copy. Solve for n.

	a	*b*	
7.	$84 \div 4 = n$	$81 \div 3 = n$	(97)
8.	$95 \div 5 = n$	$68 \div 2 = n$	(97)
9.	$228 \div 76 = n$	$324 \div 36 = n$	(104)

Self-Evaluation

Part 1 Copy. Find a replacement for n which will make each of the following sentences true.

	a	*b*
1.	$80 \div 20 = n$	$n = 42 \div 21$
2.	$85 \div 5 = n$	$n = 132 \div 6$
3.	$72 \div 24 = n$	$n = 84 \div 14$
4.	$480 \div 12 = n$	$n = 0 \div 27$
5.	$780 \div 26 = n$	$n = 950 \div 50$

Part 2 Copy. Find each quotient and remainder.

	a	*b*	*c*
1.	$20\overline{)80}$	$30\overline{)65}$	$10\overline{)26}$
2.	$43\overline{)86}$	$31\overline{)72}$	$56\overline{)73}$
3.	$17\overline{)57}$	$41\overline{)80}$	$12\overline{)96}$
4.	$37\overline{)92}$	$25\overline{)87}$	$13\overline{)65}$
5.	$42\overline{)93}$	$29\overline{)90}$	$15\overline{)63}$
6.	$30\overline{)210}$	$40\overline{)280}$	$70\overline{)420}$
7.	$28\overline{)105}$	$23\overline{)111}$	$47\overline{)199}$
8.	$35\overline{)738}$	$17\overline{)291}$	$82\overline{)172}$
9.	$3\overline{)1824}$	$4\overline{)3056}$	$7\overline{)3007}$

Part 3 Copy. Perform the indicated operations.

	a	*b*	*c*
1.	$\begin{array}{r}321\\ \times 7\\ \hline\end{array}$	$\begin{array}{r}576\\ \times 22\\ \hline\end{array}$	$\begin{array}{r}892\\ -576\\ \hline\end{array}$
2.	$\begin{array}{r}5604\\ +7297\\ \hline\end{array}$	$\begin{array}{r}6725\\ \times 42\\ \hline\end{array}$	$\begin{array}{r}9682\\ -897\\ \hline\end{array}$
3.	$\begin{array}{r}376\\ \times 85\\ \hline\end{array}$	$\begin{array}{r}9857\\ +3496\\ \hline\end{array}$	$\begin{array}{r}8002\\ -7915\\ \hline\end{array}$
4.	$\begin{array}{r}4070\\ \times 705\\ \hline\end{array}$	$\begin{array}{r}4444\\ +6789\\ \hline\end{array}$	$\begin{array}{r}5009\\ \times 416\\ \hline\end{array}$

Part 4 Write the correct answer for each of the following questions.

1. In the equation, $52 \times 121 = 6292$, which numbers are the factors?

2. What is the most convenient way to check your work in a division?

3. In the equation, $58 - 27 = 31$, which number is the subtrahend?

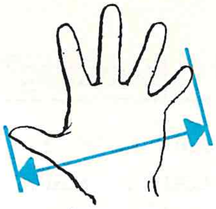
4. What is the inverse of dividing by 12?

5. Would the quotient of $321 \div 47$ contain one, two, or three digits?

6. How do you find a trial quotient for $231 \div 47$?

7. How many possible remainders are there when 12 is the divisor?

7



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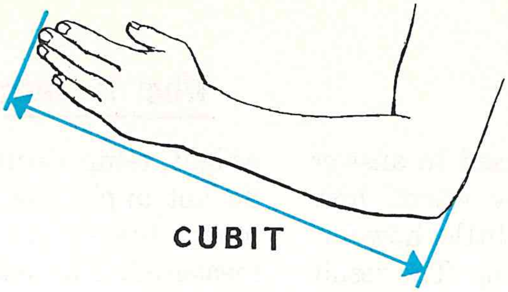

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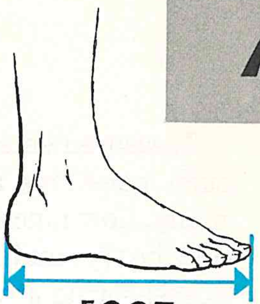


FOOT

The Importance of Measurement

Measurement is so often used in our everyday lives that we are apt to overlook its importance. Men have always measured the objects in the world about them.

In ancient times man used units of measure that compared to parts of his body. Three of these units of measure are shown above.

A *span* was the distance from the tip of the thumb to the tip of the little finger when the fingers were widespread.

Measure the length of your desk in spans. Was the length of your desk an exact number of spans? If your teacher measured your desk in spans, would you expect her to get the same answer? Why or why not?

A *cubit* was the distance from the elbow to the tip of the middle finger when the hand was outstretched. Is your measure of a cubit the same as that of your father? Why? Were these good units to use? Why?

The names of some old-time units of measure are still used today. One of the most common terms is *foot.* It originated as the length of a person's foot. Not everyone had the same size foot and arguments arose about the lengths of objects.

Unfortunately, the ancient units of measure had no particular relationship to each other. As a man's knowledge increased, there was a need to have each unit of measure mean the same thing to more and more people.

Standard units of measure are now defined by law. Units are set by the United States Congress or by the individual states.

Even today, with missiles, rockets, and space travel, man is in need of new units of measure, especially to measure very small quantities and very large quantities.

Something to do Use a dictionary to find the meaning of *fathom, furlong, fourscore,* and *league.*

What Is Meant by Measuring?

Measurements are used to answer such questions as how warm, how many, how much, how little, how far, how heavy, and how long. The result of measuring any quantity is a number of units. To express the measurement, write the number of units and the kind of units, such as 3 feet or 8 hours.

You usually read the number of units from a measuring instrument like a scale, clock, ruler, thermometer, or meter. Can you name other measuring instruments?

In daily life you constantly use both direct measurement and indirect measurement.

Direct measurement is used whenever you apply the measuring instrument directly to the quantity being measured. For example, when you measure the length of a room in feet, you apply a ruler directly to a line on the floor to determine how many times the length of the ruler is contained in the length of the room. In a similar way, when you measure an amount of water in cups, you can actually use a measuring cup to find the number of cups of water in the container.

Indirect measurement is used when measuring such things as time, weight, temperature, and speed. You do not apply the measuring instrument directly to the thing being measured. For example, when you step on a scale the force (gravity) pulling you toward the earth causes the scale platform and gears to move. This in turn makes the scale dial show that numeral which represents your weight in pounds.

Can you name other instruments for indirect measurement?

Oral Would you use direct or indirect measurement to measure each of the following?

1. Your height

2. The temperature in Paris

3. The weight of a book

4. The speed of an automobile

5. The distance in miles from the earth to the sun

6. How long it takes a satellite to go around the earth

7. How many gallons of oil in a barrel of oil

8. The time which has passed since you had breakfast

9. The age of a person

Approximate Nature of Measurement

To count the objects in a set, you need only the whole numbers or counting numbers. If you count the number of people in your room, the result will be a whole number. There might be 24 people, not $24\frac{1}{2}$ people or $23\frac{1}{4}$ people.

When you measure something, the situation is different. Use a ruler to measure the length of the line segment below.

Is it exactly 2 inches long? 3 inches? Is the length closer to 2 inches or 3 inches? What is the length of the line segment to the nearest inch?

Now use the half-inch marks on your ruler to measure the line segment above. What is its length to the nearest $\frac{1}{2}$ inch? Which unit of measure gave a more nearly exact length of the line segment?

When measuring the line segment above, the endpoint of the segment appears to be at the $2\frac{1}{4}$-inch mark on the ruler. However, if you looked at it through a magnifying glass, this would not be quite true.

In fact, the measurement of any object is always approximate. You could express the length of your room to the nearest yard, nearest foot, or nearest inch. Which of these measures gives the length of the room most precisely?

Measurement is used with all kinds of quantities. You state your weight to the nearest pound, or the nearest ounce, but never exactly. You give a better indication of your true weight by using the smaller unit.

The smaller the unit, the greater is the **precision** of the measurement. Measuring to the nearest inch gives greater precision than measuring to the nearest foot. Measuring to the nearest second gives greater precision than measuring to the nearest minute.

What units of distance give greater precision than a mile?

Oral Use a ruler to measure the length of the line segment below.

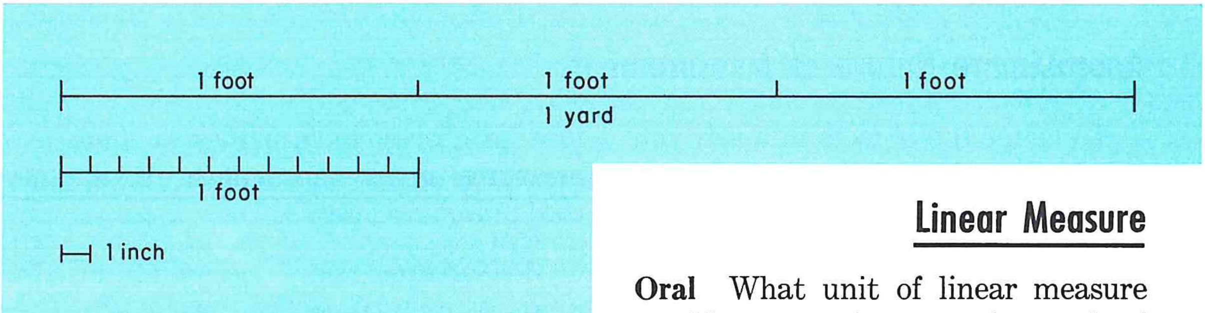
---

1. What is its length to the nearest inch?

2. What is its length to the nearest $\frac{1}{2}$ inch?

3. What is its length to the nearest $\frac{1}{4}$ inch?

4. Do you think you could ever give its length exactly?



Linear Measure

You are already familiar with the most commonly used units of linear measure—the foot and the yard. These are called units of **linear measure** because you measure in a straight line, such as along the edge of this page.

If the length of an object is 1 yard, its length may also be given as 3 feet. To say 1 yard = 3 feet is a short way of saying that 1 yard measures the same length as 3 feet.

Whenever the sign of equality (=) is used with measurements, it means ***measures the same as.***

Study the following relationships of the linear measure units.

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12 inches (in.) = 1 foot (ft.)
36 in. = 1 yard (yd.)
3 ft. = 1 yd.
$16\frac{1}{2}$ ft. = 1 rod (rd.)
$5\frac{1}{2}$ yd. = 1 rd.
5280 ft. = 1 mile (mi.)
1760 yd. = 1 mi.
320 rd. = 1 mi.

Oral What unit of linear measure would you use in measuring each of the following?

1. The height of a telegraph pole

2. The distance from your home to Fairbanks, Alaska

3. The width of your classroom

4. The length of your pencil

5. The distance from the earth to the moon

6. The width of a football field

7. The thickness of a brick

Written Copy. Write a numeral in each blank so that each statement becomes true.

1. 2 ft. = ___ in.

2. 57 ft. = ___ yd. = ___ in.

3. 3 ft. 2 in. = ___ in.

4. 3 yd. 2 ft. = ___ ft.

5. 1 yd. 1 ft. 3 in. = ___ in.

6. 4 mi. = ___ rd. = ___ ft.

7. 6 rd. = ___ yd. = ___ ft.

Estimation in Measurement

When you state or record a measurement, be sure to give the unit used. For instance, if you say your desk is 30, someone might wonder if you mean 30 inches, 30 feet, 30 years, or 30 pounds.

It is good to have a visual concept of distance. This means that you should be able to estimate the measure by just looking. This takes a lot of practice. The man in the meat market can take a quarter of beef and cut off a 3-pound roast with very little error. He has had much experience with the weight of meat.

Oral Estimate each of the following distances as indicated. Record your estimates.

1. Length of your arithmetic book in inches

2. Height of your desk in inches

3. Width of a window in feet

4. Length of your room in yards

5. Height of the classroom door in feet

6. Width of your room in feet

Measure each of the above. Compare your results with the estimates you have already recorded.

Written Solve each problem.

1. The boys ran a 50-yard race. How many feet did they run?

2. Don is 4 feet 7 inches tall, and Ed is 58 inches tall. What is the difference in their heights?

3. Mr. Roberts bought a board 16 feet long and cut it into pieces each 32 inches long for book shelves. How many shelves did he make?

4. It is 2 miles from Linda's house to the new supermarket. This is how many rods? Yards? Feet?

5. Mrs. Jacobs had 12 yards of clothesline rope. She cut it into 18-foot lengths. How many lines did she make?

6. William jumped 1 yard 8 inches and Melvin jumped 43 inches. Who jumped farther?

7. Roger had 5 ft. 8 in. of rope, Bill had 3 ft. 7 in., and Dennis had 4 ft. 4 in. How much rope did they have altogether?

8. Which is longer, a distance of 86 ft. or 29 yd.?

Something to do Collect pictures or drawings of measuring instruments, new and old, for a bulletin board display.

Liquid Measure

There are two systems of **liquid measure** in common usage today. The system you will discuss in this lesson has units called fluid ounces, cups, pints, quarts, and gallons. It is used to measure the amounts of liquids such as gasoline, milk, oil, paint, or water.

The other system is called the **metric system**; it is used by scientists to measure such things as amounts of medicine, drugs, and chemicals. You will study this system of measures as you need it for the study of science. It has become the most universal system of measures, not only for liquids, but for other kinds of things as well.

Study the relationships of the units of liquid measure below.

16 fluid ounces (fl. oz.) = 1 pint (pt.)
2 cups (c.) = 1 pt.
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.)

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You realize that in this system of measures, the word *cup* means a measuring cup like the one your mother may have in her kitchen for baking. When someone has a cup of tea, he usually has less than a measuring cup of liquid.

Fluid ounces are used in packaging many items you can find in stores. Soft drinks, ink, liquid soap, vanilla extract, and liquid shoe polish are examples of items measured in fluid ounces.

Oral Which of the units of liquid measure would you use to measure each of the following?

1. Amount of gasoline in a loaded transport truck

2. Amount of juice in an orange

3. Amount of milk you drink in one day

4. Amount of lemonade in a jar

Written Copy. Write a numeral in each blank so that each statement becomes true.

	a	*b*
1.	4 gal. = ___ qt.	12 pt. = ___ qt.
2.	1 gal. = ___ pt.	1 qt. = ___ cups
3.	1 cup = ___ fl. oz.	5 gal. = ___ pt.
4.	32 fl. oz. = ___ pt.	48 pt. = ___ qt.
5.	80 pt. = ___ gal.	3 gal. = ___ cups
6.	20 qt. = ___ pt.	1 gal. = ___ fl. oz.
7.	7 gal. = ___ qt.	216 qt. = ___ gal.

Dry Measure

A system of *dry measure* was developed for many of the crops that man raises. Of course, all of these have weight and could be measured by units of weight.

In a grocery store, you may be able to buy bananas by the bunch, by the pound, or by the dozen. Which do you think is the best way of selling them? Why?

You might be able to buy strawberries by the dozen, by the pound, or by the quart. Which unit do you think is best to use? Why?

The pint, quart, peck, and bushel are units of dry measure. In the United States a dry quart is a larger amount than a liquid quart.

Below is a chart showing the relationships of the common units of dry measure.

2 pints (pt.) = 1 quart (qt.)
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

A farmer sells his grain or fruit crops by the bushel. Bushels of different things do not weigh the same amount. For instance, in many states a bushel of apples weighs 50 pounds, a bushel of wheat weighs 60 pounds, and a bushel of potatoes weighs 60 pounds. Certainly a truckload of wheat is not measured by continuously filling a bushel basket. Instead, the load is weighed. This weight is then divided by 60 to determine the number of bushels.

Oral Answer the following.

1. What examples can you give of dry measures used in your home?

2. What examples can you give of the use of the peck measure?

3. Why is there a difference in the weight of a bushel of different things?

Written Copy. Write a numeral in each blank so that each statement becomes true.

	a	*b*
1.	8 bu. = ___ pk.	12 pt. = ___ qt.
2.	56 qt. = ___ pk.	21 pk. = ___ qt.
3.	1 pk. = ___ pt.	112 pt. = ___ pk.
4.	232 pk. = ___ bu.	82 qt. = ___ pt.
5.	160 qt. = ___ bu.	9 bu. = ___ qt.
6.	37 pk. = ___ qt.	210 bu. = ___ pk.
7.	14 pk. = ___ pt.	480 qt. = ___ bu.
8.	2 bu. = ___ pt.	48 pt. = ___ pk.

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Solve each of the following problems, and give your answers in the appropriate units. Refer to the relationships between the units of measure on the preceding pages if necessary.

1. The Morris family uses 6 gallons of milk each week. What is their weekly milk bill if milk costs $.22 a quart?

2. Marlys canned one peck of peas in pint jars. How many jars of peas did she can?

3. Mr. King bought 5 gallons of motor oil for his farm tractor. He used 5 quarts for each oil change. How many times can he change oil with the 5 gallons?

4. When a car travels at a speed of 30 miles an hour, how many yards per hour is it traveling?

5. How many pints of milk are contained in 15 gallons of milk?

6. During a day Richard drank 1 pint of milk, 1 quart of water, 1 cup of lemonade, and 1 cup of chocolate milk. How many quarts of liquid did he drink?

7. If a cow gives 5 gallons of milk per day, she gives how many quarts per day? How many pints?

8. If you walked 15 blocks, each 350 feet long, have you walked more or less than one mile? How much more or less than one mile?

9. Mr. Smith's garden is 4 rods long and 15 yd. wide. How many feet of fence will he need to enclose the garden?

10. Mrs. Sawyer bought 2 qt. of raspberries, 1 bushel of apples, and 3 pecks of potatoes. How many quarts of food did she buy?

A quick review Copy. Write a numeral in each blank so that each statement becomes true.

1. 4 gal. 2 qt. = __qt.

2. 15 qt. 3 pt. = __pt.

3. 7 yd. 2 ft. = __ft.

4. 2 yd. 2 ft. 7 in. = __in.

5. 1 rod $4\frac{1}{2}$ yd. = __yd.

6. 4 bu. 36 pk. = __bu.

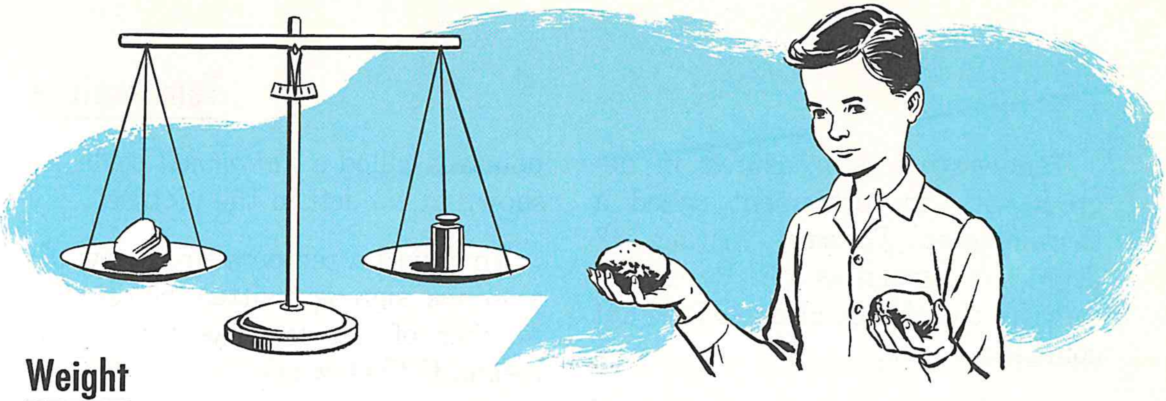
7. 2 mi. 1100 yd. = __yd.

8. 1 gal. 1 qt. = __cup

9. 216 in. = __ft.

10. 39 in. = __ft. and __in.

11. 2175 yd. = __mi. and __yd.



Weight

The first measurement of weight was probably the comparison of two objects by placing one in each hand, and guessing which was heavier. Next, someone got the idea of balancing objects by using a rod or stick, rather than by hand. This idea of balancing is still used in science laboratories.

A *weighing scale* is an instrument which indirectly indicates the weight of an object.

The units of weight, as given below, were chosen arbitrarily.

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 ton (T.)

Oral Answer each question below.

1. What are some things measured in ounces? In pounds? In tons?

2. When might a unit of weight less than an ounce be used?

Written Copy. Fill each blank with a numeral which makes the statement true.

1. 1 lb. 11 oz. = __oz.

2. 112 oz. = __lb.

3. 169 oz. = __lb. and__oz.

4. 5 tons = __lb.

5. 2 tons = __cwt.

6. 5615 lb. = __T. and__lb.

7. 3 T. 5 cwt. = __lb.

Solve each of these problems.

8. A truck loaded with grain weighed 7856 pounds. The empty truck weighed 4837 pounds. What was the weight of the grain?

9. A box of pepper was labeled "Contents, 2 oz." One pound of pepper will fill how many boxes?

10. Mr. Bowman sold a 500-pound calf for $23.25 a cwt. How much money should he receive?

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Temperature

Temperature is measured in degrees with an instrument called a thermometer. To save writing, 47 degrees is written as 47°. Be careful to write the abbreviation so it is not confused with 0.

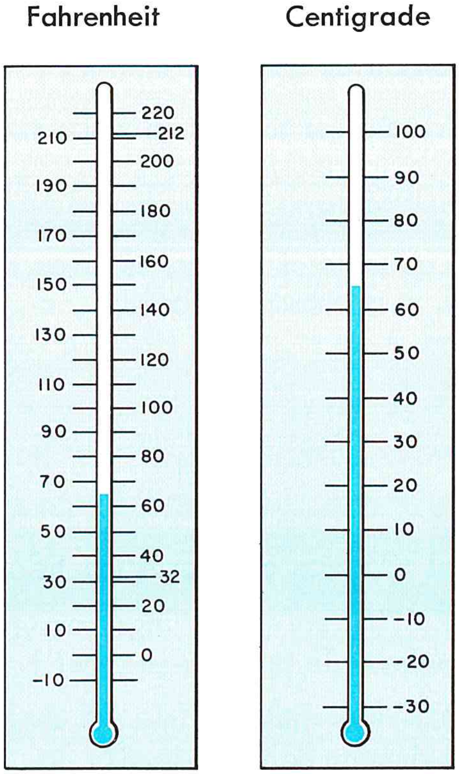
You have used two types of thermometers. One is made from a glass tube enlarged at the bottom into a bulb and partly filled with liquid. The liquid is either mercury or red-colored alcohol. As the temperature rises, the liquid expands and is forced up into the very small opening in the tube. This type thermometer is used when a doctor takes your body temperature.

In the other type, strips of two different metals, probably iron and brass, are joined together to make but one strip. As the temperature changes, one metal expands faster than the other and causes the strip to bend, which in turn moves a pointer on the thermometer dial. This type is used in some desk thermometers and in thermostats.

In making a thermometer, man had to pick some temperatures as reference points. The *freezing point* of water is labeled 32°, and the *boiling point* of water is labeled 212°. A scale with these temperatures as fixed points is called a *Fahrenheit* scale, as shown at the left in the picture.

To record a temperature below 0°, a minus sign is written before the number of degrees, like this: −15° means 15° below zero.

Thermometers measure the temperature of different things. You may want a thermometer with a scale for high temperatures, for low temperatures, or for both high and low temperature readings.



Oral Answer each of the following questions using the Fahrenheit scale shown on page 120.

1. Do you think the thermometer pictured is for outdoor or indoor use? Could it be used for both?

2. What temperature readings would be necessary on a cook's thermometer? A clinical thermometer?

3. What temperature does the thermometer in the picture show?

Written Solve each problem below.

1. The weather forecast stated that the high temperature would be 72° and the low temperature 54°. What is the difference between the predicted high and low?

2. Homes should be kept at a temperature near 68°. This is how many degrees above freezing?

3. One January morning the temperature in San Diego, California, was 62° while in Bismarck, North Dakota, it was −28°. What was the difference in temperature?

4. At noon the temperature was 57°. By 6 o'clock the temperature had dropped 19°. What was the temperature at 6 o'clock?

5. Your body temperature is normally about 99°. How much warmer is your body than freezing water?

Another way Temperature is also read from a *Centigrade* scale. This is part of the metric system of measurement used by scientists.

In the Centigrade system, water freezes at 0° and water boils at 100°. This is easier to remember than our usual Fahrenheit system.

42° Centigrade is written 42°C., and 17° Fahrenheit is written 17°F. In the United States, if neither letter is used, it usually means Fahrenheit temperature.

Oral Use the Centigrade scale pictured on page 120 in answering the following questions.

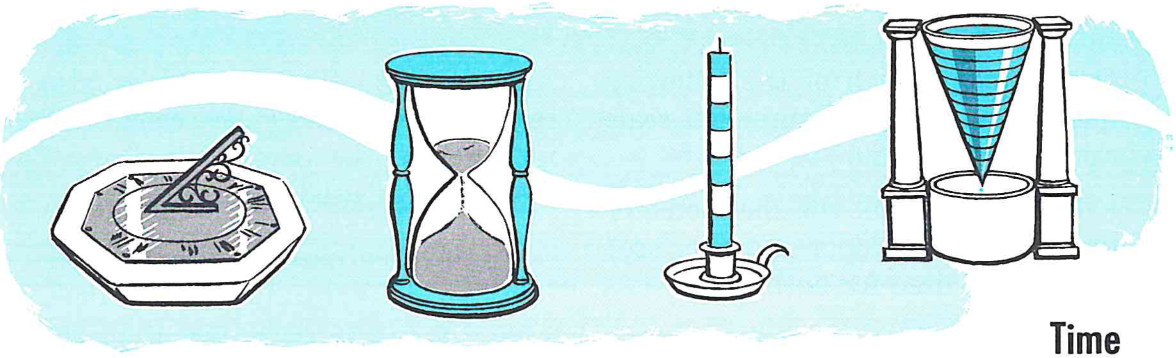
1. What reading does the Centigrade scale in the picture show?

2. Which would be warmer, water at 30°F. or water at 30°C.?

3. If the temperature in your room was 68°, was the temperature read from a Fahrenheit or a Centigrade thermometer?

4. A thermometer in a bucket of ice had a temperature reading of 32°. Was it a Centigrade or a Fahrenheit thermometer? Why?

Something to do Use a Fahrenheit scale to keep a record of the temperature at the same time each day for one week.



Time

Time has always been important to man. In ancient times man measured time by *sunrises, suns, moons,* or *sleeps*. Eventually, crude forms of clocks were invented to measure shorter periods of time. Among these were a sundial, an hourglass with sand, a marked candle, and a water clock.

What are the disadvantages of each of the above-named timepieces?

As man's knowledge increased, he invented gears, he discovered the laws of operation of gears and pendulums, and he found use for electricity. Today people have elaborate and precise instruments for measuring time. The measure of radar and radio waves is done to the nearest millionth of a second.

Most of the present units of time are defined according to the movement of the earth among the stars. The time it takes the earth to complete its orbit is called a *year,* and the time it takes the earth to make one rotation on its axis is called a *day*. From these basic units, the measures of time have developed.

60 seconds (sec.) = 1 minute (min.)
60 min. = 1 hour (hr.)
24 hr. = 1 day (da.)
7 da. = 1 week (wk.)
52 wk. = 1 year (yr.)
365 da. = 1 yr.
366 da. = 1 leap year
12 months = 1 yr.
10 yr. = 1 decade
100 yr. = 1 century

Astronomers have used precise measurement to determine one year to be 365 da. 5 hr. 48 min. and 46 sec. Such an interval of time cannot be shown on the calendar.

Instead, the calendar shows 365 days per year except for every fourth year, which has 366 days and is called a leap year. Most of the leap years are determined by the following rule: When the number of the year is divisible by 4, the year is called a leap year.

Among most Western peoples, centuries are counted from the birth of Christ. The time before Christ's birth is called B.C., and the time after his birth is called A.D. The year 200 B.C. means 200 years before Christ's birth. The year A.D. 80 means 80 years after his birth and is in the 1st Century. You are living in the 20th Century which began January 1, 1901.

Oral Answer each of the following questions.

1. Which months have 30 days? 31 days? 28 or 29 days?

2. Describe a situation where you would measure time in seconds. In minutes. In hours. In years.

Written Solve each of the following problems. Use the chart showing the units of time if necessary.

1. John began mowing the lawn at 1:20 p.m. and finished at 3:40 p.m. How long did it take him to mow the lawn?

2. The Burns family left home at 10:20 a.m. for a trip to the zoo. They got to the zoo at 1:15 p.m. How long did the trip take?

3. Mary practices her flute for 36 minutes each school day. How many hours does she practice her flute each week?

4. A car is traveling at an average rate of 30 miles an hour. How far will the car go in 4 hours?

5. If Mr. Benson walks at the rate of 50 yd. a minute, how long will it take him to walk 200 yd.?

6. Which indicates the greater speed, 25 yd. per minute or 85 ft. per minute?

7. If a jet flies at the speed of 420 miles an hour, it travels how many miles each minute?

We speak so often of time units that it is well for you to be able to convert from one kind of time unit to another. Use the table on the preceding page if necessary in completing these statements.

8. 7 min. = ___ sec.

9. 5 min. 42 sec. = ___ sec.

10. 3 hr. = ___ min.

11. 2 hr. 52 min. = ___ min.

12. 8 yr. = ___ mo.

13. 13 yr. 10 mo. = ___ mo.

14. 1 wk. = ___ hr.

15. 3 decades = ___ mo.

16. 160 yr. = ___ decades

17. 7 yr. 16 da. = ___ da.

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Money and Making Change

The development of money systems has been closely related to systems of measurement. In fact, a money system measures the value of the things people have and use.

The first coins were actually marked according to the value of the metal they contained. A gold coin was worth more than a silver coin of the same size. Under such conditions, there were coins of all sizes and of various materials.

Our coins today have a value greater than the metal they contain. For example, a dime contains less than 10 cents worth of silver. Paper money is valuable only because the government promises to pay its value to the bearer.

A country without a money system has difficulty trading with other countries. The government defines the basic money unit and issues currency accordingly.

People who travel in foreign countries, or do business with firms in foreign countries, must learn the value of foreign currency as compared with American money.

You will constantly use money and give or receive change. For example, at the store you buy a loaf of bread for 29¢ and give the clerk $1.00. When you are given the change, the clerk says, "Thirty, forty, fifty, one dollar."

Which operation did the clerk use in figuring your change? What coins did he give you as change?

Written Find the answer to each of the following by the counting method, and then by using subtraction.

1. Doris bought a sweater for $3.70 and gave the clerk $10.00. How much change did she receive?

2. If you buy a book for $1.95 and a pen for $.98, how much change should you receive from $5.00?

3. While at the Hobby Store, Gene bought a model airplane for $.35, 4 flashlight batteries at $.13 each, and a tube of glue for $.19. How much change did he receive from the $2.00 that he gave the clerk?

4. Which is the greater amount: 3 dimes, 5 nickels, and 7 pennies, or 2 quarters and 1 dime?

Something to do Use the dictionary, encyclopedia, or almanac to find the value of each of the following foreign money units and the country in which each is used.

peso, franc, shilling, ruble

Using Measures

Copy. Find a replacement for n so that each statement is true.

	a	b
1.	72 ft. = n yd.	65 lb. = n oz.
2.	420 min. = n hr.	5 mi. = n ft.
3.	15 yr. = n mo.	8 da. = n hr.
4.	13 gal. = n qt.	168 in. = n ft.
5.	216 in. = n yd.	21 bu. = n pk.
6.	64 pk. = n bu.	720 sec. = n min.
7.	6 cups = n fl. oz.	37 wk. = n da.
8.	336 oz. = n lb.	6 T. = n lb.
9.	17 pt. = n cups	49 yd. = n ft.
10.	1600 rd. = n mi.	7 mi. = n yd.
11.	400 pt. = n gal.	3 cwt. = n lb.
12.	19 qt. = n cups	26 pk. = n qt.
13.	1 wk. = n hr.	14 yr. = n wk.
14.	1 hr. = n sec.	288 qt. = n bu.
15.	1 T. = n oz.	3906 ft. = n yd.
16.	320 pk. = n bu.	288 hr. = n da.
17.	144 oz. = n lb.	516 qt. = n gal.
18.	248 pt. = n gal.	17 mi. = n ft.
19.	700 lb. = n cwt.	384 in. = n ft.

Solve the following problems and record your answer with the correct unit of measure.

1. John's father earns $730 each month. What is his salary for a one-year period?

2. Robert ran around the school track in 1 min. and 35 sec. George ran the same distance in 99 sec. Who ran the faster?

3. Shirley went out to play at 3:45 p.m. Her mother said she could play until 5 p.m. How much time does Shirley have for play?

4. Carolyn saves $.15 a week from her allowance. How much does she save during one year?

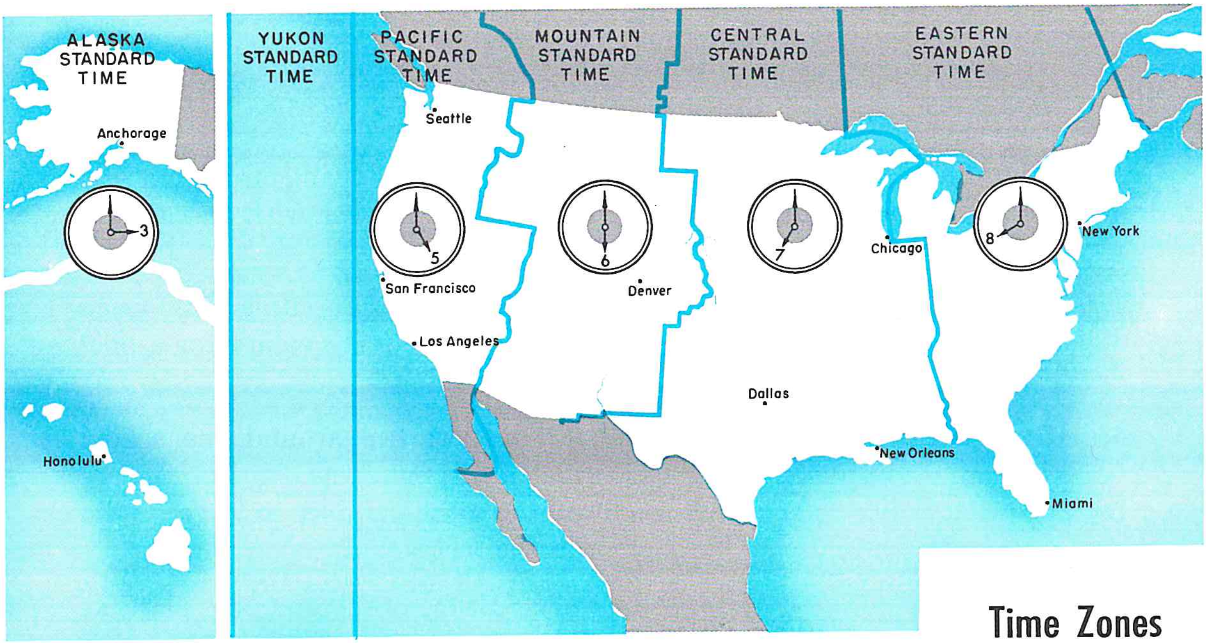
5. Sandra's mother sent a gallon of lemonade to the picnic. How many cups of lemonade did she send?

6. If 4 oz. of meat is used for one sandwich, how many sandwiches can be made from 2 lb. of meat?

7. How many quart jars will Mrs. Thompson need to can 5 bushels of tomatoes?

8. The postman told Merlin that the postage for his package would be 2¢ an ounce. The package weighed 1 lb. 5 oz. How much did it cost Merlin to mail the package?

Time Zones



What time is it when the sun appears at its highest point? Knowing the exact moment of noon is important in keeping time.

Imagine a curved line in space beginning at the north pole, passing directly overhead, and ending at the south pole. When the sun appears to cross this line, it is noon, or midday, or 12 o'clock noon. If the sun is east of this line, you say it is forenoon, or a.m., and when the sun is west of this line, you say it is afternoon, or p.m.

Is it noon at exactly the same instant in all parts of the world? How many hours are between noon today and noon tomorrow? How long does it take the earth to make one rotation on its axis?

Instead of having everyone set his watch when the sun appears at its highest point, man has imagined the places on earth as being separated into 24 separate areas. Each area corresponds to an hour of the day and is called a **time zone**.

People in a certain time zone set their watches to read the same time. In the next time zone, the time will be different by one hour.

The time zones are named. Alaska and Hawaii both use Alaska Standard Time, which is 2 hours earlier than Pacific Standard Time.

The clocks pictured on the map show the standard time in each time zone when it is 8 o'clock Eastern Standard Time.

To record a given time, you can use numerals with a colon separating the number of hours from the number of minutes. For instance, four fifteen can be written as 4:15; and 10 minutes before 2 o'clock can be written as 1:50. Why?

Oral Use the map on the preceding page to answer these questions.

1. When traveling west, do you set your watch ahead or back when entering a different time zone?

2. Would you have to reset your watch in traveling from Los Angeles to Seattle?

3. What time is shown now by your classroom clock? What time is it in Denver? In Honolulu?

Written Determine the correct answer for each question below.

1. A program is televised in New York at 6 p.m. Pacific Standard Time (PST). What is the Mountain Standard Time (MST) when people in Denver see the program?

2. If you lived in Dallas and wanted to call a friend in Seattle, at what time should you call in order to reach him at noon his time?

3. A plane left Chicago at 3:30 p.m. CST. It flew for 3 hours and landed at Denver. What time was it in Denver when the plane landed?

4. The President of the United States is scheduled to speak before the United Nations in New York at 10 a.m. EST. At what standard time would you hear it?

5. A fast jet plane left New York at 5 a.m. EST and landed in San Francisco at 7 a.m. PST. How many hours did it take the jet to complete the flight?

Write each of the times given below by using numerals.

	a	*b*
6.	Five thirty	Six forty
7.	Two twenty	Five ten
8.	Ten forty-three	One o'clock

Write each of the following in a shorter form by using numerals.

9. Half past twelve

10. 12 minutes after 11

11. 15 minutes before 3

12. 47 minutes after 8

13. 23 minutes before 6

14. 10 minutes before 1

15. A quarter of 9

Can you do this? Could it be Sunday in one time zone and Monday in the next? Describe the sun's position when this occurs.

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Review of Basic Operations

Part 1 Copy. Find each sum.

	a	*b*	*c*
1.	7026 +877	4972 +805	5873 +2567
2.	2871 3367 +5498	4354 5851 +2622	6752 2791 +8057
3.	7496 58520 70705 +2619	81354 9099 56452 +15118	70098 6907 20703 +57668

Part 2 Copy. Find each product.

	a	*b*	*c*
1.	3720 ×5	4271 ×8	9726 ×7
2.	764 ×25	893 ×64	475 ×89
3.	4763 ×36	7684 ×47	8675 ×56

Part 3 Copy. Find each difference.

	a	*b*	*c*
1.	4805 −36	3472 −2756	4090 −876
2.	7080 −5765	4009 −399	2003 −1986
3.	2134 −678	4723 −2090	3076 −809

Part 4 Copy. Solve each of the following equations.

	a	*b*
1.	$560 \div 2 = n$	$1736 \div 7 = n$
2.	$544 \div 32 = n$	$966 \div 46 = n$
3.	$(5 \times 0) + n = 13$	$n + 8 = 26$
4.	$n - (2 \times 4) = 7$	$n \times 5 = 75$
5.	$8 + n = 21$	$15 + n = 37$
6.	$32 \times n = 352$	$540 \div n = 36$
7.	$(5 - 2) + n = 17$	$(2 \times 3) \times n = 42$
8.	$47 - n = 19$	$n \times n = 16$
9.	$13 + n = 37$	$n \div 4 = 17$
10.	$19 = 7 + (2 \times n)$	$33 + n = 49$

Part 5 Solve these problems.

1. Judith sold 57 tickets for the play, Betty sold 39 tickets, and Joan sold 42 tickets. What was the average sale for the three girls?

2. James worked for three hours at $.70 for each hour. How much money did he earn?

3. Ralph saved $2.67 and still needed $1.88 in order to buy a new baseball glove. What was the price of the glove?

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The result of measuring any physical quantity is a number of units of measure. (112)

2. Choose the unit of measure according to the quantity and the precision with which you want to measure. (113, 114, 116)

3. Whenever you use the sign of equality (=) with measurements, this sign means *measures the same as.* (114)

4. When you record a measurement, always state the unit of measure used. (115)

5. Units of time are determined by the movement of the earth among the stars. (122)

Words to Know

1. Direct measurement (112)
2. Indirect measurement (112)
3. Precision of measurement (113)
4. Linear measure (114)
5. Fahrenheit (120-121)
6. Centigrade (120-121)
7. Time zone (126)

Questions to Discuss

1. How does measuring differ from counting? (113)

2. What is really meant by saying 1 lb. = 16 oz.? (114)

3. How do you record below zero temperature readings? (120)

4. How many time zones are there on the earth? (126)

5. What names are given to the time zones in the United States? (126)

Written Practice

Write a numeral in each blank so that each statement becomes true.

1. 28 yd. = __ft. (114)
2. 72 gal. = __qt. (116)
3. 13 bu. = __pk. (117)
4. 8 lb. = __oz. (119)
5. 7 da. = __hr. (122)
6. 100 in. = __ft.__in. (114)
7. 83 pt. = __qt.__pt. (116)
8. 39 qt. = __pk.__qt. (117)
9. 6130 lb. = __T.__lb. (119)

Self-Evaluation

Part 1 Copy. Find a replacement for the blanks which make each of the following sentences true.

1. 17 mi. = ___ ft.
2. 12 yr. = ___ da.
3. 313 min. = ___ hr. ___ min.
4. 132 mo. = ___ yr.
5. 65 lb. = ___ oz.
6. 324 in. = ___ yd.
7. 77 pt. = ___ qt. ___ pt.
8. 56 pt. = ___ gal.
9. 10 T. = ___ lb.
10. 89 pk. = ___ bu. ___ pk.
11. 108 da. = ___ wk. ___ da.
12. 60 fl. oz. = ___ cups ___ fl. oz.

Part 2 Copy. Find a replacement for n which makes each sentence true.

	a	*b*
1.	$32 \times 15 = n$	$n = 512 + 89$
2.	$828 \div 23 = n$	$n = 451 - 263$
3.	$286 \times 9 = n$	$n = 368 \div 8$
4.	$4261 + 287 = n$	$n = 707 - 49$
5.	$303 \times 72 = n$	$n = 693 \div 21$

Part 3 Write the correct answer for each of the following questions.

1. How many hours difference is there between Central Standard Time and Mountain Standard Time?

2. What is the difference in hours between 3 p.m. and 9 p.m.?

3. At what Fahrenheit temperature does water freeze?

4. Name three things that can be measured by counting.

5. What is the difference in degrees between the freezing and the boiling temperatures of water?

Part 4 Solve the following problems and give your answers with the correct unit of measure.

1. Mr. Jones left for work at 8 a.m. and returned home at 3 p.m. How many hours was he away?

2. The first game of the World Series started at 12 noon in New York City. At what hour, Mountain Standard Time, would someone in Colorado turn on his TV set in order to see the game begin?

3. Each of the 520 pupils in Madison School received 1 pint of milk a day. How many gallons of milk were needed each day?

8

Arithmetic Sentences

$n+7=15$ $\frac{n}{3}-13=6$ $(42\div7)+n=217$ $(54+16)\div\frac{3}{n}=70$

$(n\div15)+71=90$ $(n\div5)-31=9$

Arithmetic Sentences

Your knowledge of arithmetic is useful and worthwhile if you can use it to solve problems in everyday life. Unfortunately, problems in daily life are not stated with numerals and operation signs ready for you to solve. Instead, most problems are stated in the English language.

When solving a problem, you can write the English sentence as an arithmetic sentence. Examples of arithmetic sentences follow.

$2+6=8$ $n\times7=56$

$37-5=32$ $63\div9=n$

The only symbols used in these sentences are numerals, a letter (n) to represent a number, operation signs, and the equal sign. Other symbols, such as the inequality signs ($\neq$, $<$, and $>$), may also be used.

Every arithmetic sentence could represent many problems about many different kinds of things. The arithmetic sentence $5+4=9$ could be written as "Five plus four is equal to nine." These numerals could represent 5 apples, 4 apples, and 9 apples; 5 people, 4 people, and 9 people; or they could represent 5 miles, 4 miles, and 9 miles. Can you give other possible meanings of $5+4=9$?

A story problem usually tells *how many things* and *what kind of things.* Knowing the *number* of things in the problem is necessary in order to write the problem in the language of arithmetic. The *kind* of things only indicates the everyday life situation from which the problem arose.

Oral Use the arithmetic sentences below in answering the questions.

$10+6=16$ $17-9=8$

$7+7=14$ $32\div4=8$

1. Which sentence refers to joining two sets with unequal numbers?

2. In which sentence are two numbers compared to find their difference?

3. Which sentence refers to joining two sets with an equal number?

4. Which sentence indicates that a set is separated into 4 sets?

Placeholders for Numerals

A sentence like $n+5=8$ is called a sentence of arithmetic. The letter n holds the place for a numeral. In the English language, $n+5=8$ could be read as "Some number plus five is equal to eight." If you were asked to write "Some number minus two is equal to seven" in the language of arithmetic, you might write $n-2=7$.

Before you change complete sentences from the English language to the arithmetic language, it is a good plan to practice doing this for word phrases. You usually let a letter, such as n, represent the number which is not given, or the number asked for, in the problem.

Example 1: Express the following in the language of arithmetic:

six problems more than Jim worked

The number of problems Jim worked is not given; so represent this number by using n. Then the above expression could be written: $n+6$.

Example 2: Express the following in the language of arithmetic:

12 less rockets than we fired last year

Let n stand for the number of rockets we fired last year. Then the above expression could be written: $n-12$.

Example 3: Express the following in the language of arithmetic:

twice as many doughnuts as we ate

Let n stand for the number of doughnuts we ate. Then the above expression could be written: $2\times n$.

Oral Tell what you would let n represent in each of the following.

1. Ten more fish than I caught

2. Eight points less than we made in Friday's game

3. Two dollars less than I earned last week

4. Two thousand people more than the population of Podunk

5. Three times as many cookies as mother baked

6. Five less pounds than I weighed last year

Written Write each of the above phrases in the language of arithmetic. State what n represents in each case. Then write a phrase in English which may be represented by each of the following.

	a	*b*	*c*
7.	$7+n$	$n+11$	$23-n$
8.	$n-10$	$5\times n$	$n\div 4$

Solving Problems by Using Equations

Arithmetic sentences are useful when you think about numbers. They help you show relationships in short, easy form. If a sentence uses the = sign, it is called an **equation.**

You have no way of knowing whether an arithmetic sentence or equation containing n is true or false until you replace n by a numeral. Is the equation $n+18=25$ true when n is replaced by 2? By 3? By 6? By 7?

What does it mean to solve an equation? To solve $12-n=8$, you think: $12-4=8$; therefore $n=4$. Why is $n=4$ correct? Why is $n=7$ not a correct answer? Does any other number, except 4, make the sentence true? To solve an equation means to find a replacement for n which makes the sentence true.

In solving a story problem, you first change the English sentences to equation form. Then you must solve the resulting equation.

In each of the following examples two steps are used: first, the English sentences are written as equations; and next, the equations are solved.

Example 1: Seven more than the number of pupils enrolled last year is forty-nine. How many pupils enrolled last year? Let n represent the number of pupils enrolled last year. Then the
equation is $n+7=49$,
and $n=49-7$,
so $n=42$.

Since n represents the number of pupils enrolled last year, you know that 42 pupils enrolled last year.

Example 2: Bob bought five boxes of candy with six candy bars in each box. How many candy bars did he buy? Let n represent the number of candy bars he bought. Then the
equation is $5\times6=n$
and $30=n$.

Bob bought 30 candy bars.

Oral What would you let n represent in each of the following problems?

1. Joe entered the store and spent 72¢ for treats. He still had 48¢. How much money did Joe have when he entered the store?

2. The Lincoln School has filled 156 of 210 Red Cross boxes. How many more boxes are to be filled?

3. Bert weighed 63 pounds last year. Now he weighs 76 pounds. How many pounds did he gain?

Written Solve each of the above problems by using equations. Tell what n represents in each case.

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Problems Using Addition and Subtraction

Many problems in arithmetic involve the use of more than one operation. Read the following problem to determine the operations you would use in solving the problem.

John and James had 195 papers to deliver. John delivered 72 and James delivered 81. How many papers did they still have to deliver?

One way to solve this problem is: The two boys had already delivered (72+81) papers. Subtract this amount from the total they had to deliver. Let n stand for the number of papers still to be delivered. The equation

is $195-(72+81)=n$,

or $195-153=n$,

so $42=n$.

They still had 42 papers to deliver.

Oral Another equation to use in solving the above problem is shown.

$$(195-72)-81=n$$

1. What does (195−72) represent?

2. What does n stand for in this equation?

3. How would you think about the problem to get this equation?

4. Would $(72+81)+n=195$ be a correct equation for the problem?

Written Solve each of the following problems by writing an equation containing n. Write what n represents in each equation.

1. Polly made 26 party favors and Jan made 29. They need 70. How many more must they make?

2. Nancy saved 75¢ and earned 90¢ more. Then she spent 39¢. How much money did she have left?

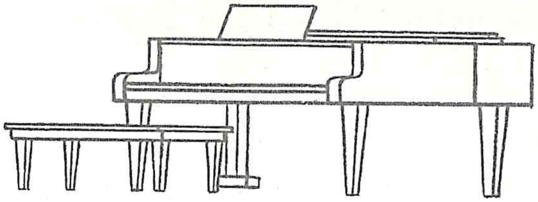
3. George bought a tube of toothpaste for $.65 and a toothbrush for $.43. How much change would he get from $2.00?

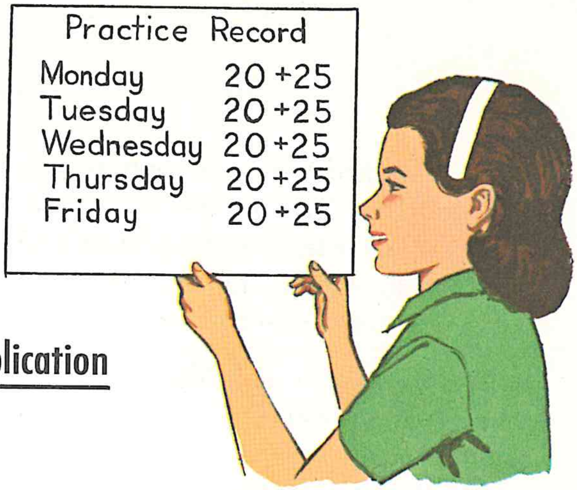
4. Bill had 43 fish. He sold 9 of them, some of them died, and he had 27 fish left. How many fish died?

5. The pupils sold 17 tickets on Monday and 23 on Tuesday. After Wednesday's sales they had sold a total of 65 tickets. How many tickets did they sell Wednesday?

6. Bob scored 14 points in the first football game and 6 points in the second game. He still needed 24 points to tie the school record. How many points is the school record?

7. Tom weighs 62 pounds, Jack weighs 71 pounds, and Bill weighs 66 pounds. What is the combined weight of the three boys?





Problems Using Addition and Multiplication

Linda practiced her piano lesson 20 minutes each morning and 25 minutes each afternoon. She practiced for 5 days. How many minutes did she practice in the 5 days?

To answer the question asked in the problem you could think: She practiced 20+25 minutes each day. Then multiply this sum by 5. Let n represent the number of minutes practiced in 5 days. The equation and the solution is given below.

$$(20+25)\times 5=n$$
$$45\times 5=n$$
$$225=n$$

She practiced 225 minutes in 5 days.

Oral Answer the questions below.

1. What does (20+25) represent in the equation given above?

2. Would $(20\times 5)+(25\times 5)=n$ be a correct equation for the problem?

3. How would you think about the problem to arrive at the equation given in question 2?

Written Solve each of the following problems by writing an equation containing n. Write what n represents in each equation.

1. Mr. Jones drove to work each day. He drove 11 miles to work and 16 miles home by another route. How many miles did he drive in going to and from work in 10 days?

2. For use in art class the pupils brought 9 boxes of crayons and the school furnished 14 boxes. If each box contained 12 crayons, how many crayons were available for art class?

3. Robert worked for 4 hours at 50¢ an hour. He still needs 45¢ in order to buy a new sweater. What is the cost of the sweater?

4. First-class postage in 1960 was 4 cents an ounce and airmail was 7 cents an ounce. How much more did it cost to airmail a 6-ounce parcel than to send it first class?

Problems Using Addition and Division

It is sometimes necessary to use addition and division to solve a problem. You have used both addition and division in finding the average when the addends are given separately.

For an example, consider the following problem. On 3 arithmetic tests Don's scores were 79, 85, and 91. What was his average score?

You may remember that the average is found by dividing the sum by the number of addends. Let n represent Don's average score. Then the following equation is used.

$$(79+85+91)\div 3=n$$
$$255\div 3=n$$
$$85=n$$

Don's average score was 85.

Another example is: The Girls Club had 50 safety posters and made 62 more. They distributed them equally among 16 classrooms. How many posters were left in each room?

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Let n represent the number of posters left in each room.

$$(50+62)\div 16=n$$
$$112\div 16=n$$
$$7=n$$

Oral Answer these questions about the second example problem.

1. What does $(50+62)$ represent?

2. What tells you to add before you divide in solving the equation?

3. Why would you divide by 16?

Written Solve each of the following problems by writing an equation containing n. Write what n represents in each equation.

1. The four watermelons at the fruit stand weighed 18 lb., 20 lb., 17 lb., and 21 lb. What was the average weight of each melon?

2. Three girls agreed to share equally the expenses of a trip to the zoo. Bus fare was $.90 and lunches cost $1.95. What was each girl's share of the expenses?

3. Thomas had saved 60¢. Then he earned 75¢ on Monday evening and 90¢ on Saturday morning. How many 25¢ show tickets could he buy?

4. The boards Mr. Halt used for his fence were 14 ft., 8 ft., and 5 ft. long. What was the average length of the boards he used?

Can you do this? Marilyn and two of her friends shared 42 stamps equally. Then Marilyn and her brother had 27 stamps. How many stamps did her brother have?

Subtraction and Multiplication

Paul spent 4 days of his 3-week vacation at home and the rest of it at camp. How many days did Paul spend at camp?

One person might think: In a 3-week vacation there are 3×7 days. Then subtract the 4 days he spent at home, and I will find the number of days at camp. Let n represent the number of days at camp. The equation and solution are shown below.

$$(3 \times 7) - 4 = n$$
$$21 - 4 = n$$
$$17 = n$$

So Paul spent 17 days at camp.

You might think differently about the problem and write the equation below. Again, n represents the number of days Paul spent at camp.

$$4 + n = 3 \times 7$$

Oral Answer the following about the equation $4+n=3\times7$.

1. What does $4+n$ represent?

2. What does 3×7 represent?

3. Explain how you think about the problem in order to arrive at this equation.

4. Can you give another equation suitable for the problem?

Written Solve each of the following problems by writing an equation containing n. State what n represents in each case.

1. A lady bought 3 bushels of apples at $2.75 a bushel. How much change did she receive from $10?

2. The weight of 4 baskets of apples was 212 pounds. If each empty basket weighs 3 pounds, what was the actual weight of the apples?

3. Jane is 14 years old today. Her grandmother is 5 times as old. In what year was her grandmother born?

4. Junior spends 300 minutes each day at school. This time includes 30 minutes used for recess. How many minutes does he spend in class during one week of school?

5. Robert bought 3 dozen cookies for his mother. He ate 4 cookies on the way home. How many cookies did he have when he got home?

Can you do this? Solve each of the following equations for n.

1. $(16 \times n) + 42 = 378$

2. $(n \div 4) - 12 = 20$

3. $n \div (6 \times 7) = 9$

4. $(n + 6) \times 8 = 64$

Mr. Meyer had 275 chickens. He sold 51 chickens and separated the rest equally among 4 pens. How many chickens did he have in each pen?

To solve the problem, think: The set of 275 chickens is separated into two unequal sets. Therefore, the subtraction, 275 − 51, gives the number of chickens Mr. Meyer kept. Then this set must be separated into four sets of equal size. So you must divide the number of chickens he kept by 4.

Let n represent the number of chickens he had in each pen. An equation and solution for the problem is given below.

$$(275-51) \div 4 = n$$
$$224 \div 4 = n$$
$$56 = n$$

Mr. Meyer had 56 chickens in each of the pens.

Oral Answer the following questions about the problem above.

1. What does (275 − 51) represent in the above equation?

2. What tells you to do the subtraction before the division?

Written Solve each of the following problems by writing an equation containing n. State what n represents in each equation.

1. Mrs. Anderson baked 104 cookies. She kept 32 and packed the rest in boxes which held one dozen each. How many boxes did she use?

2. The distance from Mr. Black's home to Centerville is 263 miles. The first hour he drove 55 miles. What was the average number of miles per hour for the remainder of the trip if he drove it in 4 hours?

3. June had $1.20. She spent $.45 for thread and used the balance to buy ribbon at $.15 a yard. How many yards of ribbon did she buy?

4. The creamery sold cheese at 4 lb. for 72¢. John bought 1 lb. How much change did he receive from 25¢?

5. Tom had 38 tropical fish. He sold 11 of them and separated the rest of them equally among 3 bowls. How many did he put in each bowl?

6. John and two of his friends shared equally the 72 arrowheads they had collected at summer camp. John mounted his share on 4 display cards. How many arrowheads did he mount on each display card?

7. Roger raised 65 pounds of carrots. He sold 40 pounds at the grocery store, and put the rest into 5-pound sacks to keep for winter use. How many sacks did he use?

Multiplication and Division

Kenneth bought 48 feet of rope to make a swing. Rope sold for 12¢ a yard. What did 48 feet of rope cost?

In both solutions below, n represents the amount paid for 48 feet of rope. In A, think: The number of feet times the price per foot would give me n. To find the price of one foot of rope, I must divide the price of one yard by 3, or 12÷3. Why?

In B, think: The number of yards times the price per yard would give me n. To change feet to yards, I divide the number of feet by 3. The number of yards is shown by 48÷3.

A	B
$(12\div 3)\times 48 = n$	$(48\div 3)\times 12 = n$
$4\times 48 = n$	$16\times 12 = n$
$4\times(40+8) = n$	$16\times(10+2) = n$
$160+32 = n$	$160+32 = n$
$192 = n$	$192 = n$

Kenneth paid $1.92 for the rope.

Oral Answer the following about the problem above.

1. Explain the steps used in solving either equation above.

2. What unit of money is used throughout the solutions?

Written Solve each of the following problems by writing an equation containing n. Tell what n represents in each equation.

1. Bill's father bought 160 pounds of oats for seed. He paid $1.05 per bushel. A bushel of oats weighs 32 pounds. What did he pay for the seed?

2. Tom put 96 eggs into cartons that held 1 dozen eggs each. He sold the cartons of eggs for 42¢ each. How much did he get for the eggs?

3. James stamped 78 envelopes in 6 minutes. At this rate, how many envelopes can he stamp in 4 minutes?

4. Mary, Sue, and Dorothy each painted 14 place cards. They placed the same number of cards on each of 7 tables. How many place cards were on each table?

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Reviewing Basic Operations in Problems

In everyday life situations you are not told which operation or operations to use. You must study the problem, decide what to do with the sets, decide which operation to use, and write an equation.

Remember: The kind of things given in the problem are not part of the equation, but are given as part of the answer.

The following problem is solved according to the above suggestions.

Dennis picked 5 pecks of apples, and his father picked 3 bushels. How many pecks of apples did they pick together?

You are asked to give the answer in pecks; so change all the amounts of apples to peck measurement. The amount of apples that Dennis' father picked is given as 3 bushels. To change bushels to pecks you must multiply the number of bushels by four. Why? Dennis' father picked 3×4 pecks.

The set of pecks Dennis picked must be joined with the set of pecks his father picked. Joining them means addition. The equation would be $(3 \times 4) + 5 = n$, where n represents the number of pecks they picked together.

The equation is solved below.

$$\begin{aligned} (3\times4)+5&=n \\ 12+5&=n \\ 17&=n \end{aligned}$$

They picked 17 pecks together.

Oral What question or questions are answered by addition? Subtraction? Multiplication? Division?

Written Use the suggestions given on this page to solve the following problems. Write an equation for each problem, tell what n represents, and label your answer according to the kind of things in the problem.

1. John saved half of his weekly allowance of 60 cents. How much money did he save in 10 weeks?

2. Mary and Ann baked 7 dozen chocolate-chip cookies. They took 50 cookies to the bake sale. How many cookies did they have left?

3. Charles and Tim worked together and made 18 baskets. Charles sold 3 of his baskets and Tim sold 2 of his baskets. How many baskets did they have left?

4. Mr. Simpson bought 65 dozen eggs at 27¢ a dozen and sold them at 34¢ a dozen. How much money did he make on the eggs?

5. Betty walks 5 blocks going to school, and 7 blocks going home on another route. How many blocks does she walk each week in going to and from school?

6. Paul bought a notebook for $1.25. He also bought 3 packs of notebook paper at $.21 a pack. How much money did he spend?

7. A grocer bought 5 boxes of squash for his store. Each box contained 14 squash. He sold the squash for 15¢ each. How much money did he receive for all the squash?

8. When milk sells for 92¢ a gallon, what is the cost of 3 quarts?

9. Mrs. Bloom bought 3 yards of ribbon for wrapping packages. When she finished, she had 5 inches of ribbon left. How many inches of ribbon did she use?

10. The painter used 5 gallons of white paint, 3 quarts of black paint, and 1 gallon of porch paint on his house. How many quarts of paint did he use?

11. Last year 281 girls joined the Girl Scouts in Mayville. Since then, 23 new members have been added. What is the average size of each of the 16 troops this year?

12. When apples cost $3.00 a bushel, what is the cost of 5 pecks?

Can you do this? John weighs 6 lb. more than Tom. They stand on the scales together. Their weight is 152 lb. How much does each boy weigh?

Let n represent Tom's weight. Then $n+6$ represents John's weight. Their combined weight could be given by $n+(n+6)$.

$$n+(n+6)=152$$

$$(n+n)+6=152 \qquad \text{Why?}$$

$$n+n=152-6 \qquad \text{Why?}$$

$$2\times n=146 \qquad \text{Why?}$$

$$n=146\div 2 \qquad \text{Why?}$$

$$n=73$$

Since n represents Tom's weight, you know that Tom weighs 73 lb. John's weight, which was represented by $n+6$, is $73+6$, or 79 lb.

Solve the following problems.

1. In the fifth grade there are 33 pupils. There are 7 more boys than girls. How many boys and how many girls are in the class?

2. Four years ago Mary was 7 years old. How old is she now?

3. Joe's mother gave him $3.80 to buy groceries at the supermarket. The cost of the groceries was 3 times the amount of change he received. How much did the groceries cost?

4. Richard is half as old as Mark. Their combined age is 21 years. How old is Richard?

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Estimating Answers—Addition and Subtraction

For many problems you are able to give an approximate answer without doing any computation. When you do this, you **estimate** the answer.

You can estimate the answer mentally by rounding the numbers given in the problem to the nearer 10, 100, or 1,000. When the answer obtained from the computation is very near to your estimated answer, you have probably done the work correctly.

Study the following example and its explanation.

Eric and his father had 215 chickens. They sold 56 of them. How many chickens did they have left?

Comparing the two sets to find how many were left shows that you must subtract the numbers of the sets. The equation, $215-56=n$, illustrates the problem where n represents the number of chickens they had left.

You could think of either of the equations

$$200-50=n \qquad \text{or} \qquad 210-60=n$$

to estimate the answer as 150. Explain how to get these equations.

Or you could think of the equation

$$220-60=n$$

to estimate the answer as 160.

The previous examples tell you that the answer to the problem should be about 150 or 160. Actual computation of $215-56=n$ shows that $n=159$.

Oral Estimate mentally the value of n in the following equations.

	a	*b*
1.	$12+38+47=n$	$127+83+49=n$
2.	$(37+23)-33=n$	$(76-29)+18=n$

Written Write an equation for each of the following problems. Estimate an answer. Then solve the equation and compare the correct answer with the estimated answer.

1. On their vacation the Olson family drove 213 miles on Monday, 287 miles on Tuesday, and 357 miles on Wednesday. How many miles did they drive in the three days?

2. The early morning temperature was 27°. At noon it was 53°. How much warmer was it at noon than in early morning?

3. There are 291 boys and 347 girls at Rose School. How many fewer boys than girls are enrolled?

4. Janice bought a pair of shoes for $6.75 and a dress for $11.98. How much change did she receive from a twenty-dollar bill?

Estimating Answers—Multiplication and Division

When estimating the answer for a problem, you can often say that the answer is greater than some number but less than another number.

The Carr family used 14 loaves of bread each week. Bread sold for 24¢ a loaf. How much did the family spend for bread each week?

To solve the problem you could use the equation $24\times14=n$, where n represents the amount of money spent each week for bread.

You could replace each factor by the next smaller multiple of 10. That is, think of the equation

$$20\times10=n$$

to estimate the answer as 200¢ or $2.00. The answer must be larger than $2.00, since both factors were replaced by smaller numbers.

You could replace each factor by the next greater multiple of 10. That is, think of the equation

$$30\times20=n$$

to estimate the answer as 600¢ or $6.00. Since each factor was replaced by a larger number, this estimate is too large.

Hence the answer to the problem must be between $2.00 and $6.00.

If you multiply 24 by 14, you will get a product of 336, or an answer of 336¢, or $3.36. Is this answer between the above estimates?

Oral Estimate mentally the value of n in each of the equations below.

	a	*b*
1.	$6\times37=n$	$8\times23=n$
2.	$32\times12=n$	$62\times94=n$
3.	$158\div21=n$	$792\div43=n$

Written Write an equation for each of the following problems. Estimate an answer. Solve the equation and compare the answers.

1. James scored 144 points in 18 basketball games. What was his average score per game?

2. At $.27 a foot, what is the cost of a board 18 feet long?

3. A farmer picked 29 melons that had a combined weight of 609 pounds. What was the average weight of the melons?

4. Mr. Barton drove 13,670 miles in 14 months. What was the average number of miles driven per month?

5. Paul raised 4 bushels of potatoes. He sold them for $.39 a peck. How much did he get for them?

MORE PRACTICE PAGE 306

Review and Practice

You should be able to do all the exercises on this page without referring to other pages in the book.

Copy. Write a numeral in each blank so that the statements below become true. (= means *measures the same as.*)

1. 18 wk. = ___ da.

2. 1 lb. 4 oz. = ___ oz.

3. 4 hr. 11 min. = ___ min.

4. 3450 lb. = ___ T. ___ lb.

5. 141 in. = ___ ft. ___ in.

6. 67 pk. = ___ bu. ___ pk.

7. 14 min. = ___ sec.

8. 17 da. = ___ hr.

9. 418 oz. = ___ lb. ___ oz.

Copy. Find the correct answer for each exercise by performing the indicated operations.

	a	*b*	*c*
10.	$8\overline{)302}$	$15\overline{)823}$	$43\overline{)701}$
11.	305 ×709	926 −357	879 +506
12.	4257 −982	8769 +945	5671 ×524

Copy. Find a replacement for n which makes each of the following a true statement.

	a	*b*
13.	$2315 \div 5 = n$	$3008 \times 907 = n$
14.	$78421 - 3946 = n$	$576 \div 48 = n$
15.	$54696 + 8325 = n$	$702 \times 308 = n$
16.	$2664 \div 37 = n$	$4003 - 507 = n$
17.	$(5 \times 7) + 8 = n$	$(12 \div 3) - 2 = n$
18.	$(42 + 3) \div 5 = n$	$7 + (5 \times 9) = n$
19.	$(24 \div 6) - n = 1$	$n + (72 \div 9) = 13$
20.	$n + 5 = 19$	$n \div 7 = 12$
21.	$7 \times n = 98$	$52 - n = 36$

Write which basic operation comes to mind when you hear or see each of the following words.

	a	*b*
22.	Factor	Subtrahend
23.	Sum	Addend
24.	Product	Multiplier
25.	Minuend	Quotient
26.	Multiplicand	Difference
27.	Divisor	Minus
28.	Plus	Remainder

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The language of arithmetic contains sentences. (131)

2. A letter, such as n, used in an arithmetic sentence, holds the place for a numeral. (132)

3. Arithmetic sentences using the = sign are called equations. (133)

4. To solve an equation means to find a replacement for n which makes the sentence true. (133)

Words to Know

1. Equation (133)

2. Estimate (142)

Questions to Discuss

1. What kinds of symbols are used in an arithmetic sentence? (131)

2. Why should you state what n stands for in an equation? (131)

3. Is it possible to state a written problem with more than one equation? (134, 135)

4. What suggestions can you give for solving a problem? (133, 140)

5. Of what use is an estimated answer to a problem?(142)

Written Practice

Solve each problem below by using an equation containing n. Tell what n represents in each equation.

1. Martha had 14 postcards and bought 20 more. Then she sent 26 of them in the mail. How many did she have left after mailing them? (134)

2. A touchdown counts 6 points. Last Friday the high school team scored 4 touchdowns and 3 extra points. What was their score? (135)

3. Last week Ed kept a record of how long it took him to walk home. His record read: Monday, 12 min.; Tuesday, 14 min.; Wednesday, 9 min.; and Thursday, 9 min. What was the average time? (136)

4. Jack delivers papers after school. He pays 4¢ for each paper and sells it for 7¢. Last week he sold 113 papers. How much did he make? (137)

5. Lois had 17 pieces of candy. She kept 2 pieces for herself and divided the rest equally among 5 of her friends. How many pieces did each friend get? (138)

6. John's father bought 6 gallons of motor oil for 40¢ a quart. How much did the oil cost? (139)

Self-Evaluation

Part 1 Each of the problems is followed by four equations. Which equations correctly describe the problem? In each case more than one equation may be correct.

1. Craig weighs 63 pounds and his brother weighs 51 pounds. How much heavier than his brother is Craig?

a. $63+51=n$ **c.** $51+n=63$

b. $63-51=n$ **d.** $51-63=n$

2. Mary made 36 visits to the public library this year. What was the average number of visits per month?

a. $n\times12=36$ **c.** $12+n=36$

b. $12\div36=n$ **d.** $36\div12=n$

3. George bought 2 gallons of ice cream for 40¢ a quart. How much did the ice cream cost?

a. $(2\times4)-40=n$ **c.** $(2\times4)\times n=40$

b. $(2\times4)\times40=n$ **d.** $(40\times4)\times2=n$

Part 2 Write an estimate for the value of n in each of the following. Then compute each value of n.

	a	*b*
1.	$29+7=n$	$47\times13=n$
2.	$654\div6=n$	$88-57=n$
3.	$104\times37=n$	$39+24+47=n$
4.	$92\times21=n$	$127-76=n$
5.	$98\times9=n$	$77+19+31=n$

Part 3 Write an equation for each of the following problems. Estimate an answer and record it. Solve the equation, and compare the results with your estimated answer.

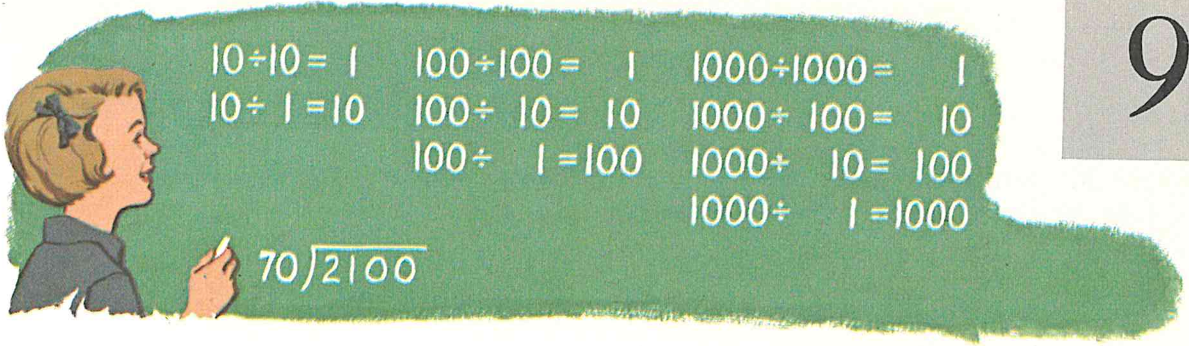
1. Rita practiced her music lesson for 15 minutes each morning and 30 minutes each afternoon. She did this for 4 days. How many minutes did she practice in the 4 days?

2. At school 84 pupils were divided into 3 class sections. The largest section had 31 pupils and the smallest had 25 pupils. How many pupils were in the remaining section?

3. Every day at Scout Camp each of the 52 boys drank 5 cups of milk. How many quarts of milk are used each day?

4. Each of the 15 members in the Science Club pays 10¢ a month dues. How much money is paid to the club for dues in one year?

5. For the games at the picnic, Richard brought 8 yards of rope and Bill brought 6 yards. How many feet of rope did the two boys bring for the picnic games?



9

Division with Larger Numbers

Thus far you have used two-digit divisors and three-digit dividends. Many problems, however, require the use of larger numbers.

Suppose the equation for some story problem was 2100÷70=n. To find n, you divide. If you ask yourself some questions about this division, you can reach some conclusions about the quotient.

Could the quotient contain 4 digits? If it contained 4 digits, there would be a digit in thousands place. The smallest possible digit in thousands place would be 1. But 1000×70=70000, which is larger than the dividend. Therefore, the quotient cannot have 4 digits.

Could the quotient have 3 digits? Explain why not.

Could the quotient have only 1 digit? Explain why not.

In the division, 3742÷62=n, the quotient must be greater than 10 because 62×10=620. Also the quotient must be less than 100 since 62×100=6200. Hence the quotient contains two digits. Your knowledge of trial quotients may cause you to try 60. Since 62×60=3720, you see that the quotient is at least 60.

Oral In each division below, tell:

(**a**) Whether the quotient has 1, 2, or 3 digits, and why.

(**b**) In which place-value position you would write the first digit of the quotient.

(**c**) What you would use as the first trial quotient.

	a	*b*	*c*
1.	$8\overline{)1720}$	$28\overline{)936}$	$17\overline{)639}$
2.	$5\overline{)157}$	$71\overline{)3652}$	$26\overline{)7416}$
3.	$4\overline{)842}$	$44\overline{)2175}$	$37\overline{)2999}$
4.	$3\overline{)5262}$	$32\overline{)3416}$	$53\overline{)5872}$

Dividing Tens in Four-Digit Numbers

Mr. Smith sold 1,200 boxes of pears in 30 days. What was the average number of boxes sold each day?

To find the average, divide as shown. The first digit of the divisor is greater than the first digit of the dividend; that is, 3 is greater than 1.

$$\begin{array}{r} 40 \\ 30\overline{)1200} \\ \underline{120} \\ 0 \end{array}$$

In this division, find a trial quotient by dividing the 12 hundreds in the dividend by the 3 tens in the divisor. A short way of writing this is 12 H÷3 T=4 T.

Try 4 as a trial quotient. Why is 4 written in tens place in the quotient? Why is 0 written in ones place in the quotient?

Mr. Smith sold an average of 40 boxes of pears each day.

Study the division below. Why is the value of n in the trial quotient equation more than 6 tens but less than 7 tens?

$$\begin{array}{r} 60 \\ 42\overline{)2520} \\ \underline{252} \\ 0 \end{array}$$

Trial Quotient
25 H÷4 T=n

MORE PRACTICE PAGE 307

Oral Answer the following questions about 2520÷42=60.

1. What do 25 H and 4 T stand for in the trial quotient equation?

2. What would you use for a trial quotient?

3. Why should you write 6 in tens place in the quotient?

4. Why should 0 be written in ones place in the quotient?

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$80\overline{)4000}$	$60\overline{)2400}$	$20\overline{)1100}$
2.	$70\overline{)4200}$	$90\overline{)7200}$	$22\overline{)1540}$
3.	$80\overline{)3200}$	$40\overline{)2800}$	$60\overline{)5400}$
4.	$43\overline{)1720}$	$90\overline{)1350}$	$34\overline{)1440}$
5.	$26\overline{)1300}$	$74\overline{)4440}$	$30\overline{)2100}$
6.	$70\overline{)3500}$	$60\overline{)4200}$	$28\overline{)1680}$
7.	$70\overline{)1400}$	$60\overline{)1800}$	$90\overline{)4500}$
8.	$52\overline{)3640}$	$34\overline{)2040}$	$47\overline{)1410}$
9.	$50\overline{)2000}$	$30\overline{)1500}$	$57\overline{)2280}$
10.	$85\overline{)4250}$	$64\overline{)4480}$	$72\overline{)3600}$

Dividing Tens and Ones in Four-Digit Numbers

Mr. Black raised 2,240 bushels of wheat on a 64-acre field. How many bushels per acre did he raise?

In order to find the answer you divide 2,240 by 64. Before doing the division, ask yourself: Does the quotient have 3 digits? 2 digits?

100×64=6400, which is larger than the dividend. So the quotient must be less than 100.

10×64=640, which is less than the dividend. So the quotient must be greater than 10.

Hence the quotient is between 10 and 100, and contains two digits. The first digit of the quotient must be written in tens place. Why?

The completed division is shown below with the equations for finding the trial quotients.

	Trial Quotients
35	
64)2240	22 H÷6 T=n
192	
320	32 T÷6 T=n
320	
0	

In the equation, 22 H÷6 T=n, the value of n is more than 3 T but less than 4 T. Why? Why is 3 used as the first trial quotient? Why is it written in tens place in the quotient?

You find a second trial quotient in order to determine the digit in ones place in the quotient. To find a second trial quotient, you must solve the equation, 32 T÷6 T=n. In this case, the value of n is between 5 and 6. Why? You use 5 as a trial quotient for ones place.

Oral Answer the following questions about 1752 divided by 73.

1. Does the quotient have 3 digits? 2 digits?

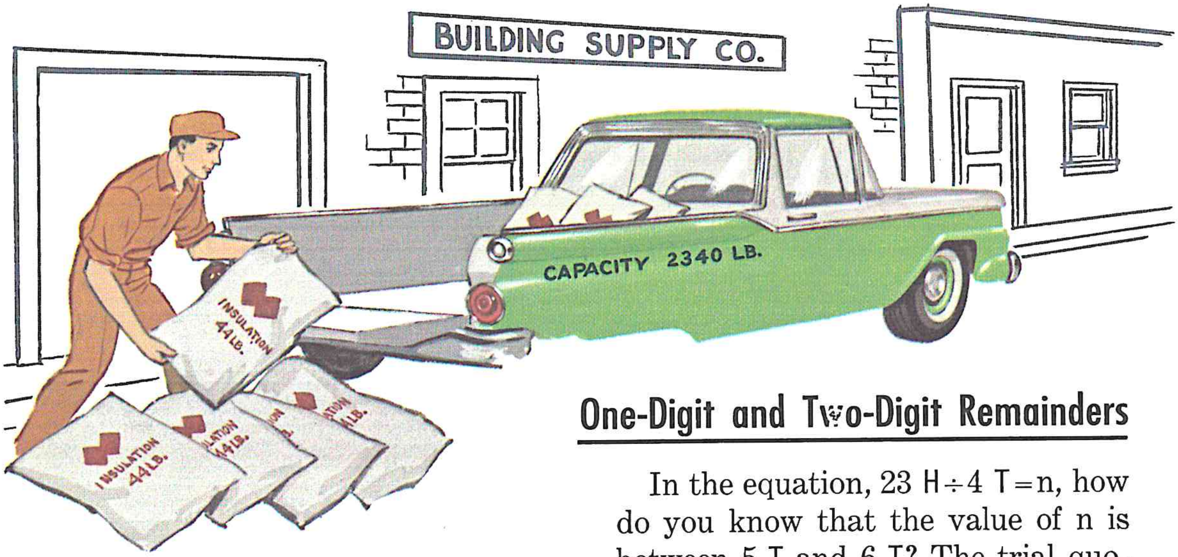
2. What trial quotient would you use for tens place?

3. In what place-value position would you write the first trial quotient?

Written Copy. Find each quotient.

	a	*b*	*c*
1.	50)2300	20)1180	90)1980
2.	37)2997	81)3402	54)1944
3.	45)2115	38)1976	67)2942
4.	27)2025	44)2332	87)3306
5.	35)3220	91)3367	73)3066

MORE PRACTICE PAGE 307


BUILDING SUPPLY CO.
CAPACITY 2340 LB.
INSULATION 44 LB.

One-Digit and Two-Digit Remainders

Mr. Harper's truck has a load capacity of 2340 pounds. How many 44-pound bags of insulation can he haul in one load? How many pounds below capacity will the load be?

To find how many sets of 44 are contained in 2340, divide 2340 by 44. The remainder will be the number of pounds below capacity.

Study the division and trial quotient equations below. Mr. Harper can haul 53 bags and still be 8 pounds below load capacity.

$$\begin{array}{r} 53\ \text{r}8 \\ 44\overline{)2340} \\ \underline{220} \\ 140 \\ \underline{132} \\ 8 \end{array}$$

Trial Quotients

23 H÷4 T=n

14 T÷4 T=n

In the equation, 23 H÷4 T=n, how do you know that the value of n is between 5 T and 6 T? The trial quotient is 5 T, which tells you the numeral and the place value of the trial quotient.

To find a trial quotient for ones place, use the equation, 14 T÷4 T=n. Since n is between 3 and 4, try 3 as the trial quotient.

The example shows that the quotient is 53 and the remainder is 8.

Study the following division and the check. Notice that the remainder is a two-digit number.

$$\begin{array}{r} 15\ \text{r}35 \\ 70\overline{)1085} \\ \underline{70} \\ 385 \\ \underline{350} \\ 35 \end{array} \qquad \begin{array}{r} 70 \\ \underline{\times 15} \\ 350 \\ \underline{70} \\ 1050 \\ \underline{+35} \\ 1085 \end{array}$$

Oral Answer the questions below.

1. In the division 1085÷70, how would you determine the tens digit of the quotient? The ones digit of the quotient?

2. Can the remainder be as large as the divisor? Why?

3. Can the remainder be as large as the quotient? Why?

4. If the divisor has 2 digits, when can the remainder be 0? A 1-digit number? A 2-digit number?

Written Copy. Find each quotient and remainder. Check for accuracy by using multiplication.

	a	*b*	*c*
1.	$36\overline{)1948}$	$24\overline{)1519}$	$42\overline{)3410}$
2.	$47\overline{)2116}$	$52\overline{)1984}$	$75\overline{)2034}$
3.	$16\overline{)1103}$	$49\overline{)2589}$	$50\overline{)1639}$
4.	$26\overline{)1499}$	$99\overline{)1390}$	$84\overline{)1945}$
5.	$31\overline{)1369}$	$42\overline{)2678}$	$35\overline{)1587}$
6.	$15\overline{)1000}$	$12\overline{)1196}$	$18\overline{)1376}$
7.	$61\overline{)2994}$	$75\overline{)2030}$	$81\overline{)3000}$
8.	$15\overline{)1303}$	$57\overline{)4678}$	$13\overline{)1277}$
9.	$68\overline{)3462}$	$29\overline{)4500}$	$33\overline{)2016}$

A quick review Answer the following questions.

1. In the number 2587, what does the digit 8 represent? The digit 5? The digit 2?

2. If you already know that 418×7=2926, then what is the answer for 2926÷7=n?

3. Sally removed 32 envelopes from a full box which was labeled "4 dozen envelopes." How many envelopes were left in the box?

4. What equation could you use to find a trial quotient in the division 4281÷76? 3724÷52?

5. Would 6000 be a sensible estimate for the answer of 205×31?

6. What is the correct way to read 1,315,027?

7. Which is the larger number: 2011 or 2009? 21,514 or 21,504?

8. Is the answer to 4120÷61 greater than or less than 60?

9. What is the largest possible 3-digit number?

10. What is the smallest 4-digit whole number?

11. How many inches equal 1 yard?

12. If you travel at an average speed of 60 miles per hour, how far do you travel in 1 minute?

MORE PRACTICE PAGE 307

Using Division to Solve Problems

After you read a story problem carefully, think about it before you start writing. In most cases common sense will tell you how to solve the problem.

Ask yourself such questions as: Which operation should I use? What equation can I write? What is a sensible answer?

Write an equation for each of the following problems. Solve the equation and check the correctness of your solution.

1. Mark's father went on a 2,115-mile trip. The driving time was 47 hours. What was his average speed in miles per hour?

2. A factory had 3,300 toys to pack into boxes. Each box held 60 toys. How many boxes were needed?

3. The truckload of grain weighed 3,402 pounds. How many 42-pound bags of grain were on the truck?

4. When a car travels at the speed of 1 mile every minute, or 60 seconds, how many feet does it travel in 1 second?

5. Mr. Johnson harvested 3,306 bushels of corn from a 38-acre field. What was the average number of bushels per acre?

6. Jim's father planned a 1,620-mile trip. If his average speed was 45 miles an hour, how many hours of driving did the trip take?

7. The Red Star Cannery packs 24 cans per case for shipping. How many cases are needed to pack 1,512 cans for shipping?

8. Mr. Goldman gathered 1,005 eggs during March. He sold them in cartons of 1 dozen each. How many cartons of eggs did he sell and how many eggs were left over?

9. Ann's mother used 1,224 inches of ribbon in wrapping gifts for the holidays. This is equivalent to how many yards of ribbon?

10. In order to check how many miles he could go on a gallon of gasoline, Bob's father kept a record. He used 87 gallons of gasoline to travel 1,479 miles. How many miles did he travel on each gallon?

11. A transport truck is loaded with 6,600 gallons of fuel oil. How many 55-gallon barrels can be filled with this amount of fuel oil?

12. The Maytown School District provides 14 schools for the 5,110 pupils living in the district. What is the average number of pupils attending each school?

Dividing Hundreds and Tens in Four-Digit Numbers

As you study the divisions below, do the computation yourself and compare your work with that shown.

A

$$\begin{array}{r} 230 \\ 30\overline{)6900} \\ \underline{60} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

B

$$\begin{array}{r} 210 \\ 31\overline{)6510} \\ \underline{62} \\ 31 \\ \underline{31} \\ 0 \end{array}$$

In division A, notice that the first digit of the two-digit divisor is less than the first digit of the four-digit dividend. Whenever this is the case, the quotient is a three-digit number.

In division B, round the divisor, 31, to 30 when finding trial quotients. How many times would you find trial quotients in completing the division?

C

$$\begin{array}{r} 240 \\ 37\overline{)8880} \\ \underline{74} \\ 148 \\ \underline{148} \\ 0 \end{array}$$

D

$$\begin{array}{r} 130 \\ 21\overline{)2730} \\ \underline{21} \\ 63 \\ \underline{63} \\ 0 \end{array}$$

In division C, why is 37 rounded to 40 when finding trial quotients?

In division D, the first digit of the divisor is the same as the first digit of the dividend. In such cases, compare the size of the divisor with the number formed by the first two digits of the dividend to help you decide on a trial quotient.

Oral Answer the questions below.

1. How do you find each trial quotient in division C?

2. How would you determine the first trial quotient in division D?

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$30\overline{)6000}$	$20\overline{)6400}$	$30\overline{)3600}$
2.	$40\overline{)7200}$	$80\overline{)8800}$	$50\overline{)6500}$
3.	$30\overline{)5400}$	$40\overline{)7600}$	$60\overline{)6600}$
4.	$13\overline{)1690}$	$11\overline{)1210}$	$12\overline{)1680}$
5.	$31\overline{)4030}$	$22\overline{)7260}$	$12\overline{)5280}$
6.	$23\overline{)8050}$	$21\overline{)5460}$	$32\overline{)3840}$
7.	$40\overline{)8050}$	$32\overline{)6720}$	$21\overline{)2940}$
8.	$31\overline{)6820}$	$29\overline{)9280}$	$38\overline{)5700}$

MORE PRACTICE PAGE 307

Practicing for Division

You have probably noticed that you make a multiplication and a subtraction after writing each digit in the quotient. You cannot do good work in division unless you can multiply and subtract accurately.

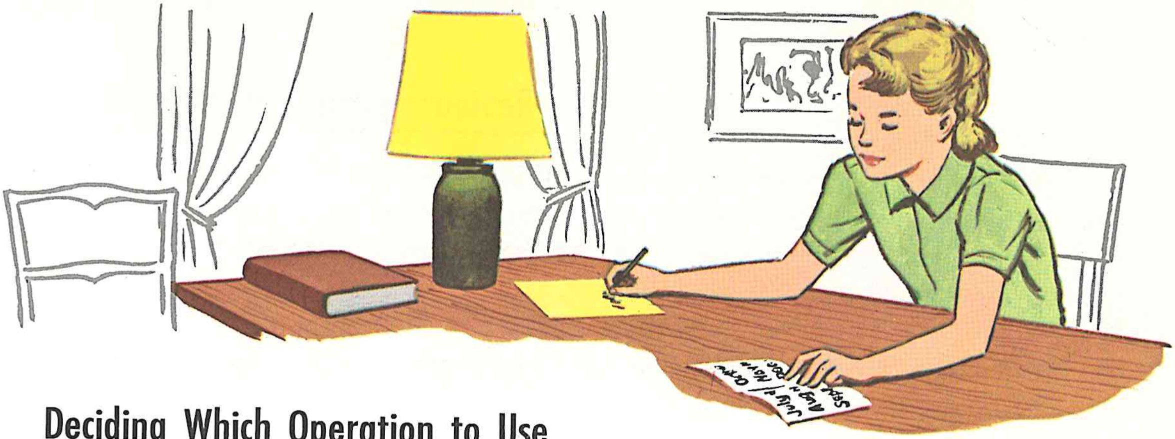
In order to do division rapidly, you must multiply and subtract mentally. Mental computation also helps you check a trial quotient.

Oral The multiplications and subtractions below will help you do the divisions which follow. If you cannot find a product or difference mentally, use pencil and paper to determine the answer.

	a	*b*	*c*
1.	20 ×7	34 ×4	58 ×2
2.	91 ×3	40 ×8	58 ×3
3.	16 ×6	79 ×2	49 ×6
4.	38 ×8	92 ×6	56 ×4
5.	57 ×2	24 ×3	24 ×6
6.	60 ×5	53 ×4	72 ×4
7.	72 ×8	16 ×8	34 ×6
8.	173 −158	112 −96	185 −174
9.	298 −294	330 −304	580 −552
10.	234 −212	346 −288	143 −128

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$20\overline{)1400}$	$34\overline{)1365}$	$58\overline{)1163}$
2.	$79\overline{)1738}$	$16\overline{)1127}$	$40\overline{)1551}$
3.	$91\overline{)1183}$	$58\overline{)1859}$	$49\overline{)2989}$
4.	$38\overline{)3306}$	$92\overline{)3220}$	$56\overline{)1344}$
5.	$57\overline{)4674}$	$24\overline{)1517}$	$60\overline{)3311}$
6.	$53\overline{)2340}$	$72\overline{)3460}$	$16\overline{)1103}$
7.	$34\overline{)2087}$	$18\overline{)1376}$	$84\overline{)1945}$
8.	$44\overline{)5280}$	$26\overline{)8060}$	$39\overline{)9750}$
9.	$52\overline{)8840}$	$86\overline{)9460}$	$72\overline{)9360}$
10.	$13\overline{)1248}$	$57\overline{)4731}$	$34\overline{)3128}$


July / Aug. / Sept. / Oct. / Nov. / Dec.

Deciding Which Operation to Use

Deciding which operation to use means deciding whether to add, subtract, multiply, or divide. The problems on this page do not give all the numbers necessary to find an answer. Decide whether to add, subtract, multiply, or divide, and tell how you would solve the problem.

Study the following example, the explanation, and answer you would be expected to give.

Peggy keeps a record of the money she saves. How can she find the amount she saves in 6 months?

You are not told whether she saves the same amount each month or not. Whether she saves the same amount each month or not, six sets of money are to be combined. Addition would always give the answer.

Hence the answer would be as follows: I would add the six numbers which represent the amount of money she saves each month.

Write a statement telling how you would solve each of the following story problems.

1. At a bake sale there are 7 dozen cookies for sale. How would you find the number of cookies?

2. You know the cost of 1 movie ticket. You buy tickets for yourself and 3 of your friends. How do you find the cost of all the tickets?

3. Joe can buy new shoe skates for $12.95 or used skates for less. How can you find the difference between the prices of the skates?

4. At Monroe School, a fifth-grade pupil may be assigned to any of 3 classrooms. How do you find the average number of pupils assigned to a fifth-grade room?

5. Jack, Polly, and Roger each brought pictures for the bulletin board. How can you find the total number of pictures brought by the three pupils?

Reviewing the Basic Operations

In doing any problem or computation in arithmetic, strive for accuracy. A problem done incorrectly is almost a waste of time. With practice, your speed in computation should increase. Do not sacrifice accuracy for speed.

Oral What numeral must replace n so that each of the following equations becomes true?

	a	*b*
1.	$4\times8=n$	$15\times10=n$
2.	$43\times8=n$	$75-8=n$
3.	$200\div10=n$	$2000\div4=n$
4.	$12\times0=n$	$173+0=n$
5.	$512\times1=n$	$12\times6=n$
6.	$81\div9=n$	$76+12=n$
7.	$62\times5=n$	$143-23=n$
8.	$254-43=n$	$1200\div30=n$
9.	$284\div4=n$	$n\times7=56$
10.	$(3\times2)\times n=42$	$91-n=91$
11.	$48\times4=n$	$618+32=n$
12.	$2400\div80=n$	$7200\div9=n$
13.	$(4\times7)-n=25$	$5+6+17=n$
14.	$237-18=n$	$31+27=n$

Written Copy. Find the sum, difference, product, or quotient as indicated by the operation sign.

	a	*b*	*c*
1.	$47\overline{)705}$	$7\overline{)2282}$	$15\overline{)5100}$
2.	$26\overline{)312}$	$72\overline{)2232}$	$30\overline{)3690}$
3.	$\begin{array}{r}412\\-259\\\hline\end{array}$	$\begin{array}{r}3417\\-976\\\hline\end{array}$	$\begin{array}{r}5291\\-3757\\\hline\end{array}$
4.	$\begin{array}{r}4296\\356\\+2473\\\hline\end{array}$	$\begin{array}{r}752\\98\\+4327\\\hline\end{array}$	$\begin{array}{r}8762\\3594\\+4276\\\hline\end{array}$
5.	$\begin{array}{r}315\\\times27\\\hline\end{array}$	$\begin{array}{r}4672\\\times56\\\hline\end{array}$	$\begin{array}{r}5017\\\times84\\\hline\end{array}$
6.	$\begin{array}{r}730\\-504\\\hline\end{array}$	$\begin{array}{r}501\\-87\\\hline\end{array}$	$\begin{array}{r}4009\\\times302\\\hline\end{array}$
7.	$\begin{array}{r}7092\\307\\+4060\\\hline\end{array}$	$\begin{array}{r}3427\\5090\\+4675\\\hline\end{array}$	$\begin{array}{r}4982\\38\\+756\\\hline\end{array}$
8.	$\begin{array}{r}576\\\times342\\\hline\end{array}$	$\begin{array}{r}384\\\times517\\\hline\end{array}$	$\begin{array}{r}453\\\times269\\\hline\end{array}$

Find the replacement for n which makes each of the following statements true.

	a	*b*
9.	$(42\times8)\div6=n$	$(57+18)-41=n$
10.	$(17+34)\times9=n$	$(124-39)\div5=n$

Solving Problems

The following suggestions for solving a problem should enable you to solve the problems on this page by yourself.

a. Read the problem carefully.

b. Decide which operation or operations to use, and in which order you will use them.

c. Write a suitable equation.

d. Estimate the answer.

e. Solve the equation.

f. Compare the answer with the estimated answer.

g. Write the answer and give the kind of units or things asked for in the problem.

1. During 1958 there were 1,382 traffic deaths in Michigan, and in 1959 there were 1,467 traffic deaths. How many more people were killed in 1959 than in 1958 by traffic accidents in Michigan?

2. Lake Superior has a depth of 1,302 feet. Lake Huron is 750 feet deep. What is the difference in depth of the two lakes?

3. The Empire State Building is 1,472 feet high. In 1950 a 222-foot television sending tower was built on top of it. What is the distance from the ground to the top of the television tower?

4. Rhode Island has an area of about 1,215 square miles. The state is divided into 5 counties. What is the average number of square miles per county?

5. Lake Baikal in Asia is 5,712 feet deep. This depth is how many feet greater than one mile?

6. The Statue of Liberty weighs approximately 225 tons. What is its approximate weight in pounds?

7. A plane flew from New York to Miami, a distance of 1,092 miles, in 3 hours. How many miles an hour did the plane average?

8. A rocket reached a height of 250 miles above the earth's surface. What is this height in feet?

9. Golden Gate Bridge has a span of 4200 ft.; Mackinac Bridge, 3800 ft.; and George Washington Bridge, 3500 ft. What is the combined length of the spans of the 3 bridges?

10. The length of Lake Superior is 383 miles; Lake Huron, 206 miles; Lake Michigan, 322 miles; Lake Erie, 241 miles; and Lake Ontario, 193 miles. What is the average length in miles?

Dividing Hundreds, Tens, and Ones in Four-Digit Numbers

As you study the following divisions, do the computation yourself and compare your work with that shown.

A

$$\begin{array}{r} 251 \\ 32\overline{)8032} \\ \underline{64} \\ 163 \\ \underline{160} \\ 32 \\ \underline{32} \\ 0 \end{array}$$

B

$$\begin{array}{r} 124 \\ 27\overline{)3348} \\ \underline{27} \\ 64 \\ \underline{54} \\ 108 \\ \underline{108} \\ 0 \end{array}$$

In division A, the quotient must be a 3-digit number. Why? (Hint: Try 100 and 1000 as quotients.) Round 32 to 30 and find the first trial quotient: 8 Th÷3 T=2 H r2 Th. So use 2 as the first trial quotient. In which place-value position is it written? Why? How would you find the next trial quotient? How many trial quotients must you find in completing division A?

In division B, round 27 to 30 to find trial quotients. Why? 3 Th÷3 T=1 H, so the first quotient digit is 1. Why is it written in hundreds place? The next trial quotient is found from 6 H÷3 T=2 T. Why? However, the trial quotient for ones place, 10 T÷3 T=3 r1 T, does not give the correct digit for ones place. How can you tell that 3 is incorrect for ones place in the quotient?

MORE PRACTICE PAGE 308

Oral Answer the following questions about the solution of the equation 9030÷42=n.

1. Does n represent a value greater than 200? Greater than 300? Why?

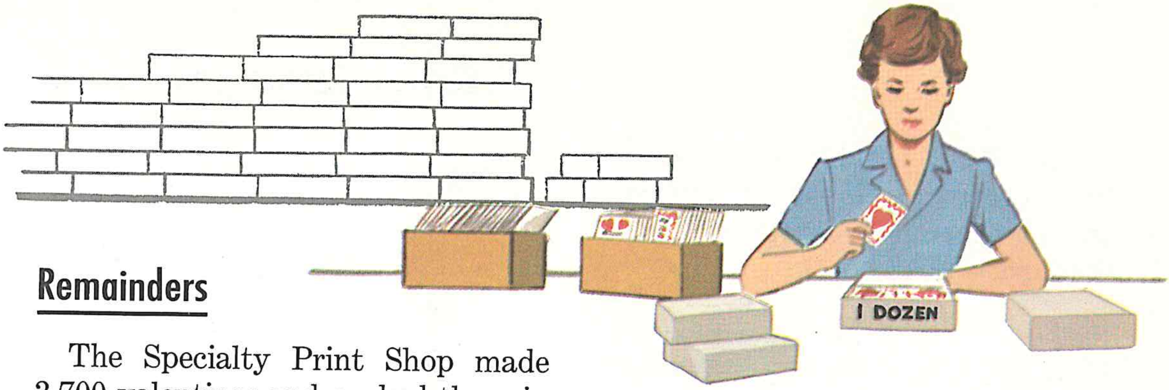
2. Is the value of n nearer to 200 or 300? Why?

3. How would you find the first trial quotient?

4. What is the hundreds digit of the quotient?

Written Copy. Find each quotient.

	a	*b*	*c*
1.	$30\overline{)6960}$	$21\overline{)2982}$	$31\overline{)6851}$
2.	$26\overline{)8346}$	$60\overline{)7380}$	$39\overline{)8463}$
3.	$20\overline{)6480}$	$41\overline{)6081}$	$36\overline{)7452}$
4.	$32\overline{)3904}$	$29\overline{)9454}$	$41\overline{)5082}$
5.	$37\overline{)8991}$	$72\overline{)8064}$	$17\overline{)5457}$
6.	$56\overline{)6832}$	$13\overline{)4290}$	$83\overline{)9296}$
7.	$15\overline{)6720}$	$12\overline{)4944}$	$24\overline{)7536}$
8.	$25\overline{)5375}$	$91\overline{)9737}$	$19\overline{)8037}$



Remainders

The Specialty Print Shop made 3,700 valentines and packed them in boxes of 1 dozen each. How many boxes were needed and how many valentines were left over?

The division for this problem is shown in A below.

A	B
$\begin{array}{r} 308\ \text{r}4 \\ 12\overline{)3700} \\ \underline{36} \\ 100 \\ \underline{96} \\ 4 \end{array}$	$\begin{array}{r} 308\ \text{r}10 \\ 15\overline{)4630} \\ \underline{45} \\ 130 \\ \underline{120} \\ 10 \end{array}$

In A, using 3 as the hundreds digit in the quotient, multiply 3×12=36. This actually means 3 H×12=36 H or 300×12=3600. Then subtract: 3700−3600=100. You still need to divide 100 by 12. You have already divided hundreds, so think of 100 as 10 T. Since 12 will not divide 10 T a whole number of times, write 0 in tens place. Now think of 10 T as 100 ones. Since 8×12=96, write 8 in ones place. The remainder is 100−96 or 4.

The Specialty Print Shop needed 308 boxes, and 4 valentines were left unpacked.

Oral Explain the steps used in division B. Also tell how you would check this division.

Written Copy. Find the quotient and remainder in each division.

	a	*b*	*c*
1.	$40\overline{)8375}$	$20\overline{)4113}$	$48\overline{)9876}$
2.	$29\overline{)6050}$	$14\overline{)4328}$	$68\overline{)7051}$
3.	$17\overline{)6889}$	$52\overline{)5695}$	$16\overline{)1677}$
4.	$11\overline{)1200}$	$35\overline{)7292}$	$22\overline{)8895}$
5.	$75\overline{)8034}$	$84\overline{)9094}$	$36\overline{)7251}$
6.	$27\overline{)8291}$	$15\overline{)6110}$	$25\overline{)7629}$
7.	$15\overline{)1610}$	$32\overline{)9864}$	$20\overline{)8189}$

MORE PRACTICE PAGE 308

Division Review and Practice

You should be able to answer all the questions and do the computation on this page without referring to other parts of the book.

Part 1 Discuss the following questions about the division, 8748÷27, without doing the computation.

1. Which number is the dividend?

2. Can the quotient be a 4-digit number? Why or why not?

3. How many digits are in the quotient?

4. Is the quotient nearer 300 or 400? Why?

5. What is your estimate of the quotient?

Part 2 Check each division by multiplying the divisor by the quotient and adding the remainder. If the division is not correct, explain the mistake which was made.

A

$$\begin{array}{r} 31\ \text{r}17 \\ 41\overline{)1298} \\ \underline{123} \\ 58 \\ \underline{41} \\ 17 \end{array}$$

B

$$\begin{array}{r} 43\ \text{r}3 \\ 18\overline{)7257} \\ \underline{72} \\ 57 \\ \underline{54} \\ 3 \end{array}$$

C

$$\begin{array}{r} 246 \\ 37\overline{)9102} \\ \underline{74} \\ 170 \\ \underline{148} \\ 222 \\ \underline{222} \\ 0 \end{array}$$

D

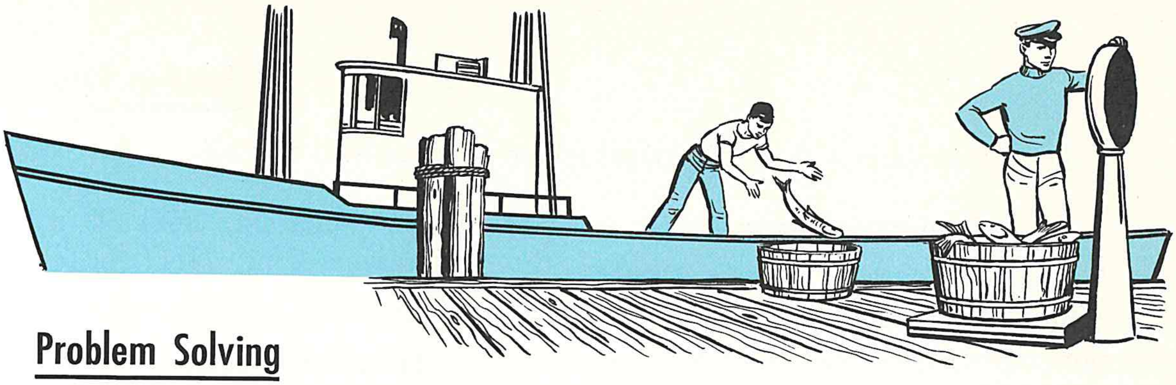
$$\begin{array}{r} 256\ \text{r}20 \\ 23\overline{)5897} \\ \underline{46} \\ 129 \\ \underline{115} \\ 148 \\ \underline{128} \\ 20 \end{array}$$

Part 3 Write the answers to the following divisions. Do not copy—find the quotient mentally.

	a	*b*	*c*
1.	$20\overline{)600}$	$10\overline{)250}$	$40\overline{)280}$
2.	$50\overline{)2500}$	$20\overline{)1240}$	$13\overline{)130}$
3.	$21\overline{)4200}$	$90\overline{)7200}$	$4\overline{)288}$
4.	$5\overline{)2505}$	$25\overline{)1000}$	$15\overline{)450}$
5.	$16\overline{)4800}$	$8\overline{)3216}$	$7\overline{)224}$

Part 4 Copy. Find each answer.

	a	*b*	*c*
1.	$36\overline{)6120}$	$54\overline{)3080}$	$25\overline{)5760}$
2.	$44\overline{)2344}$	$28\overline{)2792}$	$37\overline{)3000}$
3.	$31\overline{)3514}$	$18\overline{)5849}$	$15\overline{)6109}$



Problem Solving

Read each problem on this page carefully. Decide upon the operation to use. For each problem write a suitable equation, the computation, and the answer to the problem.

1. Mr. Crow handled 3055 insurance claims last year. His average number of claims per day was 13. How many days did he work on insurance claims last year?

2. The United States Capitol Building is located on a plateau 88 feet above sea level. The distance from the ground to the top of the building is 287 feet. What is the distance in feet from sea level to the top of the building?

3. The Elk Mountain Ski Lodge is open 23 weeks each winter. Last year 2047 persons stayed at the lodge. What was the average number of persons there each week?

4. Jay's Electric Shop ordered 155 cartons of light switches. Each carton contained 15 switches. How many switches were ordered?

5. Some men caught 4340 pounds of fish one day, 5172 pounds the next day, and 3374 pounds the third day. How many pounds of fish did they catch in the three days?

6. A statute mile has a length of 5280 feet and a nautical mile has a length of 6080 feet. What is the difference in length?

7. If your heart beats 68 times each minute, how many times does it beat in 1 hour?

8. The planet Mercury has 88 days in each of its years. Our year has 365 days. How many of its years does Mercury have during one of our years? How many days are left over?

9. The planet Jupiter has about 4380 days in its year. This length of time is equivalent to how many of our weeks?

Something to do Make up 5 problems like the ones on this page. Then write an equation and find the answer for each of your problems.

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Estimate the quotient before doing the computation for any division. (147, 149, 157, 158, 160)

2. For each digit of the quotient you must find a trial quotient. (150, 153, 158)

3. When using a 2-digit divisor the remainder is either a 1-digit or 2-digit number. (150, 151, 159)

4. When working a story problem, take time to think carefully about the problem before you start writing the solution. (152, 157)

5. You cannot do good work in division unless you can multiply and subtract accurately. (154)

6. Strive for accuracy in all arithmetic computations. (156)

Questions to Discuss

1. How many digits are in the quotient of 3641 ÷ 78? (147, 149)

2. Before writing the equation for a story problem, what questions should you ask yourself about the problem? (147, 152)

3. What trial quotient is used for tens place in 3756 ÷ 86? (148)

4. In the equation, 36H ÷ 5T = n, the n represents 7Tr1, 7Tr1T, or 7T r1H? (150)

Written Practice

Copy. Complete each division.

	a	*b*	
1.	$80\overline{)3200}$	$52\overline{)4160}$	(148)
2.	$76\overline{)2584}$	$22\overline{)1518}$	(149)
3.	$50\overline{)2162}$	$37\overline{)1599}$	(151)
4.	$30\overline{)8402}$	$47\overline{)5930}$	(153)
5.	$26\overline{)5538}$	$40\overline{)8920}$	(158)
6.	$30\overline{)9415}$	$28\overline{)6920}$	(159)
7.	$14\overline{)3626}$	$45\overline{)8640}$	(159)

Solve each of the following written problems.

8. The average weight of a king salmon is 23 pounds. A fishing boat brought in a 4922-pound load of salmon. How many salmon were in the load? (152)

9. Mr. Jones drove 1352 miles in 26 hours. What was the average number of miles per hour? (152)

Self-Evaluation

Part 1 Copy. Find a replacement for n which makes each of the following equations true.

	a	*b*
1.	$3200 \div 40 = n$	$1600 \div n = 20$
2.	$6200 \div 20 = n$	$9300 \div n = 30$
3.	$3570 \div 70 = n$	$2520 \div n = 60$
4.	$9000 \div 15 = n$	$1600 \div n = 32$

Part 2 Each division statement is followed by four statements. Which statements are true about the division? In each case, two statements are correct.

1. 3721 divided by 87

a. The quotient has 3 digits.

b. The first trial quotient should be 4.

c. 3721 is the dividend.

d. The remainder could be 87.

2. 6571 divided by 30

a. 30 is the divisor.

b. The quotient has 3 digits.

c. The first digit in the quotient should be in tens place.

d. The remainder could be 0.

3. 4872 divided by 25

a. The quotient is a number between 10 and 100.

b. The quotient is a number between 100 and 200.

c. The quotient is nearer 100 than 200.

d. The remainder is not 0.

Part 3 Copy. Find each sum, product, difference, or quotient as indicated by the operation sign.

	a	*b*	*c*
1.	$58\overline{)1624}$	$36\overline{)3492}$	$84\overline{)5796}$
2.	$32\overline{)8032}$	$29\overline{)3074}$	$76\overline{)9324}$
3.	$40\overline{)9080}$	$70\overline{)6930}$	$13\overline{)7891}$
4.	$17\overline{)1394}$	$55\overline{)2530}$	$91\overline{)4186}$
5.	$\begin{array}{r}7258\\+5926\\\hline\end{array}$	$\begin{array}{r}6581\\-4753\\\hline\end{array}$	$\begin{array}{r}3796\\\times 507\\\hline\end{array}$
6.	$\begin{array}{r}3090\\\times 764\\\hline\end{array}$	$\begin{array}{r}2827\\-1958\\\hline\end{array}$	$\begin{array}{r}4754\\\times 865\\\hline\end{array}$

Part 4 Solve each of the following problems by using an equation.

1. The bakery made 6480 loaves of bread in 16 hours. What was the average number of loaves per hour?

2. Mr. Bowman needed 3460 plastic wall tile to finish decorating his kitchen. The tile are packed 48 per carton. How many cartons and how many extra tile did he need?

Midyear Review

On your paper, write the answer to each exercise on these two pages. Then you will have answers to check during the class discussion. The numerals in ( ) refer to pages where you can turn for help. Use these pages to check your answers.

Number Meanings

1. The decimal system is based on what number? (5)

2. What does each digit in the numeral 321,674 represent? (7)

3. Name the largest number possible by using each of the digits 2, 3, 6, 7, and 0 only once. (7)

4. How do you round a number to the nearer 10? Nearer 100? (13)

Addition

1. Why is zero called the *identity number of addition?* (19)

2. Why is addition called an *operation of arithmetic?* (19)

3. Give an example to show that the order of the addends does not change the sum. (19)

4. Show that, with three addends, either the first two or last two may be grouped. (21, 22, 23)

Subtraction

1. What steps would you use for solving a story problem? (25, 38, 40, 101)

2. Explain a good way to check a subtraction? (33)

3. What is meant by *minuend, subtrahend,* and *difference?* (33)

4. What is meant by an *inverse operation?* (33)

Multiplication

1. Why is 1 called the *identity number of multiplication?* (50)

2. What is the product of two factors if one of them is 1? If one of the factors is 0? (49, 50)

3. Give an example to show that the product is unchanged when the order of the factors is changed, or when the grouping of the factors is changed. (50, 66)

Division

1. Why are multiplication and division inverses of each other? (74)

2. Why is the number 0 never used as a divisor? (75)

3. How do you find the average of five different numbers? (84)

4. How do you proceed if the trial quotient is too large? (99, 105)

5. How do you find a trial quotient for $362 \div 87 = n$? (102) For $571 \div 42 = n$? (105) For $1215 \div 23 = n$? (153)

6. Why must the remainder have 2 digits or less when the divisor has 2 digits? (150, 151)

Measures

1. What does it mean to measure something? (112)

2. What is different about direct and indirect measurement? (112)

3. How does measuring differ from counting? (113)

4. What is meant by 2 lb. = 32 oz. or 5 gal. = 20 qt.? (114)

Problem Solving

1. What symbols are used to write an arithmetic sentence? (131)

2. Why should you state what n represents in an equation? (131)

3. What does a letter, such as n, stand for in an equation? (132)

4. What is an equation? (133)

5. What does it mean to solve an equation? (133)

6. Could 2 equations be correct for a written problem? (134, 135)

Written Practice

Copy. Find each answer as indicated by the operation sign.

	a		*b*	
1.	$\begin{array}{r}79162\\+34589\\\hline\end{array}$	(30)	$\begin{array}{r}48213\\-5764\\\hline\end{array}$	(43)
2.	$\begin{array}{r}586\\\times73\\\hline\end{array}$	(62)	$\begin{array}{r}3576\\\times854\\\hline\end{array}$	(65)

Copy. Find each quotient.

	a		*b*	
3.	$8\overline{)5464}$	(87)	$23\overline{)161}$	(103)
4.	$47\overline{)3055}$	(148)	$43\overline{)5332}$	(153)

Find a replacement for n which makes each of the following true.

5. $(30+9)+(40+4)=n$ (23)

6. $7\times(5+10)=n$ (55)

7. $30\times87=n$ (66)

8. $(70+65)\div15=n$ (136)

9. $(21\div7)\times68=n$ (139)

Fill each blank so the resulting statement is true.

10. 684 ft. = __yd.__ft. (114)

11. 59 qt. = __gal.__qt. (116)

12. 55 pk. = __bu.__pk. (117)

13. 127 sec. = __min.__sec. (122)

Midyear Tests

Test 1 Copy. Find a replacement for n which makes each of the following statements true.

	a	*b*
1.	$94+0=n$	$26\times0=n$
2.	$96+37=n$	$6+11+7+3=n$
3.	$51-8=n$	$62-47=n$
4.	$492+36=n$	$(82-60)-17=n$
5.	$36\times12=n$	$(47+53)\div20=n$
6.	$636\div6=n$	$4824\div4=n$
7.	$432\times7=n$	$(5\times8)+16=n$
8.	$5\times3510=n$	$(42\div6)+28=n$

Test 2 Copy. Write a numeral in each blank so that the statement becomes true.

1. 251 in. = __ft.__in.

2. 12 mi. = __ft.

3. 157 hr. = __da.__hr.

4. 63 pk. = __bu.__pk.

5. 18 yd. = __in.

6. 369 sec. = __min.__sec.

7. 5 lb. 7 oz. = __oz.

8. 9 qt. 1 pt. = __pt.

Test 3 Copy. Find each answer as indicated by the operation sign.

	a	*b*	*c*
1.	$\begin{array}{r}543\\-36\\\hline\end{array}$	$\begin{array}{r}631\\-482\\\hline\end{array}$	$\begin{array}{r}12476\\-8397\\\hline\end{array}$
2.	$\begin{array}{r}475\\86\\351\\+37\\\hline\end{array}$	$\begin{array}{r}3218\\427\\5849\\+632\\\hline\end{array}$	$\begin{array}{r}5761\\2840\\6037\\+1909\\\hline\end{array}$
3.	$\begin{array}{r}572\\\times7\\\hline\end{array}$	$\begin{array}{r}56\\\times34\\\hline\end{array}$	$\begin{array}{r}724\\\times59\\\hline\end{array}$
4.	$\begin{array}{r}3057\\\times46\\\hline\end{array}$	$\begin{array}{r}4008\\\times305\\\hline\end{array}$	$\begin{array}{r}2153\\\times346\\\hline\end{array}$

Copy. Find each answer.

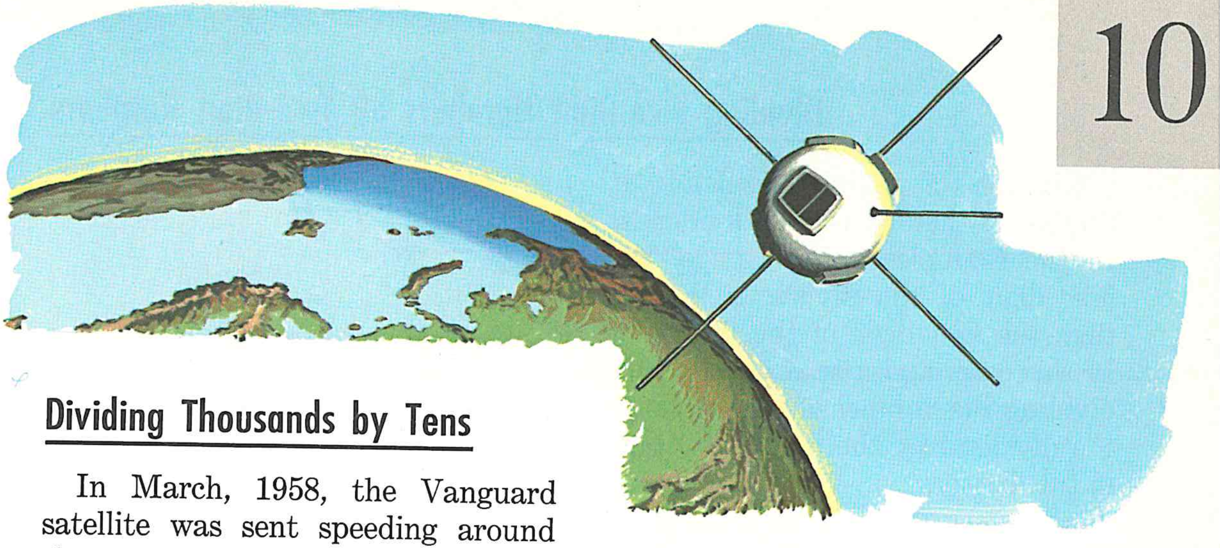
	a	*b*	*c*
5.	$8\overline{)608}$	$7\overline{)2674}$	$4\overline{)6988}$
6.	$73\overline{)532}$	$35\overline{)840}$	$31\overline{)6721}$
7.	$76\overline{)4408}$	$32\overline{)6689}$	$31\overline{)3561}$

Test 4 Solve each problem. State the equation used.

1. Mrs. Reed took 4 dozen candy bars to the picnic. The children ate 39 of them. How many were left?

2. Pam read 54 names in 3 minutes. At this rate, how many names can she read in 5 minutes?

10



Dividing Thousands by Tens

In March, 1958, the Vanguard satellite was sent speeding around the earth at approximately 18,000 miles an hour. This was how many miles each minute?

You are asked to find the number of miles per minute. Think of 1 hour as 60 minutes. The satellite traveled 18,000 miles in 60 minutes. By dividing 18,000 by 60, as shown, you find that it traveled 300 miles each minute.

$$\begin{array}{r} 300 \\ 60\overline{)18000} \\ 180 \\ \hline 00 \end{array}$$

The 6 in the divisor and the 18 in the dividend probably indicate to you that there would have to be a 3 in the quotient. But how do you find the place-value position for 3 in the quotient?

Multiplying by 100 and 1000 is easy, so think of the following equations to find the number of digits in the quotient.

$$\begin{array}{rr} 100\times 60 = 6000 & 6000 \div 60 = 100 \\ n\times 60 = 18000 & 18000 \div 60 = n \\ 1000\times 60 = 60000 & 60000 \div 60 = 1000 \end{array}$$

Hence the quotient must have three digits. Why? Then you can think: 18 Th ÷ 6 T = 3 H. Write 3 in the hundreds place in the quotient. Why do you write 0 in the tens place and 0 again in the ones place?

Oral Give the quotient for each of the following divisions.

	a	*b*
1.	$60\overline{)30000}$	$70\overline{)21000}$
2.	$40\overline{)24000}$	$80\overline{)40000}$
3.	$50\overline{)25000}$	$30\overline{)15000}$
4.	$90\overline{)36000}$	$60\overline{)54000}$
5.	$80\overline{)64000}$	$20\overline{)14000}$
6.	$30\overline{)12000}$	$70\overline{)35000}$

Dividing Five-Digit Numbers by Two-Digit Numbers

Division of 5-digit dividends is similar to that of 4-digit dividends. In the examples on this page, the first digit of the divisor is greater than the first digit of the dividend. In this case, find a trial quotient by dividing the number of thousands in the dividend by the number of tens in the divisor.

$$\begin{array}{rl} 656 & \text{Trial Quotients} \\ 40\overline{)26240} & 26\text{ Th}\div 4\text{ T}=n \\ \underline{240} & \\ 224 & 22\text{ H}\div 4\text{ T}=n \\ \underline{200} & \\ 240 & 24\text{ T}\div 4\text{ T}=n \\ \underline{240} & \\ 0 & \end{array}$$

The equations for trial quotients are solved below.

$26\text{ Th}\div 4\text{ T}=6\text{ H r}2\text{ Th}$

$22\text{ H}\div 4\text{ T}=5\text{ T r}2\text{ H}$

$24\text{ T}\div 4\text{ T}=6$

MORE PRACTICE PAGE 308

Remember that the remainders have nothing to do with determining the trial quotients. The quotients, 6 H, 5 T, and 6, tell what to use as the trial quotients and their place value in the quotient.

$$\begin{array}{rl} 321\text{ r}11 & \text{Trial Quotients} \\ 54\overline{)17345} & 17\text{ Th}\div 5\text{ T}=n \\ \underline{162} & \\ 114 & 11\text{ H}\div 5\text{ T}=n \\ \underline{108} & \\ 65 & 6\text{ T}\div 5\text{ T}=n \\ \underline{54} & \\ 11 & \end{array}$$

Oral Answer the following questions about the division above.

1. Why was 5 used as a divisor in finding trial quotients?

2. What does each trial-quotient equation represent?

Written Copy. Find each quotient and remainder.

	a	*b*
1.	$50\overline{)15250}$	$70\overline{)21700}$
2.	$42\overline{)85260}$	$20\overline{)14040}$
3.	$12\overline{)10824}$	$19\overline{)15276}$
4.	$64\overline{)44864}$	$45\overline{)13590}$
5.	$40\overline{)32385}$	$41\overline{)33049}$
6.	$86\overline{)43516}$	$37\overline{)25718}$
7.	$32\overline{)17642}$	$48\overline{)42670}$

Division Review and Practice

For each division below, tell:

a. How many digits there will be in the quotient.

b. Where you write the first digit of the quotient.

c. What you would use as the first digit in the quotient.

	a	*b*	*c*
1.	$57\overline{)314}$	$8\overline{)5462}$	$6\overline{)86501}$
2.	$42\overline{)862}$	$12\overline{)3008}$	$37\overline{)14263}$
3.	$62\overline{)637}$	$72\overline{)3256}$	$52\overline{)41598}$

Each of the following statements is followed by three possible answers. Choose the correct answer.

4. If you change the 4 to 0 in 564,000, is the number smaller, larger, or the same?

5. If $4215 \div 92 = n$, does n represent a number of 1, 2, or 3 digits?

6. If 56 is the divisor, can the remainder be 10, 56, or 72?

7. If 23 is divided by 0, is the quotient 0, 23, or meaningless?

8. If $0 \div 6 = n$, does n represent 6, 0, or no number?

9. Is $4270 \div 61 = 7$, 70, or 700?

Copy. Find the answer for each division below.

	a	*b*
10.	$82\overline{)7954}$	$7\overline{)22514}$
11.	$9\overline{)7232}$	$32\overline{)19968}$
12.	$92\overline{)3775}$	$61\overline{)42768}$
13.	$33\overline{)6785}$	$66\overline{)19808}$

Find and correct the mistakes in these divisions.

a *b*

14.

$$\begin{array}{r} 4\ \text{r}\,67 \\ 91\overline{)421} \\ \underline{364} \\ 67 \end{array} \qquad \begin{array}{r} 6\ \text{r}\,9 \\ 58\overline{)327} \\ \underline{348} \\ 9 \end{array}$$

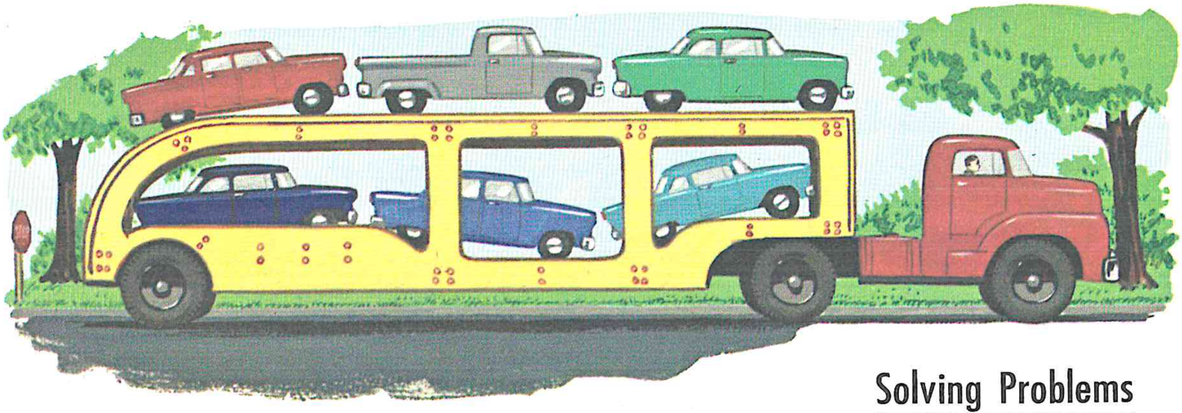
15.

$$\begin{array}{r} 53\ \text{r}\,31 \\ 43\overline{)21831} \\ \underline{205} \\ 133 \\ \underline{129} \\ 31 \end{array} \qquad \begin{array}{r} 62\ \text{r}\,37 \\ 52\overline{)32601} \\ \underline{312} \\ 140 \\ \underline{104} \\ 37 \end{array}$$

Can you do this? Replace each dot in the divisions below.

a *b* *c*

$$\begin{array}{r} 65 \\ \bullet\overline{)58\bullet} \\ \underline{\bullet 4} \\ 4\bullet \\ \underline{4\bullet} \\ 0 \end{array} \qquad \begin{array}{r} \bullet 1 \\ 7\overline{)3\bullet\bullet} \\ \underline{\bullet\bullet} \\ \bullet \\ \underline{\bullet} \\ 0 \end{array} \qquad \begin{array}{r} \bullet\bullet \\ 7\overline{)1\bullet\bullet} \\ \underline{\bullet\bullet} \\ 1\bullet \\ \underline{\bullet\bullet} \\ 0 \end{array}$$



Solving Problems

The following suggestions for solving a problem should enable you to solve the problems on this page by yourself.

a. Read the problem carefully.

b. Decide which operation or operations to use.

c. Write an equation.

d. Estimate the answer.

e. Solve the equation.

f. Compare your answer with the estimated answer.

g. Write the answer and give the kind of units or things asked for in the problem.

1. A plane flew the 2226 miles from Honolulu to Los Angeles in 7 hours. What was its average speed in miles per hour?

2. When John's father ran for mayor he received 1821 votes in Ward 1, 2015 in Ward 2, 879 in Ward 3, and 1758 in Ward 4. How many votes did he get from all four wards?

3. Ronald's father drove 10,140 miles in 12 months. What was the average number of miles he drove each month?

4. The length of the earth's equator is about 24,900 miles. Mars' equator is about 13,195 miles. What is the difference between these lengths?

5. A transport truck carried six new automobiles. The average weight of each automobile was 3750 pounds. What was the weight of the six automobiles?

6. At the start of a trip the speedometer on Mr. Smith's car read 32,412 miles. After 11 hours of driving, it read 33,017 miles. What was his average speed?

7. The weight of 3 melons is 4 lb. 8 oz. What is the average weight of each melon in ounces?

Four-Digit Quotients

The population of North Dakota decreased by about 69,000 people from 1930 to 1950. What was the average decrease per year?

To find the average decrease per year, you divide 69,000 by 20. Why do you divide by 20?

Think of the following products before doing the division.

$$\begin{aligned} 1000\times20 &= 20000 \\ n\times20 &= 69000 \\ 10000\times20 &= 200000 \end{aligned}$$

How many digits must there be in the quotient? Why?

The division below shows that the population decreased about 3,450 people per year.

$$\begin{array}{r}
3450\\
20\overline{)69000}\\
\underline{60}\\
90\\
\underline{80}\\
100\\
\underline{100}\\
0
\end{array}
\qquad\qquad
\begin{array}{r}
\text{Check}\\
3450\\
\underline{\times20}\\
69000
\end{array}$$

You can find a trial quotient from the equation $69\div20=3$ r9, or $6\div2=3$. This last equation is the number of ten-thousands in the dividend divided by the number of tens in the divisor and can be written 6 TTh ÷ 2 T = 3 Th.

Study the following division and equations for trial quotients.

$$\begin{array}{r}
3240\\
24\overline{)77760}\\
\underline{72}\\
57\\
\underline{48}\\
96\\
\underline{96}\\
0
\end{array}
\qquad\qquad
\begin{array}{l}
\text{Trial Quotients}\\
7\ \text{TTh}\div2\ \text{T}=n\\
5\ \text{Th}\div2\ \text{T}=n\\
9\ \text{H}\div2\ \text{T}=n
\end{array}$$

Oral Explain the division above. Also tell what each trial quotient equation represents.

Written Copy. Find each quotient and the remainder.

	a	*b*
1.	$20\overline{)80000}$	$30\overline{)81000}$
2.	$40\overline{)82400}$	$50\overline{)77350}$
3.	$30\overline{)72531}$	$31\overline{)74865}$
4.	$25\overline{)67500}$	$13\overline{)65117}$
5.	$36\overline{)72108}$	$19\overline{)38131}$
6.	$47\overline{)94213}$	$62\overline{)62310}$
7.	$73\overline{)85261}$	$88\overline{)92502}$

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Deciding Which Operation to Use

The problems on this page do not give all the information necessary to find an answer. Write a sentence to explain how you would solve each problem if all the information were given. Problem 1 is answered for you.

1. The school bought 18 new records for the music class. What was the average cost per record?

Answer: Divide the cost of all the records by 18.

2. The combined weight of a truck and its weight of groceries is 13,670 pounds. What is the weight of the load of groceries?

3. The Junior Red Cross members packed 243 items of clothing into boxes. What was the average number of items in each box?

4. Ed weighs 65 pounds and Mark 72 pounds. What is the combined weight of Ed, Mark, and Bill?

5. Jean walks 5 blocks to school each day, and takes a shorter route coming home. How many blocks does she walk in going to and from school in 8 days?

6. Five girls agreed to share equally the expenses for a trip to the carnival. They paid $2.00 for bus fare, had lunch, and rode the Ferris wheel. What was each girl's share of the expenses?

7. Mrs. Hector bought 3 dozen eggs. Carol used some of them to bake a cake. How many eggs were left?

8. Robert had 17 rabbits. Some of them got out and ran away. He separated the rest of them equally among 3 new pens. How many rabbits did he put in each pen?

9. Ruth paid $2.75 for a ring and also bought a bracelet. Joan paid $3.15 for a necklace. Ruth spent how much more than Joan?

10. The speed of an airplane is given in miles per hour. How far will it fly in 7 hours?

A quick review Find a value of n which makes each of the following statements true.

	a	*b*
1.	$(32 \div 8) \times 16 = n$	$(20 \div 4) \times n = 15$
2.	$72 - (14 \times 3) = n$	$n - (15 \div 3) = 18$
3.	$(14 + 88) - 76 = n$	$(7 + 13) - n = 6$
4.	$(12 \div 3) + 40 = n$	$n + (14 \times 5) = 82$
5.	$0 \div (50 \times 80) = n$	$n \div (17 - 12) = 0$
6.	$4 + (59 \div 59) = n$	$(62 \div 2) - n = 0$

Using Two Operations to Solve a Problem

The problems on this page require the use of two operations. Write an equation for each problem to help you see the relationships in short, easy form.

Study the following example before proceeding with the problems.

Fair Weather Gardens harvested 25,650 pounds of apples and sold them for $2 a bushel. A bushel of apples weighs 50 pounds. How much money was received?

Let n represent the amount of money received for the apples. This amount can be found by multiplying the number of bushels by the price per bushel. To change 25,650 pounds to bushels you must divide by 50. A suitable equation and its solution follows.

$$(25650 \div 50) \times 2 = n$$
$$513 \times 2 = n$$
$$1026 = n$$

Fair Weather Gardens received $1,026 for the apples.

Work each of the following problems. Write a sentence telling what n represents.

1. The melting temperature of copper is 1083°C.; of gold, 1063°C.; of iron, 1533°C.; of nickel, 1455°C.; and of silver, 961°C. What is the average melting temperature of these five metals?

2. A construction firm bought motor oil in 50-gallon barrels at a cost of $45 a barrel. At this rate how much did they pay for 3,750 gallons of motor oil?

3. On June 2, 1957, a manned balloon set a new altitude record by reaching an altitude of 96,000 feet. This is how many feet more than 18 miles?

4. Charles was paid $.70 an hour for helping clean his father's store. Last month he worked 2 hours a day for 18 days. How much money did he earn?

5. Mr. Stapp bought 31 dozen oranges for 42¢ a dozen. He sold them for 57¢ a dozen. How much money did he make on the sale of the oranges?

6. A truckload of wheat weighed 13,020 pounds and sold for $2.30 per bushel. A bushel of wheat weighs 60 pounds. How much was paid for the load of wheat?

7. Six boys and eight girls are going on a picnic. Barbara offered to make 3 sandwiches for each of them. How many sandwiches should she make?

2 H÷5 T=4	200÷50= 4	8 H÷4 T=2 T	800÷40= 20
2 Th÷5 T=4T	2000÷50= 40	8 Th÷4 T=2 H	8000÷40= 200
2 TTh÷5 T=4 H	20000÷50= 400	8 TTh÷4 T=2 Th	80000÷40= 2000
2 HTh÷5 T=4 Th	200000÷50=4000	8 HTh÷4 T=2 TTh	800000÷40=20000

Dividing Six-Digit Numbers by Two-Digit Numbers

An airport needed 420,000 gallons of aviation fuel. A petroleum transport company hauled the fuel in 70 trucks. What was the average number of gallons per truck?

To answer the question in the problem, divide 420,000 by 70. Ask yourself: How many digits must the quotient have?

$$1000 \times 70 = 70000$$
$$n \times 70 = 420000$$
$$10000 \times 70 = 700000$$

Since 420,000 is between 70,000 and 700,000, the quotient must have 4 digits. Why?

The division is shown below.

```
      6000
 70)420000
    420
    ------
       000
```

In the above division, you obtain a trial quotient by dividing the first two digits of the dividend by the first digit of the divisor.

This can be symbolized by the equation, 42÷7=6. This tells only the trial quotient and not the place value. In order to determine place-value position, think of the equation above as follows.

42 TTh÷7 T=6 Th

Study the divisions below.

A

```
      3410
 40)136400
    120
    ---
     164
     160
     ---
       40
       40
       --
        0
```

B

```
      3205
 43)137815
    129
    ---
      88
      86
      ---
       215
       215
       ---
         0
```

In A, 13÷4 gives a trial quotient of 3. Why was it written in thousands place? Why was 0 written in ones place in the quotient?

In B, 43 was rounded to 40 in finding trial quotients. What are the equations for finding the trial quotients?

In divisions C and D, the first digit of the divisor is less than the first digit of the dividend. In this case the trial quotient can be obtained by dividing the first digit of the dividend by the first digit of the divisor.

C	**D**
42150	15745
$20\overline{)843000}$	$31\overline{)488095}$
80	31
43	178
40	155
30	230
20	217
100	139
100	124
0	155
	155
	0

How would you find the first trial quotient in C? How many trial quotients must you find? Why is 0 written in ones place? How do you read the dividend? The quotient?

In D, what equation would give the first trial quotient? How many trial quotients must you find? How do you read the quotient?

Oral For each division tell:

a. How many digits there will be in the quotient.

b. The first trial quotient.

c. Place-value position of the first trial quotient.

	a	*b*
1.	$40\overline{)200000}$	$50\overline{)150000}$
2.	$30\overline{)900000}$	$20\overline{)120000}$
3.	$60\overline{)366000}$	$30\overline{)846000}$
4.	$72\overline{)458172}$	$29\overline{)721954}$
5.	$37\overline{)759281}$	$67\overline{)348534}$

Written Copy. Find the quotient for each of the following divisions.

	a	*b*
1.	$30\overline{)816300}$	$70\overline{)289240}$
2.	$53\overline{)125133}$	$48\overline{)249840}$
3.	$69\overline{)242328}$	$73\overline{)449461}$
4.	$26\overline{)106990}$	$13\overline{)478647}$
5.	$18\overline{)114570}$	$82\overline{)107994}$
6.	$75\overline{)242475}$	$21\overline{)725655}$
7.	$45\overline{)135180}$	$11\overline{)562364}$
8.	$23\overline{)784921}$	$17\overline{)768502}$
9.	$55\overline{)396770}$	$55\overline{)204875}$
10.	$61\overline{)285602}$	$67\overline{)579617}$

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More Six-Digit Numbers to Divide

In the divisions below, the first digit of the divisor is the same as the first digit of the dividend. In this case, compare to see which is larger: the divisor or the number formed by the first two digits of the dividend.

A

```
      9216
   ______
23)211968
   207
   ---
     49
     46
     --
      36
      23
      --
      138
      138
      ---
        0
```

B

```
     10423
   ______
47)489881
   47
   --
    198
    188
    ---
     108
      94
     ---
      141
      141
      ---
        0
```

In division A, the divisor, 23, is greater than the number formed by the first two digits of the dividend, 21. This indicates that there will not be a digit in ten-thousands place of the quotient. Why not? You might begin by using 9 in thousands place as a trial quotient.

In division B, the divisor, 47, is less than the number formed by the first two digits of the dividend, 48. The ten-thousands digit in the quotient must be 1. Why?

Avoid the common mistake of forgetting to write a 0 in the quotient when necessary. Estimating the quotient tells you how many digits the quotient must have and where to write the first trial quotient. It also helps you determine if you have written the necessary number of digits in the quotient.

Study the following divisions.

C

```
     5007 r2
   ______
31)155219
   155
   ---
      2
      0
      -
      21
       0
      --
      219
      217
      ---
        2
```

D

```
     5007 r2
   ______
31)155219
   155
   ---
      219
      217
      ---
        2
```

In division C, a multiplication was made each time a digit was written in the quotient. This shows why 0 was written in hundreds and in tens place of the quotient.

Since subtracting 0 from a number leaves it unchanged, it is unnecessary to multiply by 0 and make the subtractions. The division then appears as in D.

You have probably discovered that division with large dividends is no more difficult than division with small dividends. It only takes more time to complete.

The division process depends on these four important steps:

a. Determining a trial quotient.

b. Deciding the place-value position of the trial quotient.

c. Multiplying the divisor by the trial quotient.

d. Subtracting this product from the dividend.

These four steps are merely repeated as the number of digits in the dividend increases.

Oral For each division below tell what to use as the first trial quotient and where to write it.

	a	*b*
1.	$36\overline{)312645}$	$17\overline{)190570}$
2.	$29\overline{)290569}$	$51\overline{)504762}$
3.	$78\overline{)765432}$	$48\overline{)491572}$
4.	$30\overline{)300007}$	$70\overline{)241050}$
5.	$46\overline{)982476}$	$59\overline{)514268}$
6.	$33\overline{)329541}$	$65\overline{)672817}$

These multiplications are used in the divisions which follow. Solve each equation mentally.

	a	*b*
1.	$5\times24=n$	$32\times7=n$
2.	$47\times8=n$	$51\times6=n$
3.	$66\times7=n$	$72\times7=n$
4.	$3\times56=n$	$8\times81=n$
5.	$6\times39=n$	$5\times39=n$
6.	$26\times5=n$	$5\times92=n$
7.	$8\times92=n$	$44\times9=n$
8.	$8\times13=n$	$55\times6=n$

Written Copy. Find each quotient and remainder.

	a	*b*
1.	$24\overline{)267600}$	$32\overline{)324704}$
2.	$47\overline{)461822}$	$51\overline{)574464}$
3.	$66\overline{)462528}$	$72\overline{)720504}$
4.	$56\overline{)205762}$	$81\overline{)948156}$
5.	$39\overline{)643825}$	$26\overline{)781359}$
6.	$92\overline{)486540}$	$16\overline{)356816}$
7.	$38\overline{)279010}$	$44\overline{)612544}$
8.	$13\overline{)105920}$	$55\overline{)753525}$

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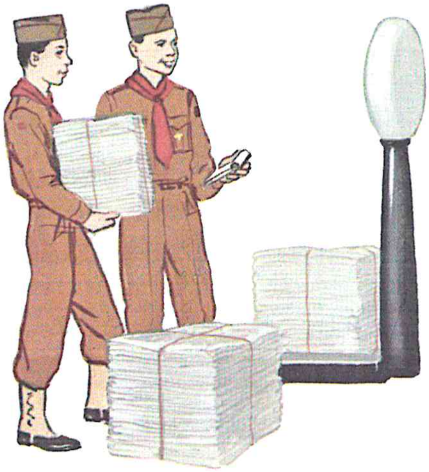
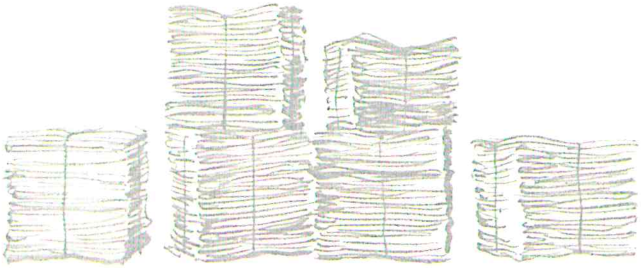




Using Division in Solving Problems

Each problem on this page requires the use of division to obtain the answer. Read each problem carefully to determine the divisor and the dividend before writing an equation. Estimate the answer. Then solve the equation.

1. A petroleum refinery sold 94,875 gallons of asphalt in 55-gallon barrels. How many barrels of asphalt did the refinery sell?

2. A truckload of grain weighed 13,440 pounds. At 56 pounds per bushel, how many bushels of grain were on the truck?

3. In 15 months Mr. Held drove 13,560 miles. At this rate, how far did he drive in 18 months?

4. Paul's father planned a 1,665-mile vacation trip. He allowed 37 hours for driving. At this rate, how many miles did he plan to travel each hour?

5. The 26 scout troops collected 37,752 pounds of paper during a paper drive. What was the average number of pounds per troop?

6. In a 9-month period during 1957 the planes of the major United States airlines flew 570,528 miles. What was the average number of miles flown per month?

7. Sound travels about 750 miles per hour in air which is at freezing temperature. About how many miles per minute does it travel under such conditions?

8. The state of Wisconsin has an area of 54,705 square miles and is divided into 71 counties. What is the average number of square miles per county?

Can you do this? If a jet plane travels 650 miles per hour, how many feet does it travel each minute? (Hint: First find how many feet per hour it travels. Then divide by 60 to change from feet per hour to feet per minute.)

Solving Problems

Solve each problem on this page with the use of an equation. Some of the problems require the use of more than one operation to obtain an answer. Write what n represents in each equation.

1. Mr. Hamilton raised 8,075 bushels of corn on an 85-acre field. What was the number of bushels he raised on each acre?

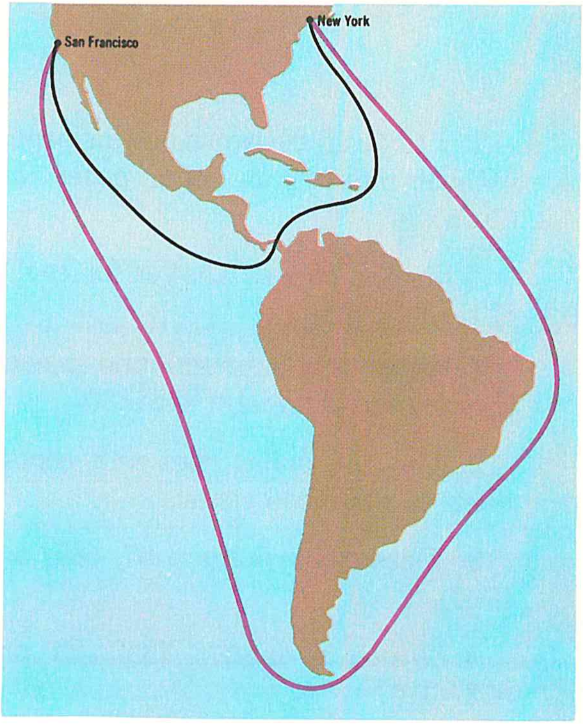
2. Light travels at a speed of 186,284 miles per second. How far does it travel in 15 seconds?

3. The deepest point in the Pacific Ocean is 35,640 feet below sea level. This is how many feet less than 7 miles?

4. In 1950 the state of Wyoming had a population of 290,536 people. The state is divided into 23 counties. What was the average population of each county?

5. Mr. Jensen's sales ticket showed that 24 of his cattle weighed 25,248 pounds. What was their average weight?

6. Suppose a bank charges 3¢ each for the first 15 checks you cash each month, and 5¢ for each additional check during the month. What would be the charge for writing 36 checks in one month?



7. Before the Panama Canal was built, ships going from New York to San Francisco traveled 13,200 miles. Now the trip is 5,200 miles. How many miles less is the trip by using the canal?

8. The Panama Canal was officially opened in 1920. Since then an average of 8,000 ocean-going ships use the canal each year. How many ships used the canal during its first 40 years of operation?

9. The attendance for 3 college football games was 27,621 at the first game; 32,407 at the next; and 33,717 at the third game. What was the average attendance?

Review and Practice

Try to do all the work on this page without referring to other pages in the book.

Write the correct answer for each of the following.

1. Copy this numeral and place the commas correctly: 36541267.

2. Use the dollar sign and cents point to write 546 cents.

3. Does $3042 \div 6$ equal 57, 507, or 570?

4. Which is larger: $1320 \div 24$ or $373 - 316$?

5. Which number divides 3745 with remainder of 0: 2, 3, or 5?

6. Jane is 11 years old and says she is more than 4000 days old. Prove that she is right.

7. Is $47256 \div 87$ about 50, 500, or 5000?

Find the mistakes made in each division below.

8.

$$\begin{array}{r} 643\ \text{r}22 \\ 63\overline{)39541} \\ \underline{378} \\ 274 \\ \underline{252} \\ 221 \\ \underline{199} \\ 22 \end{array} \qquad \begin{array}{r} 1405\ \text{r}41 \\ 37\overline{)385026} \\ \underline{37} \\ 150 \\ \underline{148} \\ 226 \\ \underline{185} \\ 41 \end{array}$$

Copy. Write a numeral in each blank so the statement is true.

9. 671 qt. = __gal.__qt.

10. 3218 sec. = __min.__sec.

11. 45 yr. = __months

12. 119 pk. = __bu.__pk.

13. 2172 ft. = __yd.__ft.

14. 15 mi. = __ft.

15. 923 oz. = __lb.__oz.

16. 723 qt. = __pk.__qt.

17. 3216 in. = __yd.__in.

18. 3 yr. 21 da. = __da.

19. 29 hr. 19 min. = __min.

Copy. Find each answer according to the operation indicated.

	a	*b*
20.	$39\overline{)34125}$	$52\overline{)876026}$
21.	$\begin{array}{r} 3285 \\ \underline{\times 47} \end{array}$	$\begin{array}{r} 51008 \\ \underline{\times 356} \end{array}$
22.	$\begin{array}{r} 49765 \\ \underline{+8997} \end{array}$	$\begin{array}{r} 357659 \\ \underline{+95277} \end{array}$
23.	$\begin{array}{r} 56821 \\ \underline{-7654} \end{array}$	$\begin{array}{r} 376594 \\ \underline{-269839} \end{array}$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Multiplying the divisor by 10, 100, 1,000, or 10,000 is an easy way to determine the number of digits in the quotient. (167)

2. If the first digit of a divisor is greater than the first digit of the dividend, find a trial quotient by dividing the first 2 digits of the dividend by the first digit of the divisor. (168, 174)

3. If the first digit of a divisor is less than the first digit of the dividend, find a trial quotient by dividing the first digit of the dividend by the first digit of the divisor. (171, 175)

4. When the first digits of a two-digit divisor and a dividend are the same, compare the divisor with the first two digits of the dividend to find a trial quotient. (176)

Questions to Discuss

1. How many digits are in the quotient when 42,815 is divided by 67? Why? (167, 168) When 325,718 is divided by 29? (175) When 582,713 is divided by 57? (176)

2. In which place-value position would you write the left-hand digit in the quotient of 768,421 divided by 42? Why? (175)

3. How would you determine a trial quotient for the division, 32517 ÷ 34? (176)

4. What is a convenient way to check that you have not omitted a digit in the quotient? (176)

Written Practice

Copy. Find each quotient and find the remainder when it is different from zero.

	a	*b*	
1.	$40\overline{)20000}$	$50\overline{)45000}$	(167)
2.	$70\overline{)33180}$	$36\overline{)16272}$	(168)
3.	$20\overline{)17075}$	$72\overline{)42560}$	(168)
4.	$30\overline{)42000}$	$40\overline{)87520}$	(171)
5.	$29\overline{)57643}$	$53\overline{)77215}$	(171)
6.	$80\overline{)320000}$	$70\overline{)294350}$	(174)
7.	$40\overline{)840000}$	$37\overline{)924038}$	(175)
8.	$23\overline{)245640}$	$46\overline{)432952}$	(176)
9.	$46\overline{)138414}$	$51\overline{)359550}$	(176)

Self-Evaluation

Part 1 Copy. Find a value for n which makes each equation true.

	a	*b*
1.	$30000 \div 60 = n$	$80000 \div 20 = n$
2.	$400000 \div 50 = n$	$600000 \div 30 = n$
3.	$27000 \div 90 = n$	$68000 \div 40 = n$
4.	$320000 \div 80 = n$	$540000 \div 20 = n$
5.	$11880 \div 90 = n$	$76200 \div 30 = n$
6.	$140000 \div 60 = n$	$974000 \div 40 = n$

Part 2 Each division statement below is followed by 4 statements, 2 of which are correct. Choose the correct answers.

1. 56481 divided by 36

a. The quotient has 4 digits.
b. The first trial quotient is 4.
c. The remainder will be 0.
d. 36 is the divisor.

2. 376543 divided by 83

a. The first trial quotient is 5.
b. The dividend is 376,543.
c. The quotient has 3 digits.
d. The first digit of the quotient is in thousands place.

3. 57641 divided by 58

a. The first trial quotient is 10.
b. The quotient has 3 digits.
c. The first digit in the quotient should be written in ten thousands place.
d. The remainder is less than 83.

Part 3 Copy. Find each answer.

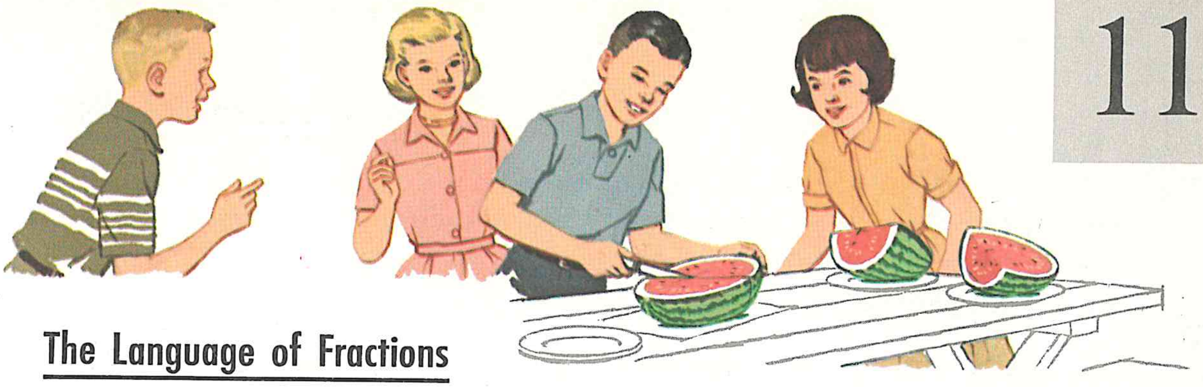
	a	*b*
1.	$60\overline{)43080}$	$40\overline{)205840}$
2.	$46\overline{)45172}$	$82\overline{)444194}$
3.	$72\overline{)44305}$	$57\overline{)488100}$
4.	$35\overline{)50007}$	$91\overline{)705062}$
5.	$12\overline{)37256}$	$18\overline{)126540}$
6.	$62\overline{)13654}$	$77\overline{)324605}$

Part 4 Solve each of the following problems. Use an equation to state the problem.

1. During the past year Mr. Hanley traveled 18,572 miles by car; 42,725 miles by air; and 8,250 miles by train. How many miles did he travel last year?

2. In January, 1957, 3 jet planes made the first non-stop, around the world flight. They flew about 24,150 miles in less than 46 hours. What was their approximate average speed?

11

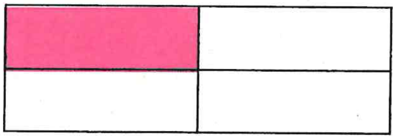


The Language of Fractions

The whole numbers are the only numbers you need for counting or computing to find *how many*. Can you use the whole numbers to measure a half cup of water, one of 3 parts of equal length of an inch, or part of a watermelon? For these purposes the whole numbers are not suitable. We need some new numbers.

In the picture above, what name would you give to each piece of watermelon?

The figure below has been divided into 4 parts of equal size. Then one of these parts was colored red. What part of the whole figure is red? In words, we say one fourth of the whole figure is red. Using numerals, one fourth is written $\frac{1}{4}$.



What part of the whole figure is left white?

The numerals, $\frac{1}{4}$ and $\frac{3}{4}$, are called **fractions.** The numbers which the fractions name are called **fractional numbers.**

In $\frac{3}{4}$, the numeral 4 is called the **denominator;** and the numeral 3 is called the **numerator.** The denominator tells the number of parts of equal size into which the whole is divided. The numerator tells how many of these parts of equal size are used.

One meaning of $\frac{2}{3}$ is that the whole is divided into 3 parts of equal size and 2 parts are used.

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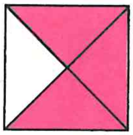
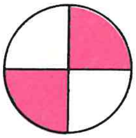
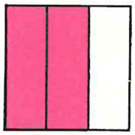
Oral What fractional part of each figure below is colored red?

a *b* *c*

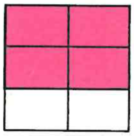
1.

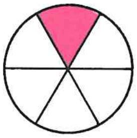


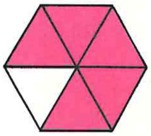




2.



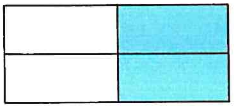




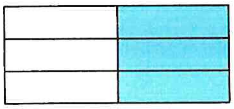
Equivalent Fractions

The figures below suggest some other ways of thinking of the fractional number $\frac{1}{2}$. From their appearance, $\frac{1}{2}$ of each figure is blue.

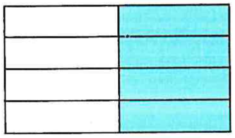
A



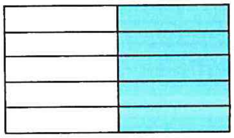
B



C



D



In A, the whole figure is divided into 4 parts of equal size, and 2 of these parts are blue. So $\frac{2}{4}$ of the figure is blue.

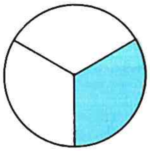
In B, the whole figure is divided into 6 parts of equal size, and 3 of these parts are blue. So $\frac{3}{6}$ of the figure is blue. Therefore, $\frac{3}{6}$ is another name for $\frac{1}{2}$.

Answer the following questions about figures C and D. Into how many parts of equal size is the figure divided? How many of these parts are blue? What fractional part of the figure is blue?

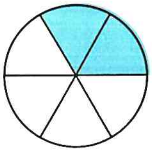
Give 3 other fractions which name the fractional number $\frac{1}{2}$.

Oral Answer questions 1 through 3 about each of the figures *a*, *b*, and *c*.

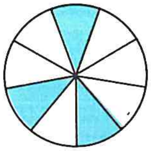
a



b



c



1. Into how many parts of equal size is the figure divided?

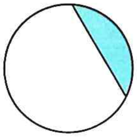
2. How many parts are blue?

3. What fractional part of the figure is blue?

4. Give three names for the fractional number $\frac{1}{3}$.

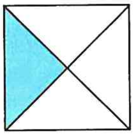
Written Write whether the statement following each figure is true or false. If the statement is false, tell why it is false.

1.



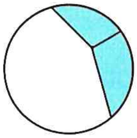
One half of the whole figure is blue.

2.



One fourth of the whole figure is blue.

3.

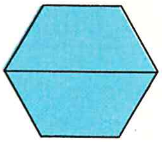


Two thirds of the whole figure is blue.

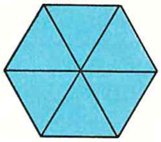
Finding Equivalent Fractions

Think of each figure below as representing 1. Then these are different ways to think about the whole number 1.

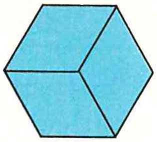
A



B



C



In A, $\frac{2}{2}$ of the figure is blue; in B, $\frac{6}{6}$ of the figure is blue; and in C, $\frac{3}{3}$ of the figure is blue. Hence, $\frac{2}{2}$, $\frac{6}{6}$, and $\frac{3}{3}$ are other names for the number 1, and you say $\frac{2}{2}=1$, $\frac{6}{6}=1$, and $\frac{3}{3}=1$. Give 3 other fractions which name the number 1.

In fact, if any whole number, other than 0, is named in both the numerator and the denominator, the resulting fraction is a name for the number 1.

Then, $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6}$

and $1=\frac{21}{21}=\frac{150}{150}=\frac{1260}{1260}$.

Since 1 is the identity number for multiplication, the product of any fractional number and 1 is the fractional number you started with. For instance, $\frac{1}{2}\times 1=\frac{1}{2}$ and $\frac{1}{3}\times 1=\frac{1}{3}$.

In the multiplication, $\frac{1}{2}\times 1$, you have many choices for naming the number 1. One choice is $\frac{2}{2}$.

$$\frac{1}{2}\times 1=\frac{1}{2}\times\frac{2}{2}=\frac{1\times 2}{2\times 2}=\frac{2}{4}$$

Another choice is $1=\frac{3}{3}$.

$$\frac{1}{2}\times 1=\frac{1}{2}\times\frac{3}{3}=\frac{1\times 3}{2\times 3}=\frac{3}{6}$$

These are examples of the general statement: Multiplying both of the numbers named in a fraction by the same number, *other than 0 or 1*, produces another name for the same fractional number.

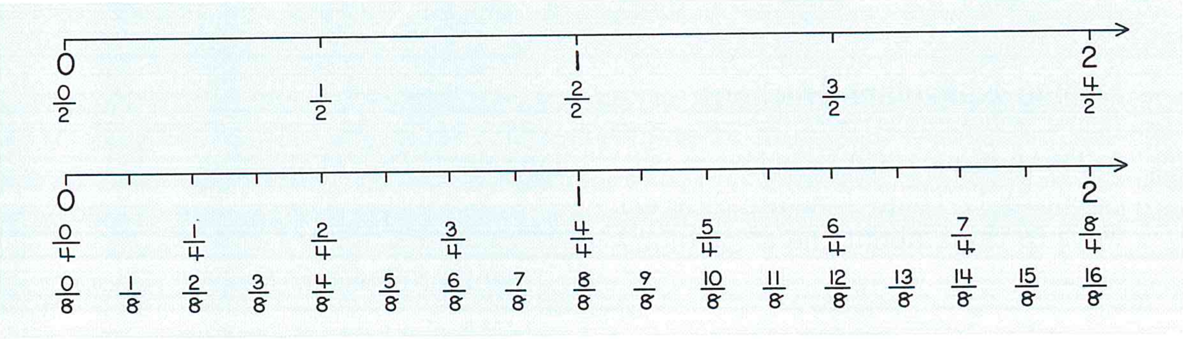
Oral Answer the questions below.

1. By what number do you multiply both numbers named in a fraction to change halves to eighths?

2. What name do you use for the number 1 in changing $\frac{2}{3}$ to $\frac{10}{15}$?

Written What numeral should replace n so that each of the following becomes a true statement?

	a	*b*	*c*
1.	$\frac{1}{2}=\frac{n}{8}$	$\frac{1}{3}=\frac{n}{9}$	$\frac{1}{4}=\frac{n}{8}$
2.	$\frac{1}{2}=\frac{n}{10}$	$\frac{1}{3}=\frac{n}{15}$	$\frac{1}{4}=\frac{n}{16}$
3.	$\frac{1}{2}=\frac{6}{n}$	$\frac{1}{3}=\frac{4}{n}$	$\frac{1}{4}=\frac{3}{n}$
4.	$\frac{2}{3}=\frac{n}{6}$	$\frac{2}{5}=\frac{n}{10}$	$\frac{3}{8}=\frac{n}{16}$
5.	$\frac{3}{4}=\frac{9}{n}$	$\frac{2}{7}=\frac{4}{n}$	$\frac{5}{6}=\frac{10}{n}$
6.	$\frac{7}{8}=\frac{n}{40}$	$\frac{4}{5}=\frac{28}{n}$	$\frac{5}{9}=\frac{n}{81}$


0
$\frac{0}{2}$
$\frac{1}{2}$
1
$\frac{2}{2}$
$\frac{3}{2}$
2
$\frac{4}{2}$
0
1
2
$\frac{0}{4}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$ $\frac{5}{4}$ $\frac{6}{4}$ $\frac{7}{4}$ $\frac{8}{4}$
$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$ $\frac{9}{8}$ $\frac{10}{8}$ $\frac{11}{8}$ $\frac{12}{8}$ $\frac{13}{8}$ $\frac{14}{8}$ $\frac{15}{8}$ $\frac{16}{8}$

Halves, Fourths, and Eighths

Fractional numbers can also be represented on a number line. On the first number line above, the points halfway between the successive points 0, 1, and 2 were marked. Beginning with 0, all the points (new and old) were labeled in order from left to right with the fractions, $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, and so on.

The numerators, in order, name whole numbers with which you are familiar. The 2 in the denominator is merely a reminder that the new unit, $\frac{1}{2}$, was obtained by dividing the original unit, 1, into 2 segments of equal length.

A fraction names the quotient of two whole numbers (divisor not 0). So $\frac{1}{2}$ means $1 \div 2$ and $\frac{4}{2}$ means $4 \div 2$.

If you divide the original unit, 1, into 4 segments of equal length, or 8 segments of equal length, and label the points as described above, you would have a diagram as shown in the second number line.

From the number lines above, you see that $\frac{0}{2}$, $\frac{0}{4}$, and $\frac{0}{8}$ are names for 0, and $\frac{2}{2}$, $\frac{4}{4}$, and $\frac{8}{8}$ are names for 1. What fractions are names for 2? If the number lines were extended to the right, what fractions would name 3? 4?

The set of fractional numbers includes the set of whole numbers.

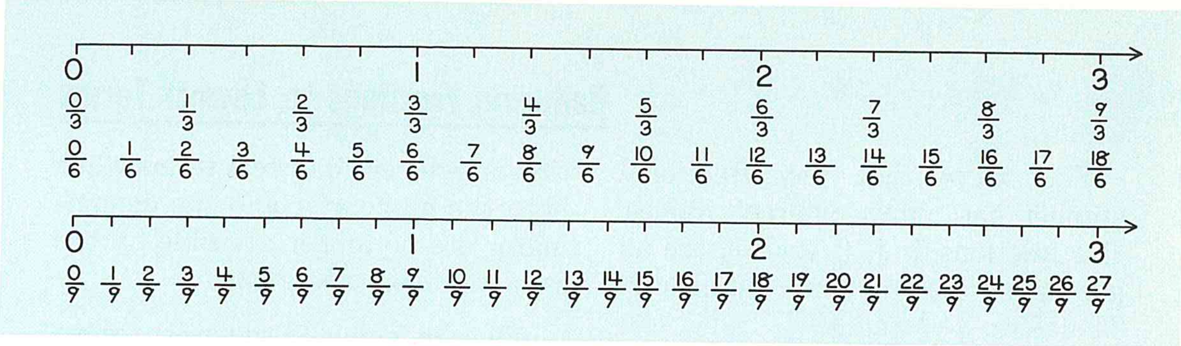
Oral Answer the following questions about the number lines above.

1. Which is farther to the right on the number line, $\frac{5}{4}$ or $\frac{7}{4}$? Which names the larger number?

2. Which is farther to the right on the number line, $\frac{13}{8}$ or $\frac{15}{8}$? Which names the larger number?

3. Which is farther to the right on the number line, $\frac{3}{2}$ or $\frac{7}{4}$? Which names the larger number?

4. Which is farther to the right on the number line, $\frac{13}{8}$ or $\frac{3}{4}$? Which names the larger number?



Thirds, Sixths, and Ninths

If you mark points on a number line so that the segment from 0 to 1 is divided into 3 parts of equal length, or 6 parts of equal length, your diagram would appear as the first number line above.

You can tell from the number line that $1=\frac{3}{3}=\frac{6}{6}$, that $\frac{5}{3}=\frac{10}{6}$, that $2=\frac{6}{3}=\frac{12}{6}$, and so on.

On the second number line above, points were marked so that the segment from 0 to 1 was divided into 9 parts of equal length.

To locate the point on a number line that corresponds to $\frac{7}{10}$, mark the points that divide the segment from 0 to 1 into 10 parts of equal length and count off 7 of them.

Can every fraction be represented by a point on a number line? How would you locate the point which corresponds to $\frac{5}{12}$? To $\frac{13}{3}$?

From the number lines above, give three names for the number 2. Give three names for the number $\frac{2}{3}$. Give three fractions which name the number $\frac{5}{3}$.

Oral Use the number lines above to answer these questions.

1. Which is farther to the right on the number line, $\frac{7}{3}$ or $\frac{12}{6}$? Which names the larger number?

2. Which is farther to the right on the number line, $\frac{5}{3}$ or $\frac{17}{9}$? Which names the larger number?

3. When two numbers are represented on the number line, how do you tell which is larger?

Written Write two other names for each of the following numbers.

	a	*b*	*c*	*d*
1.	2	$\frac{1}{2}$	$\frac{4}{3}$	$\frac{6}{6}$
2.	$\frac{5}{3}$	$\frac{2}{3}$	3	$\frac{3}{2}$
3.	$\frac{6}{9}$	1	$\frac{8}{3}$	$\frac{15}{9}$
4.	$\frac{1}{3}$	$\frac{27}{9}$	$\frac{7}{3}$	$\frac{12}{6}$

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Reducing Fractions to Lowest Terms

You know that any fractional number has many different names. The fractions $\frac{2}{3}$, $\frac{4}{6}$, $\frac{6}{9}$, and $\frac{8}{12}$ are all names for the fractional number $\frac{2}{3}$. Therefore, $\frac{2}{3}=\frac{4}{6}$ and $\frac{4}{6}=\frac{2}{3}$.

You have learned how to change $\frac{2}{3}$ to $\frac{4}{6}$ by multiplying both of the numbers named in a fraction by the same number. How do you change $\frac{4}{6}$ to $\frac{2}{3}$? In other words, how do you undo the change from $\frac{2}{3}$ to $\frac{4}{6}$?

The first change involved multiplication. Undoing this would indicate division. What is the inverse of multiplying by 1?

Remember that the number 1 has many names. Use $\frac{2}{2}$ as a name for the number 1.

$$\frac{4}{6}\div 1=\frac{4}{6}\div\frac{2}{2}=\frac{4\div 2}{6\div 2}=\frac{2}{3}$$

Another choice for naming the number 1 is $\frac{3}{3}$.

$$\frac{6}{9}\div 1=\frac{6}{9}\div\frac{3}{3}=\frac{6\div 3}{9\div 3}=\frac{2}{3}$$

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These examples indicate that dividing both of the whole numbers associated with a fractional number by the same number, *other than 0 or 1*, produces another name for the fractional number. These examples also indicate that dividing a number by one does not change the number. Therefore, as a divisor, one is the *identity number of division*. A fraction is **reduced to lowest terms** when both the numerator and the denominator are no longer divisible by the same number, *other than 1*.

When reducing $\frac{12}{30}$ to lowest terms, note that 12 and 30 are divisible by 2. Notice also that they are both divisible by 3. Since 12 and 30 are both divisible by 2 and by 3, they are divisible by 6.

$$\frac{12}{30}=\frac{12\div 6}{30\div 6}=\frac{2}{5}$$

Since 1 is the only whole number by which both 2 and 5 are divisible, $\frac{2}{5}$ is in lowest terms.

Oral By what number do you divide both numbers named in the fraction to reduce each fraction below to lowest terms?

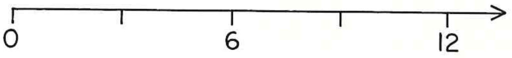
$\frac{6}{9}$, $\frac{5}{25}$, $\frac{12}{18}$, $\frac{6}{15}$, $\frac{14}{21}$, $\frac{16}{24}$, $\frac{20}{50}$

Written Copy. Reduce each fraction to lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{3}{9}$	$\frac{3}{6}$	$\frac{5}{10}$	$\frac{4}{16}$
2.	$\frac{5}{15}$	$\frac{4}{12}$	$\frac{8}{12}$	$\frac{2}{10}$
3.	$\frac{14}{16}$	$\frac{9}{12}$	$\frac{8}{10}$	$\frac{6}{8}$
4.	$\frac{18}{20}$	$\frac{3}{27}$	$\frac{6}{16}$	$\frac{15}{20}$
5.	$\frac{8}{24}$	$\frac{16}{24}$	$\frac{7}{7}$	$\frac{20}{60}$
6.	$\frac{8}{16}$	$\frac{7}{21}$	$\frac{9}{15}$	$\frac{30}{90}$

Finding Fractional Parts

A number line may also be used to show fractional parts.



One half of twelve, in terms of lengths of segments on the number line, means a length such that when you start at 0 and lay off this length twice (to the right) you arrive at the point labeled 12.

What is one half of 12? How would you label the point midway between the points labeled 0 and 6? Is $\frac{1}{2}$ of 6 the same as $\frac{1}{4}$ of 12?

What is the label of the midpoint between 6 and 12? What is $\frac{2}{4}$ of 12? What is $\frac{3}{4}$ of 12?

To find $\frac{1}{4}$ of 12, without using a number line, divide 12 into 4 parts of equal size and consider 1 of these parts.

$$\tfrac{1}{4} \text{ of } 12 = (12 \div 4) \times 1 = 3 \times 1 = 3$$

To find $\frac{3}{4}$ of 12, think:

$$\tfrac{3}{4} \text{ of } 12 = 3 \times (12 \div 4) = 3 \times 3 = 9$$

Or you might think:

$$\tfrac{3}{4} \text{ of } 12 = (12 \div 4) \times 3 = 3 \times 3 = 9$$

To find a fractional part of a number, *divide the number by the number named in the denominator* of the fraction and *multiply this quotient by the number named in the numerator.*

For example, find $\frac{2}{5}$ of 20 as shown below.

$$\tfrac{2}{5} \text{ of } 20 = (20 \div 5) \times 2 = 4 \times 2 = 8$$

Would you get the same answer if you multiplied the number, 20, by the numerator and then divided this product by 5?

$$\tfrac{2}{5} \text{ of } 20 = (2 \times 20) \div 5 = 40 \div 5 = 8$$

To find a fractional part of a number, *multiply the number by the number named in the numerator* of the fraction and *divide this product by the number named in the denominator.*

Oral Explain how to find n in each of the following by using either of the above methods.

a	*b*	*c*
$\frac{1}{3}$ of $18 = n$	$\frac{4}{5}$ of $15 = n$	$\frac{5}{3}$ of $9 = n$

Written Copy. Solve each of the following by using whichever method is most convenient.

	a	*b*
1.	$\frac{1}{3}$ of $24 = n$	$\frac{2}{3}$ of $27 = n$
2.	$\frac{3}{4}$ of $36 = n$	$\frac{5}{8}$ of $40 = n$
3.	$\frac{2}{5}$ of $30 = n$	$\frac{5}{6}$ of $54 = n$

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Fractional Parts of Measures

Knowing how to find fractional parts and how to change from one unit of measurement to another aids us in solving many problems.

For example: Nancy bought $\frac{3}{4}$ of a yard of ribbon for wrapping a birthday gift. How many inches of ribbon did she buy?

Since you are to give the answer in inches, think of 1 yard as 36 inches. One solution follows.

$$\tfrac{3}{4} \text{ of } 36 = (36 \div 4) \times 3 = 9 \times 3 = 27$$

She bought 27 inches of ribbon.

Another problem might ask you to find what fractional part one measure is of another.

For example, 2 oz. is what fractional part of 1 lb.? Think of 1 lb. as being divided into 16 oz. You are considering 2 oz. Hence the fractional part is $\frac{2}{16}$.

Can you reduce $\frac{2}{16}$? Reduce $\frac{2}{16}$ to lowest terms: $\frac{2}{16}=\frac{1}{8}$. So you can say that 2 oz. is $\frac{1}{8}$ lb. Which answer, $\frac{2}{16}$ or $\frac{1}{8}$, gives you a clearer idea of the fractional part?

What fractional part of 1 hour is 25 minutes? Think of 1 hour as 60 minutes. Then 25 minutes is $\frac{25}{60}$ hour. Since $\frac{25}{60}=\frac{5}{12}$, 25 minutes is equal to $\frac{5}{12}$ hour.

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Oral How would you find a replacement for each blank so that each statement becomes true?

	a	*b*
1.	$\frac{1}{3}$ ft. = __in.	3 in. = __ft.
2.	$\frac{1}{2}$ pk. = __qt.	4 oz. = __lb.
3.	$\frac{3}{4}$ gal. = __qt.	2 ft. = __yd.
4.	$\frac{5}{8}$ lb. = __oz.	9 min. = __hr.
5.	$\frac{2}{3}$ da. = __hr.	1 pt. = __qt.

Written Copy. Find the replacement for each blank so that the following become true statements. Give your answer in lowest terms.

	a	*b*
1.	$\frac{1}{4}$ bu. = __qt.	6 in. = __ft.
2.	$\frac{5}{6}$ ft. = __in.	9 in. = __yd.
3.	$\frac{2}{3}$ yd. = __in.	8 oz. = __lb.
4.	$\frac{3}{4}$ min. = __sec.	2 pt. = __gal.
5.	$\frac{3}{7}$ wk. = __da.	12 min. = __hr.
6.	$\frac{1}{5}$ hr. = __min.	10 oz. = __lb.
7.	$\frac{2}{3}$ yd. = __ft.	15 hr. = __da.
8.	$\frac{3}{8}$ T. = __lb.	3 pt. = __gal.
9.	$\frac{4}{9}$ yd. = __in.	48 sec. = __min.
10.	$\frac{3}{4}$ bu. = __pk.	9 in. = __ft.

Practice Using Fractional Parts

The problems on this page provide practice in finding fractional parts. Read each problem, decide which operation to use, and express the problem in equation form. Find an answer for each problem.

1. Mr. Collins bought 4 gallons of paint. He used $\frac{5}{16}$ of the paint for the back of his house. How many quarts of paint did he use on the back of his house?

2. Margie's cat got into the fishbowl and ate $\frac{1}{3}$ of the 18 fish. How many fish did the cat eat?

3. Sally was assigned 32 arithmetic problems. She completed 12 of them. What fractional part of the assignment was completed?

4. Mr. Smith paid Bob at the rate of 90¢ an hour. Bob worked $\frac{2}{3}$ hour. How much did he earn?

5. Mr. Blue said that $\frac{2}{5}$ of the 4280 people in Sunnydale were over 18 years of age. There were 2140 people in town over 18 years old. Show that he was wrong.

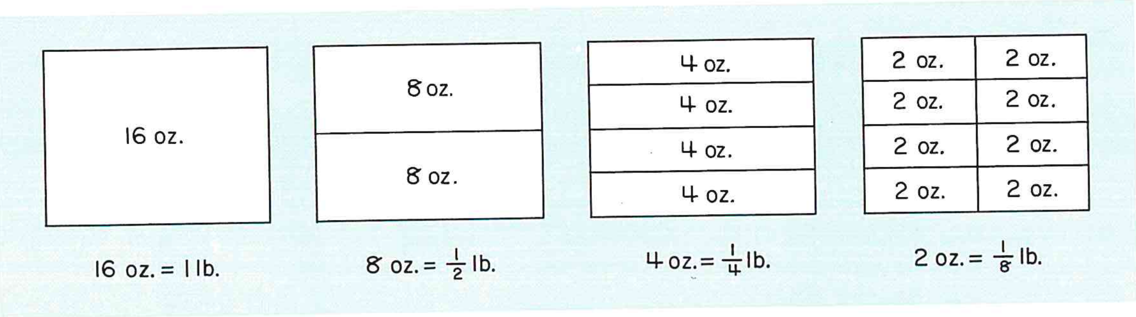
6. If candy sells for 60¢ a pound, what fractional part of a pound can you buy with 15¢?

7. Bill had 48 stamps. He traded $\frac{3}{8}$ of them to Roger. How many stamps did he trade?

8. On her way home from the store, Rita dropped a dozen eggs. When she looked inside the carton, she found $\frac{2}{3}$ of the eggs broken. How many eggs were broken?

9. Mrs. Lawler asked the butcher for $\frac{1}{2}$ lb. of meat. The piece of meat he put on the scales weighed 10 oz. This was how many ounces more than $\frac{1}{2}$ lb.?

10. James can walk home in 23 minutes. Bill walks home in $\frac{5}{12}$ hour. Which boy needs the greater amount of time to walk home?



Comparing Denominators

Imagine that each figure above represents 1 pound of some material. The first figure is not divided. The other three figures show that each pound was divided into parts of equal size.

Answer these questions about each of the last 3 figures. Into how many parts was one pound divided? Each part is what fractional part of a pound?

Since $\frac{1}{2}$ lb. = 8 oz. and $\frac{1}{4}$ lb. = 4 oz., then $\frac{1}{2}$ lb. is more than $\frac{1}{4}$ lb.

Why is $\frac{1}{4}$ lb. more than $\frac{1}{8}$ lb.? Why is $\frac{3}{8}$ lb. less than $\frac{3}{4}$ lb.?

Read the following by giving them in order from the smallest to the largest measure of weight.

$\frac{1}{2}$ lb., $\frac{1}{8}$ lb., $\frac{1}{4}$ lb., 1 lb.

In what order should you write the following fractions so that they name the numbers in order from smallest to largest?

$\frac{1}{2}, \frac{1}{4}, \frac{8}{8}, \frac{1}{8}$

Suppose you start with the number $\frac{1}{3}$ and continually increase the denominator. Some of the resulting numbers are given below.

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}$

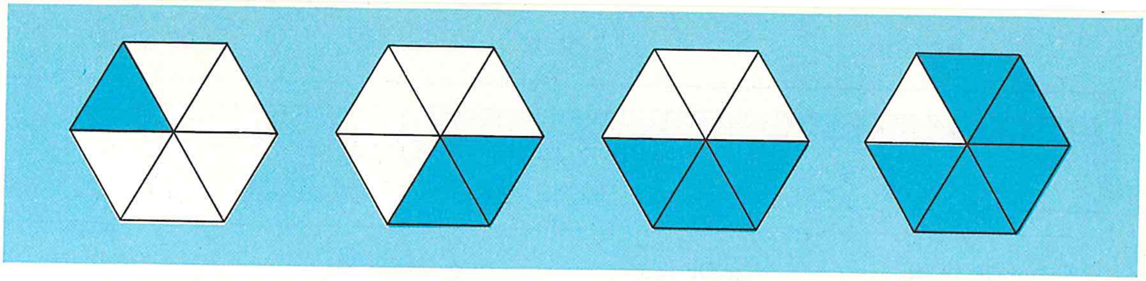
As the denominator becomes larger, the number becomes smaller.

Oral Tell which fractional part will be larger. Why is it larger?

1. $\frac{1}{2}$ or $\frac{1}{4}$ of 20 cents
2. $\frac{3}{5}$ or $\frac{3}{4}$ of 40 students
3. $\frac{5}{6}$ or $\frac{5}{8}$ of 40 books

Written Rearrange each set of the fractions so that the numbers are named from smallest to largest.

1. $\frac{1}{3}, \frac{1}{8}, \frac{1}{5}, \frac{1}{6}, \frac{1}{10}$
2. $\frac{2}{7}, \frac{2}{3}, \frac{2}{4}, \frac{2}{9}, \frac{2}{5}$
3. $\frac{3}{5}, \frac{3}{10}, \frac{3}{7}, \frac{3}{13}, \frac{3}{20}$
4. $\frac{4}{15}, \frac{4}{7}, \frac{4}{17}, \frac{4}{9}, \frac{4}{5}$
5. $\frac{6}{7}, \frac{6}{8}, \frac{6}{11}, \frac{6}{15}, \frac{6}{12}$

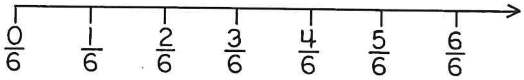


Comparing Numerators

Study the figures above. Each has been divided into 6 parts of equal size, and each has one or more of these parts colored blue.

What fractional part of each figure is colored blue? From left to right, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, and $\frac{5}{6}$ of the figure is colored blue.

These numbers are represented on the number line below.



Which is greater, $\frac{2}{6}$ or $\frac{5}{6}$? $\frac{3}{6}$ or $\frac{2}{6}$? Are the figures above arranged so that they represent numbers from the smallest to the largest?

Notice that the fractional numbers given above have equal denominators but unequal numerators.

Begin with the number $\frac{1}{7}$, and continually increase the numerator. Some resulting numbers would be $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$. As the numerator becomes larger, does the number become larger or smaller?

Given two fractions with the same denominator, how can you tell which is larger?

If two fractions have the same denominator, they have a *common denominator*. The fractions $\frac{2}{9}$, $\frac{6}{9}$, $\frac{7}{9}$, and $\frac{11}{9}$ have a common denominator. The common denominator is 9.

Oral Tell which of the fractional parts is the larger.

1. $\frac{1}{3}$ or $\frac{2}{3}$ of 18 books

2. $\frac{3}{8}$ or $\frac{5}{8}$ of 96 tons

3. $\frac{7}{9}$ or $\frac{4}{9}$ of 36 horses

Written Find both fractional parts in exercises 1 through 3 above.

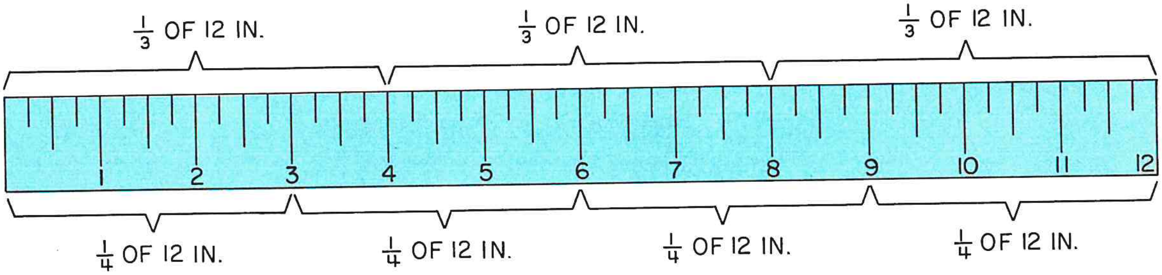
Arrange each set of fractions below so that the fractional numbers are named from the smallest to the largest.

4. $\frac{4}{7}$, $\frac{2}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, $\frac{1}{7}$

5. $\frac{5}{11}$, $\frac{7}{11}$, $\frac{9}{11}$, $\frac{2}{11}$, $\frac{4}{11}$

6. $\frac{3}{20}$, $\frac{7}{20}$, $\frac{21}{20}$, $\frac{9}{20}$, $\frac{50}{20}$

7. $\frac{2}{15}$, $\frac{7}{15}$, $\frac{5}{15}$, $\frac{17}{15}$, $\frac{6}{15}$



Comparing Fractional Numbers

Which is greater, $\frac{2}{3}$ of a foot or $\frac{5}{6}$ of a foot? Since 1 foot and 12 inches are both names for the same measure, the computation below could be used.

$$\frac{2}{3} \text{ of } 12 = (12 \div 3) \times 2 = 4 \times 2 = 8$$
$$\frac{5}{6} \text{ of } 12 = (12 \div 6) \times 5 = 2 \times 5 = 10$$

So $\frac{2}{3}$ of 1 ft. = 8 in., and $\frac{5}{6}$ of 1 ft. = 10 in. Since 10 in. is greater than 8 in., you know that $\frac{5}{6}$ of 1 ft. is greater than $\frac{2}{3}$ of 1 ft. The figure above will help you see that the above computation is correct.

Which is greater, $\frac{3}{4}$ or $\frac{7}{8}$? The question is easy to answer if you rename either or both of the numbers so that they have a common denominator. Then the fraction with the greater numerator names the greater number.

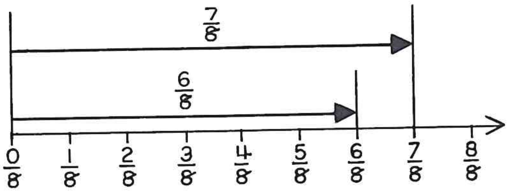
Some other names for $\frac{3}{4}$ and $\frac{7}{8}$ are given below.

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}$$
$$\frac{7}{8} = \frac{14}{16} = \frac{21}{24} = \frac{28}{32} = \frac{35}{40} = \frac{42}{48}$$

From these names you can see that both of the numbers can be expressed as eighths, sixteenths, or twenty-fourths. You could use 8, 16, or 24 as a common denominator. Which of the numbers above is best to use as a common denominator?

Study the names for these fractional numbers. Notice that the smallest number that appears as a common denominator is 8.

Which is greater, $\frac{6}{8}$ or $\frac{7}{8}$? These two numbers are shown on the number line below.



It is easy to see that $\frac{7}{8}$ is greater than $\frac{6}{8}$ because 7 is greater than 6.

To tell which of two fractional numbers is larger, *rename the fractional numbers*, if necessary, so that they have a *common denominator*. Then *compare the numerators*. The larger numerator indicates the larger number.

Suppose you want to know which is larger, $\frac{3}{4}$ or $\frac{5}{6}$. These two fractions can be renamed as shown below.

$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=\cdots$$
$$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{25}{30}=\cdots$$

The dots following each sequence of names indicate that you could go on and on giving more names for these fractional numbers.

Study the names for $\frac{3}{4}$ and $\frac{5}{6}$. You will notice that the smallest number that occurs as a common denominator is 12.

Now the question becomes: Which is larger, $\frac{9}{12}$ or $\frac{10}{12}$? Why? Since 10 is larger than 9, you know that $\frac{10}{12}$ is larger than $\frac{9}{12}$, and therefore that $\frac{5}{6}$ is larger than $\frac{3}{4}$.

You may want to arrange more than two fractions so that they name fractional numbers from smallest to largest. In this case, rename all the fractional numbers, if necessary, so the fractions have a common denominator.

Arrange the following fractions so that they name numbers from the smallest to the largest: $\frac{5}{6}$, $\frac{3}{4}$, and $\frac{5}{8}$. Some other names for these fractional numbers are given below.

$$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{25}{30}=\cdots$$
$$\frac{3}{4}=\frac{6}{8}=\frac{9}{12}=\frac{12}{16}=\frac{15}{20}=\frac{18}{24}=\cdots$$
$$\frac{5}{8}=\frac{10}{16}=\frac{15}{24}=\frac{20}{32}=\frac{25}{40}=\cdots$$

Study the names for these fractional numbers. You will notice that the smallest common denominator is 24.

Since $\frac{5}{6}=\frac{20}{24}$, $\frac{3}{4}=\frac{18}{24}$, and $\frac{5}{8}=\frac{15}{24}$, you should arrange the fractions in this order: $\frac{5}{8}$, $\frac{3}{4}$, $\frac{5}{6}$. Why?

Oral Explain how to find the smallest common denominator for these fractions.

1. $\frac{2}{3}$, $\frac{4}{5}$
2. $\frac{1}{4}$, $\frac{5}{6}$
3. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{6}$
4. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{5}{7}$

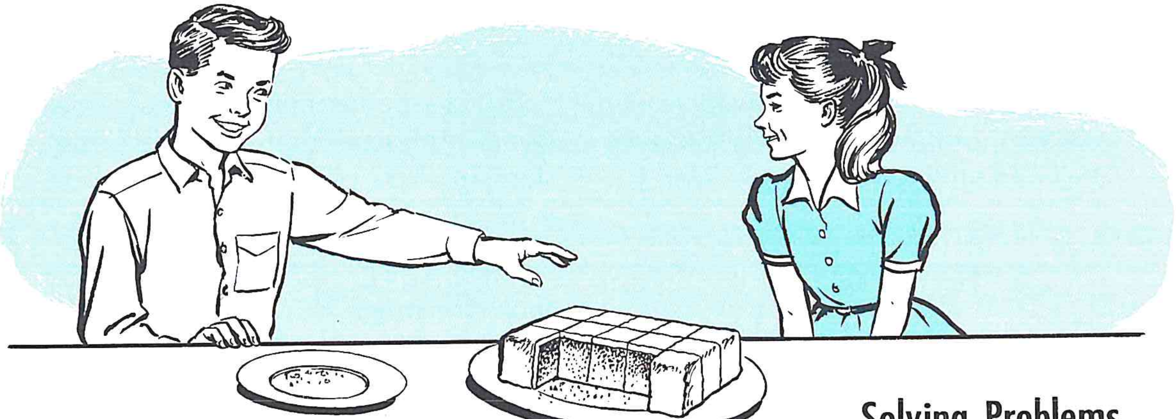
Written Show your work in determining the answers to each question below.

1. Which is larger, $\frac{3}{4}$ or $\frac{4}{5}$?
2. Which is larger, $\frac{2}{3}$ or $\frac{3}{7}$?
3. Which is smaller, $\frac{5}{9}$ or $\frac{7}{12}$?
4. Which is smaller, $\frac{2}{5}$ or $\frac{3}{8}$?

Arrange each of the following lists of fractions so that they name fractional numbers in order of size, from smallest to largest.

5. $\frac{2}{5}$, $\frac{3}{10}$, $\frac{1}{4}$
6. $\frac{2}{3}$, $\frac{12}{21}$, $\frac{4}{7}$
7. $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$
8. $\frac{3}{8}$, $\frac{1}{2}$, $\frac{7}{16}$

MORE PRACTICE PAGE 311



Solving Problems

You should be able to solve all the problems on this page without referring to other pages.

1. Jane cut a cake into 15 pieces of equal size. Tom ate 3 pieces of the cake. What fractional part of the cake did Tom eat?

2. During the summer months, Ronald earned $64.00 working at the pony farm. He saved $\frac{3}{4}$ of what he earned. How much did he save?

3. Roger worked for 25 minutes on his arithmetic assignment. For what fractional part of an hour did he work on arithmetic?

4. Bobby started to school with 21 invitations for his party. He lost $\frac{2}{7}$ of them on the way. How many invitations did he have when he arrived at school?

5. David had 24 guppies in a fishbowl with some other fish. A larger fish ate $\frac{3}{8}$ of the guppies. How many guppies were eaten?

6. Mrs. Rome bought a dozen rolls. The family ate $\frac{5}{6}$ of them for breakfast. How many rolls were eaten for breakfast?

7. Susan spends 6 hours of each day in school. During the school day she has 30 minutes for recess. What fractional part of the school day is used for recess?

8. Jan bought 15 stamps and used $\frac{2}{5}$ of them. Pam bought 18 stamps and used $\frac{2}{9}$ of them. Who used the greater number of stamps?

9. The fifth-graders and sixth-graders sold a total of 245 tickets for the music program. The sixth-graders sold $\frac{2}{5}$ of the tickets. How many program tickets were sold by each class?

10. John worked for 40 minutes and was paid $.90 an hour. Bill worked for $\frac{3}{5}$ hour and was paid $1.00 an hour. Which boy earned the greater amount of money?

Writing Remainders as Fractions

Three boys decided to share equally the 16 apples given them by Mrs. Low. How many apples should each boy receive?

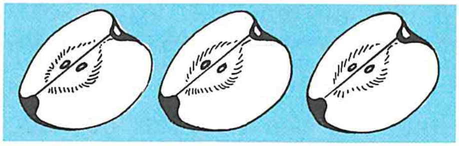
The question is answered by dividing 16 by 3. In equation form it is written as $16 \div 3 = n$.

There is no whole number replacement of n since $3 \times 5 = 15$ and $3 \times 6 = 18$. It appears that the value of n is between 5 and 6.

$$\begin{array}{r} 5\ \text{r}1 \\ 3\overline{)16} \\ 15 \\ \hline 1 \end{array}$$

If you do the division as shown at the right, the remainder is 1. Each boy should receive 5 apples, and there would still be 1 remaining apple to be divided among the boys.

You could divide the remaining apple into 3 pieces of equal size as shown below.



Each piece is what fractional part of an apple? What fractional part of an apple would each boy receive?

Each boy would receive 5 whole apples and $\frac{1}{3}$ of an apple, or $5\frac{1}{3}$ apples.

The answer can be obtained from the division by using the remainder, 1, as the numerator, and by using the divisor, 3, as the denominator, to form the fraction $\frac{1}{3}$. Reduce the fraction to lowest terms, when possible, and add it to the quotient.

Oral Explain how to solve each equation and how to express each remainder as a fraction.

	a	*b*
1.	$42 \div 5 = n$	$73 \div 8 = n$
2.	$85 \div 10 = n$	$46 \div 12 = n$

Written Copy. Solve for n. State each remainder as a fraction.

	a	*b*
1.	$46 \div 4 = n$	$38 \div 5 = n$
2.	$71 \div 6 = n$	$105 \div 6 = n$
3.	$419 \div 3 = n$	$286 \div 12 = n$
4.	$1463 \div 9 = n$	$2476 \div 10 = n$

Copy. Write each answer by stating the remainder as a fraction.

	a	*b*
5.	$20\overline{)1447}$	$36\overline{)6120}$
6.	$57\overline{)4670}$	$42\overline{)4309}$
7.	$16\overline{)1127}$	$25\overline{)5760}$

MORE PRACTICE PAGE 311

Using Common Sense in Problem Solving

Fractional numbers are very useful in arithmetic. You have seen how they can be used to solve some problems in everyday life. You also know how to state the remainder of a division in fractional form.

However, the use of fractional numbers is limited in solving everyday problems. Does it make sense to speak of $\frac{1}{2}$ an idea, $\frac{2}{3}$ of a person, or $\frac{5}{6}$ of a horse? It so happens that certain things have meaning only when we think of them as whole units but not when we think of fractional parts of them.

Twenty-two children are going to the museum in 5 cars. What is the average number of children who will ride in each car?

Finding the average implies division. The equation is $22 \div 5 = n$, where n represents the number of children per car. Solving the equation shows that $n = 4\frac{2}{5}$. Can $4\frac{2}{5}$ children ride in each car?

In this case you could say that 4 children will ride in each of 3 cars, and that 5 children will ride in each of 2 cars.

Or you might answer the problem by saying, to the nearest person, 4 children ride in each car.

Solve each of the following problems by giving the remainder in fractional form if it is sensible. Be ready to explain why you stated your answers as you did.

1. Mrs. Jensen needed 20 buns for the picnic. The buns are sold in packages of 8 each. How many packages of buns did she buy?

2. The 6 members of the safety committee presented 15 ideas for improving safety. How many ideas did each member present?

3. Mrs. Thompson used 6 cups of sugar to make 4 cakes. If each cake contained the same amount of sugar, how much sugar was used to make each cake?

4. Kathy needed 16 buttons for the dress she was making. The buttons are sold 6 on a card. How many cards of buttons did she buy?

5. An automobile dealer sold 14 cars during March, 19 during April, 26 during May, and 24 during June. What was the average number of cars sold each month?

6. On a 3-day business trip Dan Black drove the following distances: 317 miles, 262 miles, and 235 miles. What was the average number of miles he drove each day?

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Fractions are names for fractional numbers. (183)

2. A fractional number has many different names. (184, 185)

3. Multiplying both of the numbers named in a fraction by the same number, other than 0 or 1, produces another name for the same fractional number. (185)

4. Dividing both of the whole numbers associated with a fractional number by the same number, other than 0 or 1, produces another name for the fractional number. (188)

5. A fraction is reduced to lowest terms when the only number by which both the numerator and denominator are divisible is 1. (188)

Words to Know

1. Fraction (183)
2. Fractional number (183)
3. Numerator, denominator (183)
4. Identity number of division (188)
5. Reduce to lowest terms (188)
6. Common denominator (193)

Questions to Discuss

1. Paul said he had $\frac{5}{6}$ of an object. What did he mean? (183)

2. How would you reduce $\frac{72}{84}$ to lowest terms? (188)

3. Explain two methods for finding $\frac{3}{8}$ of 32. (189)

4. When do two fractions have a common denominator? (193)

5. Find a common denominator for $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{5}{6}$? (194, 195)

6. Explain how to write a remainder in fractional form. (197)

Written Practice

1. Give 3 names for each of these numbers: $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{4}{7}$. (184)

2. Reduce each of the following fractions to lowest terms.

$\frac{4}{16}$, $\frac{9}{15}$, $\frac{21}{49}$ (188)

3. Is $\frac{2}{10}$ larger than $\frac{2}{9}$? (192)

Copy. Solve for n.

	a	*b*	
4.	$\frac{1}{3}=\frac{n}{12}$	$\frac{1}{4}=\frac{5}{n}$	(185)
5.	$\frac{3}{4}$ of $20=n$	$\frac{7}{8}$ of $24=n$	(189)
6.	$52\div5=n$	$281\div16=n$	(197)

Self-Evaluation

Part 1 Copy. Write a numeral in each blank so the statement is true. Give the answer in lowest terms. (= means *measures the same as*.)

	a	*b*
1.	$\frac{3}{4}$ gal. = ___ qt.	7 in. = ___ ft.
2.	$\frac{5}{12}$ yd. = ___ in.	12 oz. = ___ lb.
3.	$\frac{7}{12}$ hr. = ___ min.	18 hr. = ___ da.
4.	$\frac{5}{8}$ lb. = ___ oz.	5 da. = ___ wk.
5.	$\frac{3}{10}$ T. = ___ lb.	4 pt. = ___ gal.
6.	$\frac{1}{4}$ mi. = ___ ft.	3 qt. = ___ bu.
7.	$\frac{1}{2}$ da. = ___ min.	55 min. = ___ hr.
8.	$\frac{4}{15}$ hr. = ___ min.	5 pt. = ___ gal.

Part 2 Copy. Solve for n.

	a	*b*	*c*
1.	$\frac{3}{4}=\frac{n}{16}$	$\frac{5}{8}=\frac{n}{24}$	$\frac{1}{6}=\frac{n}{54}$
2.	$\frac{2}{7}=\frac{n}{28}$	$\frac{3}{5}=\frac{n}{60}$	$\frac{4}{9}=\frac{n}{81}$
3.	$\frac{2}{3}=\frac{14}{n}$	$\frac{5}{6}=\frac{30}{n}$	$\frac{12}{17}=\frac{24}{n}$
4.	$\frac{8}{15}=\frac{24}{n}$	$\frac{10}{23}=\frac{100}{n}$	$\frac{7}{16}=\frac{35}{n}$
5.	$\frac{n}{12}=\frac{1}{3}$	$\frac{n}{32}=\frac{5}{8}$	$\frac{n}{24}=\frac{3}{8}$
6.	$\frac{30}{45}=\frac{n}{3}$	$\frac{9}{n}=\frac{3}{7}$	$\frac{28}{35}=\frac{4}{n}$
7.	$\frac{0}{12}=\frac{n}{24}$	$3=\frac{n}{6}$	$1=\frac{17}{n}$
8.	$\frac{n}{15}=0$	$\frac{12}{30}=\frac{n}{5}$	$\frac{100}{150}=\frac{10}{n}$

Part 3 Write the fractions given in each exercise so that the fractional numbers are named from the smallest to the largest.

1. $\frac{1}{5}, \frac{1}{4}, \frac{1}{8}, \frac{1}{7}, \frac{1}{3}$

2. $\frac{5}{7}, \frac{4}{7}, \frac{6}{7}, \frac{2}{7}, \frac{1}{7}$

3. $\frac{3}{5}, \frac{1}{3}, \frac{4}{15}$

4. $\frac{1}{3}, \frac{1}{4}, \frac{3}{8}$

5. $\frac{3}{4}, \frac{5}{6}, \frac{4}{5}$

6. $\frac{6}{5}, 1\frac{7}{8}, \frac{5}{4}$

7. $\frac{11}{2}, 2\frac{15}{8}, \frac{9}{4}$

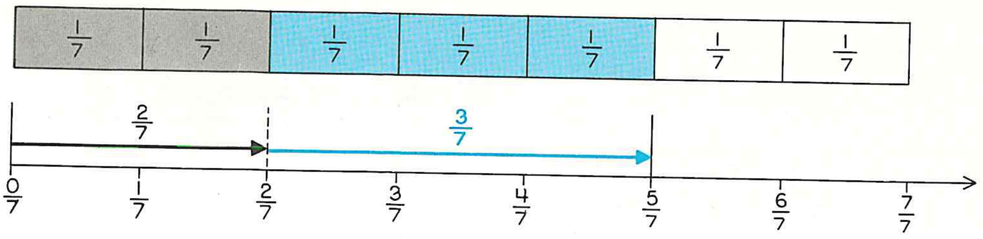
8. $\frac{7}{18}, \frac{5}{6}, \frac{1}{2}, \frac{7}{9}$

Part 4 Solve each of the following problems.

1. A truck was loaded with $\frac{7}{8}$ ton of coal. What was the weight of the coal in pounds?

2. There were 1,235 children attending Washington School. If $\frac{3}{5}$ of them were boys, how many girls attended Washington School?

3. Mr. Stark started driving from his home to Toledo, a distance of 285 miles. While driving he saw a sign which read, "Toledo—114 miles." How many miles had he driven when he saw the sign?



Addition of Fractional Numbers

The figure shown above has been divided into 7 parts of equal size. How many of these parts are gray? What fractional part of the whole figure is gray?

What fractional part of the whole figure is blue?

What fractional part of the whole figure is colored? Use addition to answer this question.

The addition of $\frac{2}{7}$ and $\frac{3}{7}$ is shown on the number line above.

The black arrow indicates the addend $\frac{2}{7}$ and the blue arrow indicates the addend $\frac{3}{7}$. Since you would arrive at the same point on the number line if you had started at 0 and laid off one segment whose length was $\frac{5}{7}$, the sum of $\frac{2}{7}$ and $\frac{3}{7}$ is $\frac{5}{7}$.

The addition below shows that you add fractional numbers just as you add whole numbers.

(gray)	2 sevenths	$\frac{2}{7}$
(blue)	+3 sevenths	$+\frac{3}{7}$
(colored)	5 sevenths	$\frac{5}{7}$

Since the numbers $\frac{2}{7}$ and $\frac{3}{7}$ have a common denominator, you merely add the numbers 2 and 3; write the sum over the common denominator.

This can be written as shown.

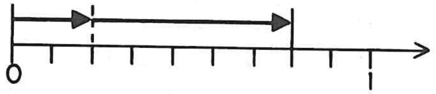
$$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$$

Oral Give the value of n in each equation below.

	a	*b*	*c*
1.	$\frac{1}{3}+\frac{1}{3}=n$	$\frac{1}{8}+\frac{2}{8}=n$	$\frac{1}{5}+\frac{3}{5}=n$
2.	$\frac{3}{7}+\frac{2}{7}=n$	$\frac{2}{15}+\frac{6}{15}=n$	$\frac{6}{9}+\frac{2}{9}=n$
3.	$\frac{1}{6}+\frac{4}{6}=n$	$\frac{5}{8}+\frac{2}{8}=n$	$\frac{4}{11}+\frac{3}{11}=n$
4.	$\frac{2}{9}+\frac{3}{9}=n$	$\frac{5}{17}+\frac{10}{17}=n$	$\frac{7}{30}+\frac{12}{30}=n$

Give the equation shown by the arrows on each number line below.

5.



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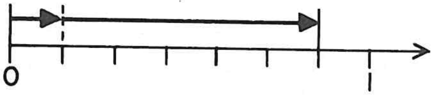

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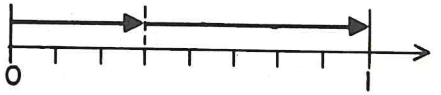

Proper and Improper Fractions

You have probably discovered by now that you really do not need a number line to compute sums of fractional numbers.

To find the sum of two fractional numbers with a common denominator, such as $\frac{2}{9}$ and $\frac{6}{9}$, write the sum of their numerators over the common denominator.

$$\frac{2}{9}+\frac{6}{9}=\frac{2+6}{9}=\frac{8}{9}$$

A fraction, such as $\frac{8}{9}$, whose numerator is smaller than its denominator is called a **proper fraction.** Other examples of proper fractions are $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{11}{12}$, and $\frac{29}{30}$.

Does a proper fraction name a number less than, equal to, or greater than 1?

Study the addition below.

$$\frac{3}{8}+\frac{7}{8}=\frac{3+7}{8}=\frac{10}{8}$$

A fraction, such as $\frac{10}{8}$, whose numerator is greater than or equal to the denominator is called an **improper fraction.** Some improper fractions are $\frac{3}{2}$, $\frac{7}{4}$, $\frac{8}{8}$, and $\frac{11}{9}$.

Does an improper fraction name a number less than, equal to, or greater than 1?

In the fraction $\frac{n}{10}$, if n is replaced by any numeral for a whole number from 0 through 9, the fraction is a proper fraction. If n is replaced by the numeral for the whole number 10 or larger, the fraction is an improper fraction.

Oral Answer each of the following questions.

1. What numbers could n stand for if $\frac{n}{3}$ is a proper fraction? If $\frac{12}{n}$ is a proper fraction?

2. What number does n stand for if $\frac{n}{20}$ is a name for 1? If $\frac{7}{n}$ is a name for 1?

3. What numbers could n stand for if $\frac{7}{n}$ is an improper fraction? If $\frac{n}{9}$ is an improper fraction?

Written Copy. Find each sum. Circle each sum which is given as an improper fraction.

	a	*b*	*c*
1.	$\frac{1}{4}+\frac{2}{4}=n$	$\frac{3}{10}+\frac{4}{10}=n$	$\frac{5}{7}+\frac{3}{7}=n$
2.	$\frac{4}{9}+\frac{5}{9}=n$	$\frac{1}{7}+\frac{5}{7}=n$	$\frac{1}{9}+\frac{4}{9}=n$
3.	$\frac{5}{16}+\frac{10}{16}=n$	$\frac{7}{12}+\frac{9}{12}=n$	$\frac{3}{13}+\frac{11}{13}=n$
4.	$\frac{16}{20}+\frac{7}{20}=n$	$\frac{3}{4}+\frac{9}{4}=n$	$\frac{3}{5}+\frac{4}{5}=n$
5.	$\frac{7}{19}+\frac{8}{19}=n$	$\frac{8}{23}+\frac{17}{23}=n$	$\frac{5}{11}+\frac{6}{11}=n$
6.	$\frac{5}{21}+\frac{8}{21}=n$	$\frac{4}{15}+\frac{13}{15}=n$	$\frac{7}{9}+\frac{8}{9}=n$
7.	$\frac{7}{12}+\frac{5}{12}=n$	$\frac{2}{15}+\frac{6}{15}=n$	$\frac{13}{20}+\frac{17}{20}=n$

Reducing Sums to Lowest Terms

The scout troop planned a Saturday hike. During the first hour the boys walked $\frac{3}{16}$ of the distance, and during the second hour they walked $\frac{5}{16}$ of the distance. What fractional part of the hike did they complete in the two hours?

To answer the question, you could add the fractional numbers as shown below.

$$\frac{3}{16}+\frac{5}{16}=\frac{3+5}{16}=\frac{8}{16}$$

You could say that they completed $\frac{8}{16}$ of the trip in two hours.

It is certainly correct to give $\frac{8}{16}$ as the answer. However, we usually get a clearer idea of the fractional part if the fraction is reduced to *lowest terms.*

$$\frac{8}{16}=\frac{8\div 8}{16\div 8}=\frac{1}{2}$$

The scouts completed $\frac{1}{2}$ of the trip in two hours.

The following additions are shown in equation form and also in column form. Each sum has been reduced to lowest terms.

$$\frac{2}{9}+\frac{1}{9}=\frac{2+1}{9}=\frac{3}{9}=\frac{1}{3} \qquad \frac{3}{8}+\frac{5}{8}=\frac{3+5}{8}=\frac{8}{8}=1$$

$$\begin{array}{r}\frac{2}{9}\\+\frac{1}{9}\\\hline\frac{3}{9}=\frac{1}{3}\end{array} \qquad \begin{array}{r}\frac{3}{8}\\+\frac{5}{8}\\\hline\frac{8}{8}=1\end{array}$$

Oral Explain the steps taken in finding each sum and in reducing the fractions to lowest terms.

	a	*b*
1.	$\frac{1}{8}+\frac{3}{8}=\frac{4}{8}=\frac{1}{2}$	$\frac{3}{20}+\frac{7}{20}=\frac{1}{2}$
2.	$\frac{1}{12}+\frac{7}{12}=\frac{8}{12}=\frac{2}{3}$	$\frac{5}{18}+\frac{7}{18}=\frac{2}{3}$
3.	$\frac{7}{10}+\frac{9}{10}=\frac{16}{10}=\frac{8}{5}$	$\frac{4}{15}+\frac{8}{15}=\frac{4}{5}$

Written Copy. Solve each of the equations for n. Write each sum in lowest terms.

	a	*b*	*c*
1.	$\frac{5}{12}+\frac{5}{12}=n$	$\frac{5}{8}+\frac{1}{8}=n$	$\frac{1}{10}+\frac{7}{10}=n$
2.	$\frac{2}{9}+\frac{4}{9}=n$	$\frac{1}{15}+\frac{4}{15}=n$	$\frac{5}{17}+\frac{12}{17}=n$
3.	$\frac{5}{18}+\frac{4}{18}=n$	$\frac{7}{12}+\frac{1}{12}=n$	$\frac{9}{20}+\frac{7}{20}=n$
4.	$\frac{7}{15}+\frac{2}{15}=n$	$\frac{5}{26}+\frac{8}{26}=n$	$\frac{1}{21}+\frac{5}{21}=n$
5.	$\frac{7}{8}+\frac{3}{8}=n$	$\frac{7}{50}+\frac{13}{50}=n$	$\frac{3}{11}+\frac{8}{11}=n$

Copy. Write each sum in lowest terms.

	a	*b*	*c*	*d*	*e*
6.	$\begin{array}{r}\frac{5}{9}\\+\frac{1}{9}\\\hline\end{array}$	$\begin{array}{r}\frac{4}{11}\\+\frac{7}{11}\\\hline\end{array}$	$\begin{array}{r}\frac{13}{20}\\+\frac{7}{20}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{18}\\+\frac{5}{18}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{10}\\+\frac{3}{10}\\\hline\end{array}$
7.	$\begin{array}{r}\frac{7}{20}\\+\frac{13}{20}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{24}\\+\frac{1}{24}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{17}\\+\frac{4}{17}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{12}\\+\frac{5}{12}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\+\frac{5}{8}\\\hline\end{array}$
8.	$\begin{array}{r}\frac{4}{15}\\+\frac{2}{15}\\\hline\end{array}$	$\begin{array}{r}\frac{9}{24}\\+\frac{6}{24}\\\hline\end{array}$	$\begin{array}{r}\frac{13}{30}\\+\frac{7}{30}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{18}\\+\frac{2}{18}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{50}\\+\frac{19}{50}\\\hline\end{array}$

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Improper Fractions and Mixed Fractions

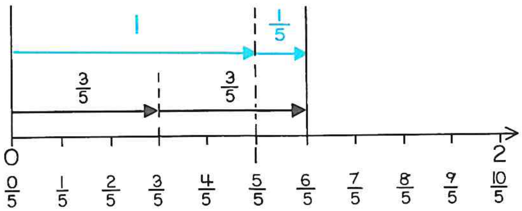
Bob checked the odometer on his bicycle and found that he lived $\frac{3}{5}$ of a mile from the swimming pool. How far does he ride in going to the pool and returning home?

The addition is shown by the equation $\frac{3}{5}+\frac{3}{5}=n$. The addition

$$\frac{3}{5}+\frac{3}{5}=\frac{3+3}{5}=\frac{6}{5}$$

shows that Bob would ride $\frac{6}{5}$ miles in going to and from the pool.

The diagram below shows this addition on a number line.



The black arrows show both addends as $\frac{3}{5}$. The blue arrows show that the sum would be the same if one addend were 1 and the other addend were $\frac{1}{5}$, as shown below.

$$\frac{3}{5}+\frac{3}{5}=\frac{6}{5}=1+\frac{1}{5}$$

Often a number is expressed as the sum of a whole number and a fractional number less than 1. A short way of writing $1+\frac{1}{5}$ is $1\frac{1}{5}$.

Then $1\frac{1}{5}$ is read "one and one-fifth." Likewise, $3\frac{5}{7}$ is read "three and five-sevenths" and means $3+\frac{5}{7}$. We call symbols like $1\frac{1}{5}$ and $3\frac{5}{7}$ **mixed fractions.**

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A mixed fraction is in lowest terms when the fractional part *is less than 1* and when the fractional part *is in lowest terms.* Why is $2\frac{5}{3}$ not in lowest terms? Why is $3\frac{6}{8}$ not in lowest terms?

To find the sum of $\frac{5}{8}$ and $\frac{7}{8}$, add the fractional numbers.

$$\frac{5}{8}+\frac{7}{8}=\frac{5+7}{8}=\frac{12}{8}$$

Then change $\frac{12}{8}$ to a mixed fraction in lowest terms.

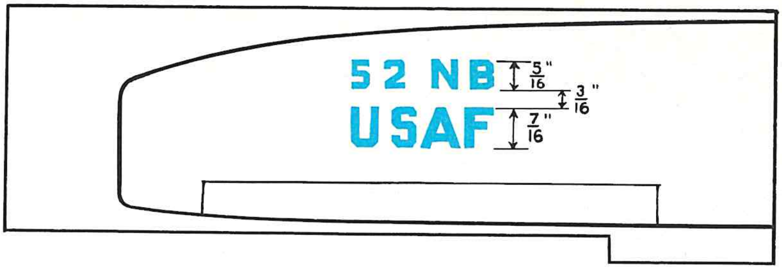
$$\frac{12}{8}=\frac{8}{8}+\frac{4}{8}=1+\frac{4}{8}=1+\frac{1}{2}=1\frac{1}{2}$$

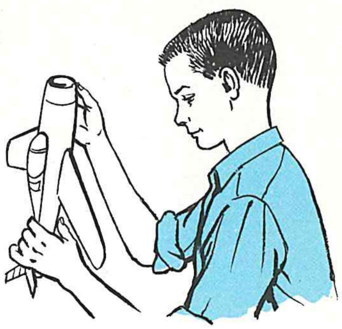
Oral Change each improper fraction to a mixed fraction.

a	*b*	*c*	*d*	*e*	*f*
$\frac{5}{4}$	$\frac{7}{5}$	$\frac{11}{7}$	$\frac{12}{5}$	$\frac{5}{3}$	$\frac{15}{9}$

Written Copy. Solve for n. Give n as a mixed fraction.

	a	*b*	*c*
1.	$\frac{7}{9}+\frac{4}{9}=n$	$\frac{4}{5}+\frac{3}{5}=n$	$\frac{7}{8}+\frac{2}{8}=n$
2.	$\frac{6}{7}+\frac{4}{7}=n$	$\frac{5}{8}+\frac{7}{8}=n$	$\frac{7}{12}+\frac{11}{12}=n$
3.	$\frac{5}{11}+\frac{9}{11}=n$	$\frac{11}{15}+\frac{7}{15}=n$	$\frac{6}{7}+\frac{5}{7}=n$
4.	$\frac{7}{20}+\frac{17}{20}=n$	$\frac{7}{16}+\frac{15}{16}=n$	$\frac{17}{21}+\frac{16}{21}=n$
5.	$\frac{9}{10}+\frac{7}{10}=n$	$\frac{17}{32}+\frac{19}{32}=n$	$\frac{34}{45}+\frac{29}{45}=n$





Using Three Fractional Addends

The plans for the model plane that Roger is making show the size of lettering on the wing. The top row of figures is to be $\frac{5}{16}$ of an inch high and the bottom row of figures is to be $\frac{7}{16}$ of an inch high. A $\frac{3}{16}$-inch space will be left between the rows. How much space does the lettering require?

To answer the question in the problem, you could solve the equation $\frac{5}{16}+\frac{3}{16}+\frac{7}{16}=n$.

The solution of the equation is given below.

$$n=\frac{5}{16}+\frac{3}{16}+\frac{7}{16}=\frac{5+3+7}{16}=\frac{15}{16}$$

The lettering requires a $\frac{15}{16}$-inch space.

To add two or more fractional numbers with a common denominator, add the numbers named by the numerators and write the sum over the common denominator.

Reduce the sum to lowest terms if possible. Also, if the sum is given as an improper fraction, change it to a mixed fraction.

Study the following addition involving three addends.

$$\frac{5}{9}+\frac{4}{9}+\frac{3}{9}=\frac{5+4+3}{9}=\frac{12}{9}$$

Then change $\frac{12}{9}$ to a mixed fraction.

$$\frac{12}{9}=\frac{9+3}{9}=\frac{9}{9}+\frac{3}{9}=1+\frac{3}{9}=1\frac{3}{9}$$

Then reduce $1\frac{3}{9}$ to lowest terms.

$$1+\frac{3}{9}=1+\frac{3\div 3}{9\div 3}=1+\frac{1}{3}=1\frac{1}{3}$$

Oral Answer the questions below.

1. In the second equation in the solution above, why is the numerator, 12, named as 9+3?

2. Would the answer be $1\frac{1}{3}$ if you add as indicated by $(\frac{5}{9}+\frac{4}{9})+\frac{3}{9}$?

Written Copy. Solve for n. Give n as a fraction or as a mixed fraction in lowest terms.

	a	*b*
1.	$\frac{3}{8}+\frac{1}{8}+\frac{1}{8}=n$	$\frac{5}{12}+\frac{1}{12}+\frac{7}{12}=n$
2.	$\frac{2}{15}+\frac{1}{15}+\frac{4}{15}=n$	$\frac{3}{20}+\frac{7}{20}+\frac{11}{20}=n$
3.	$\frac{6}{17}+\frac{3}{17}+\frac{8}{17}=n$	$\frac{7}{12}+\frac{4}{12}+\frac{5}{12}=n$
4.	$\frac{19}{21}+\frac{5}{21}+\frac{3}{21}=n$	$\frac{8}{9}+\frac{7}{9}+\frac{6}{9}=n$

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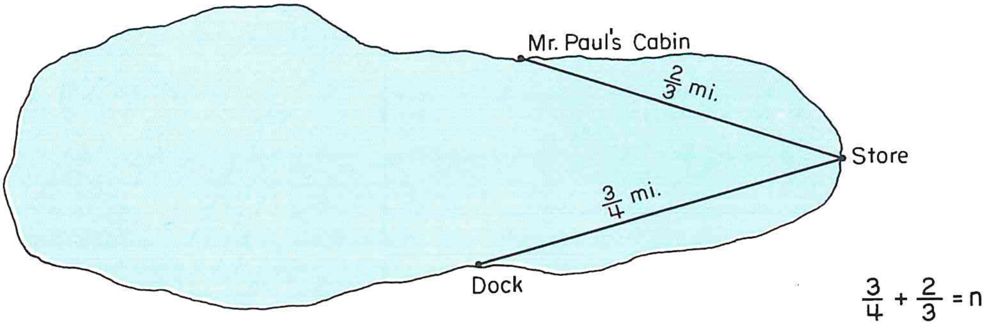

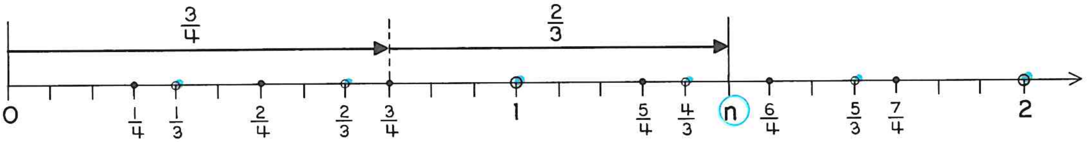

Adding Fractional Numbers

Shown above is a map of a lake. How far does Mr. Paul travel if he starts at the dock, goes to the store, and then to his cabin?

To answer the question, solve the equation, $\frac{3}{4}+\frac{2}{3}=n$.

These fractional numbers do not have a common denominator. Rename both numbers so that they have a common denominator, and add as you did before.

Finding a common denominator for $\frac{3}{4}$ and $\frac{2}{3}$ means the same as finding a number which is divisible by both denominators, 3 and 4.

Does any whole number other than 1 divide both 3 and 4? Since no whole number other than 1 divides both 3 and 4, the smallest common denominator is their product. That is, 4×3 or 12 is the smallest common denominator for $\frac{3}{4}$ and $\frac{2}{3}$.

Rewrite the equation, $\frac{3}{4}+\frac{2}{3}=n$, so that both numbers are expressed as twelfths.

$$n=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=\frac{12+5}{12}=\frac{12}{12}+\frac{5}{12}=1\frac{5}{12}$$

He would travel $1\frac{5}{12}$ miles.

The number line indicates that the segment from 0 to 1 would be divided into 12 segments of equal length. A segment of length $\frac{1}{12}$ was used to mark points between 1 and 2. Does n represent $\frac{17}{12}$, or $1\frac{5}{12}$, on the number line?

To add fractional numbers with unequal denominators, rename the numbers so that they have a common denominator. Then proceed as in the addition of fractional numbers with a common denominator.

Suppose you are to add $\frac{2}{3}$ and $\frac{1}{6}$. Find the smallest common denominator. The smaller denominator, 3, divides both denominators, so the larger denominator, 6, is the smallest common denominator. Study the addition below.

$$\frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{4+1}{6}=\frac{5}{6}$$

Suppose you are to add $\frac{5}{9}$ and $\frac{3}{4}$. There is no whole number other than 1 that divides both denominators, so their product, 4×9 or 36, is the smallest common denominator. Study the addition below.

$$\frac{5}{9}+\frac{3}{4}=\frac{20}{36}+\frac{27}{36}=\frac{20+27}{36}=\frac{47}{36}$$

Then change the answer to a mixed fraction.

$$\frac{47}{36}=\frac{36+11}{36}=\frac{36}{36}+\frac{11}{36}=1+\frac{11}{36}=1\frac{11}{36}$$

Oral What number would you use as the smallest common denominator in each addition below?

	a	*b*	*c*
1.	$\frac{2}{3}+\frac{1}{6}=n$	$\frac{1}{4}+\frac{5}{12}=n$	$\frac{2}{9}+\frac{5}{6}=n$
2.	$\frac{1}{3}+\frac{2}{5}=n$	$\frac{7}{8}+\frac{2}{3}=n$	$\frac{1}{5}+\frac{1}{6}=n$
3.	$\frac{3}{10}+\frac{1}{6}=n$	$\frac{5}{12}+\frac{4}{9}=n$	$\frac{3}{7}+\frac{1}{4}=n$
4.	$\frac{1}{7}+\frac{3}{5}=n$	$\frac{11}{12}+\frac{7}{8}=n$	$\frac{7}{8}+\frac{4}{5}=n$
5.	$\begin{array}{r}\frac{3}{5}\\+\frac{1}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{9}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\+\frac{1}{6}\\\hline\end{array}$
6.	$\begin{array}{r}\frac{17}{24}\\+\frac{8}{12}\\\hline\end{array}$	$\begin{array}{r}\frac{27}{36}\\+\frac{7}{9}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{12}\\+\frac{13}{18}\\\hline\end{array}$

Written Copy. Solve for n. Give n as a fraction or as a mixed fraction in lowest terms.

	a	*b*	*c*
1.	$\frac{1}{3}+\frac{1}{6}=n$	$\frac{3}{4}+\frac{1}{8}=n$	$\frac{3}{5}+\frac{3}{10}=n$
2.	$\frac{3}{4}+\frac{5}{12}=n$	$\frac{2}{9}+\frac{1}{3}=n$	$\frac{3}{7}+\frac{2}{3}=n$
3.	$\frac{1}{6}+\frac{3}{5}=n$	$\frac{3}{8}+\frac{1}{3}=n$	$\frac{7}{10}+\frac{3}{4}=n$
4.	$\frac{5}{12}+\frac{2}{9}=n$	$\frac{2}{7}+\frac{3}{5}=n$	$\frac{1}{2}+\frac{7}{9}=n$

Write each sum as a fraction or as a mixed fraction in lowest terms.

	a	*b*	*c*	*d*	*e*
5.	$\begin{array}{r}\frac{7}{15}\\+\frac{1}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{16}\\+\frac{3}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{7}\\+\frac{1}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{10}\\+\frac{1}{2}\\\hline\end{array}$
6.	$\begin{array}{r}\frac{5}{18}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{3}\\+\frac{5}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{12}\\+\frac{1}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{20}\\+\frac{4}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{13}{30}\\+\frac{5}{6}\\\hline\end{array}$
7.	$\begin{array}{r}\frac{7}{9}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{17}{35}\\+\frac{2}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\+\frac{3}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{27}{30}\\+\frac{3}{10}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{9}\\+\frac{4}{5}\\\hline\end{array}$

Can you do this? Solve each equation for n. Write a simpler name for each fraction before adding.

	a	*b*
1.	$\frac{5}{6\times2}+\frac{1}{4\times3}=n$	$\frac{7}{6\times3}+\frac{11}{2\times9}=n$
2.	$\frac{4}{2\times5}+\frac{2+1}{10}=n$	$\frac{4}{7+2}+\frac{2}{6+3}=n$
3.	$\frac{3+2}{12}+\frac{5-4}{12}=n$	$\frac{3\times3}{16}+\frac{2\times2}{16}=n$
4.	$\frac{1+2}{3\times3}+\frac{4-2}{7+2}=n$	$\frac{5\times3}{8\times4}+\frac{10-1}{16\times2}=n$
5.	$\frac{2\times3}{7+8}+\frac{7-5}{3\times5}=n$	$\frac{9-6}{5+3}+\frac{1+4}{2\times3}=n$

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Adding Three Fractional Numbers

The following equation could well represent a story problem.

$$\frac{1}{4}+\frac{2}{5}+\frac{1}{6}=n$$

Since the order in which you group the addends does not change the sum, add as shown below.

$$\left(\frac{1}{4}+\frac{2}{5}\right)+\frac{1}{6}=n$$

Now find the smallest common denominator for $\frac{1}{4}$ and $\frac{2}{5}$. Since no whole number other than 1 divides both 4 and 5, the smallest common denominator is their product, 4×5 or 20.

$$\left(\frac{1}{4}+\frac{2}{5}\right)+\frac{1}{6}=\left(\frac{5}{20}+\frac{8}{20}\right)+\frac{1}{6}=\frac{13}{20}+\frac{1}{6}$$

Now find the smallest common denominator for $\frac{13}{20}$ and $\frac{1}{6}$. The whole number 2 divides both 20 and 6, so the smallest common denominator is their product divided by 2. That is, $\frac{20\times 6}{2}$ or $\frac{120}{2}$ or 60 is the smallest common denominator.

$$\frac{13}{20}+\frac{1}{6}=\frac{39}{60}+\frac{10}{60}=\frac{39+10}{60}=\frac{49}{60}$$

You can use this method to find the smallest common denominator before you do any addition. The addition is shown below.

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$$\frac{1}{4}+\frac{2}{5}+\frac{1}{6}=\frac{15}{60}+\frac{24}{60}+\frac{10}{60}=\frac{15+24+10}{60}=\frac{49}{60}$$

Do the following addition as indicated by the parentheses.

$$\frac{1}{4}+\left(\frac{2}{5}+\frac{1}{6}\right)=n$$

Oral What number would you use as the smallest common denominator for adding each set of numbers?

1. Halves, thirds, fourths
2. Halves, thirds, sixths
3. Thirds, fourths, sixths
4. Thirds, sixths, ninths
5. $\frac{2}{3}, \frac{4}{5}, \frac{7}{10}$
6. $\frac{2}{5}, \frac{1}{4}, \frac{3}{8}$
7. $\frac{9}{20}, \frac{3}{4}, \frac{2}{3}$
8. $\frac{49}{100}, \frac{17}{25}, \frac{33}{50}$

Written Copy. Find a replacement for n, in lowest terms, which makes each equation true.

	a	*b*
1.	$\frac{1}{3}+\frac{1}{2}+\frac{2}{3}=n$	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=n$
2.	$\frac{3}{16}+\frac{1}{4}+\frac{5}{8}=n$	$\frac{1}{3}+\frac{5}{6}+\frac{2}{9}=n$
3.	$\frac{1}{4}+\frac{3}{5}+\frac{3}{8}=n$	$\frac{1}{3}+\frac{2}{5}+\frac{3}{10}=n$
4.	$\frac{2}{3}+\frac{4}{7}+\frac{5}{21}=n$	$\frac{1}{4}+\frac{5}{6}+\frac{3}{8}=n$
5.	$\frac{1}{2}+\frac{1}{6}+\frac{4}{9}=n$	$\frac{3}{4}+\frac{1}{2}+\frac{2}{3}=n$
6.	$\frac{1}{4}+\frac{2}{3}+\frac{3}{20}=n$	$\frac{3}{4}+\frac{1}{6}+\frac{3}{5}=n$
7.	$\frac{31}{100}+\frac{17}{50}+\frac{2}{25}=n$	$\frac{12}{80}+\frac{3}{8}+\frac{1}{10}=n$
8.	$\frac{3}{4}+\frac{19}{36}+\frac{2}{9}=n$	$\frac{3}{4}+\frac{7}{8}+\frac{1}{3}=n$

Using Fractions in Solving Problems

The problems on this page require the addition of fractional numbers. Read each problem carefully, write a suitable equation, solve the equation, and state the answer so that it answers the question asked in the problem.

1. Alice bought $\frac{1}{4}$ pound of mints and $\frac{1}{2}$ pound of bonbons. What fractional part of a pound of candy did she buy altogether?

2. Mary's mother used $\frac{2}{3}$ cup of white flour and $\frac{1}{3}$ cup of whole-wheat flour for making muffins. How many cups of flour did she use altogether?

3. Ed's mother asked him to clean his room and sweep the basement before leaving for the party. It will take him $\frac{1}{4}$ hour to clean his room and $\frac{2}{3}$ hour to sweep the basement. How many hours did he work before he left for the party?

4. Bill nailed a strip of plywood $\frac{1}{4}$ inch thick to a board which was $\frac{3}{8}$ inch thick. What was their combined thickness?

5. Bob walked $\frac{3}{4}$ mile to Tom's house, then $\frac{1}{3}$ mile to the school building, and finally $\frac{5}{6}$ mile to his home. How many miles did he walk altogether?

6. Linda helped her mother $\frac{1}{3}$ hour before school, and $\frac{2}{5}$ hour after school. What fractional part of an hour did Linda spend helping her mother that day?

7. Read problem 6 again. Express $\frac{1}{3}$ hour and $\frac{2}{5}$ hour as minutes, and find how many minutes Linda helped her mother that day.

8. Mr. Robertson has $\frac{3}{4}$ gallon of blue paint. To make it lighter blue, he added $\frac{1}{8}$ gallon of white paint. What fractional part of a gallon of paint did he have then?

9. Mrs. Johnson used $\frac{3}{4}$ cup of sugar for a cake, $\frac{1}{2}$ cup of sugar for cookies, and $\frac{1}{3}$ cup of sugar for a pie. How much sugar did she use for all the baking?

10. A can contained $\frac{1}{16}$ gallon of concentrated grape juice. The contents of the can was mixed with $\frac{1}{2}$ gallon of water. What amount of liquid resulted?

11. Mrs. Simpson used $\frac{2}{3}$ yard of blue ribbon and $\frac{5}{6}$ yard of white ribbon in making a dress for Janet. How many yards of ribbon did she use to trim the dress?

12. The rainfall during two showers on Sunday was $\frac{3}{10}$ inch and $\frac{7}{10}$ inch. How much did it rain on Sunday?

Changing Mixed Fractions to Improper Fractions

Before using an addend which is given as a mixed fraction, some practice in changing a mixed fraction to an improper fraction will be helpful.

If $4\frac{1}{2}$ is used as an addend, it may be necessary to change its form before doing the addition.

Recall that $4\frac{1}{2}$ means $4+\frac{1}{2}$, which can be written as $3+1+\frac{1}{2}$. Since $1=\frac{2}{2}$, $4\frac{1}{2}=3+\frac{2}{2}+\frac{1}{2}=3+\frac{3}{2}=3\frac{3}{2}$.

Also, $4\frac{1}{2}$ could be expressed as an improper fraction. Since the fractional part of $4\frac{1}{2}$ has a denominator of 2, express $4\frac{1}{2}$ as an improper fraction which has a denominator of 2. Two ways of doing this are given below.

Starting at 0 on a number line, how many segments of length $\frac{1}{2}$ would you use in order to reach the point labeled 4? The answer is 8 segments. This means that $4=\frac{8}{2}$. Then $4\frac{1}{2}$ may be written as shown.

$$4\frac{1}{2}=4+\frac{1}{2}=\frac{8}{2}+\frac{1}{2}=\frac{8+1}{2}=\frac{9}{2}$$

Another way to find that $4=\frac{8}{2}$ is $4=\frac{4}{1}=\frac{4\times 2}{1\times 2}=\frac{8}{2}$. Then, $4\frac{1}{2}=\frac{8}{2}+\frac{1}{2}=\frac{9}{2}$.

Oral What fraction would name the number 5 if the denominator is 2? If the denominator is 3? If the denominator is 7?

Written Write each sum as an improper fraction which has the same denominator as the fractional addend.

	a	*b*	*c*
1.	$1+\frac{2}{5}$	$1+\frac{7}{8}$	$2+\frac{1}{4}$
2.	$2+\frac{6}{7}$	$3+\frac{5}{8}$	$5+\frac{3}{9}$
3.	$8+\frac{2}{3}$	$10+\frac{3}{7}$	$21+\frac{2}{3}$

Write each of the following mixed fractions as improper fractions.

	a	*b*	*c*	*d*
4.	$1\frac{5}{6}$	$1\frac{7}{10}$	$2\frac{3}{8}$	$5\frac{3}{4}$
5.	$4\frac{1}{4}$	$5\frac{1}{6}$	$7\frac{2}{3}$	$10\frac{6}{7}$
6.	$8\frac{1}{2}$	$3\frac{8}{9}$	$12\frac{2}{3}$	$31\frac{3}{5}$
7.	$3\frac{7}{8}$	$7\frac{3}{10}$	$22\frac{1}{6}$	$50\frac{3}{4}$

Can you do this? Find a replacement for n such that each statement becomes true.

	a	*b*
1.	$\frac{1}{4}+n=\frac{3}{4}$	$n+\frac{1}{2}=\frac{5}{4}$
2.	$n+\frac{1}{7}=\frac{6}{7}$	$\frac{2}{3}+n=\frac{8}{9}$
3.	$\frac{2}{3}+n=\frac{11}{12}$	$n+\frac{1}{5}=\frac{2}{3}$
4.	$\frac{2}{3}+\frac{1}{4}+\frac{2}{3}+\frac{3}{4}=n$	
5.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{2}{3}+\frac{3}{4}=n$	
6.	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{2}{5}+\frac{1}{6}=n$	

EQUATION FORM

$$\frac{1}{5}+\frac{1}{10}+\frac{1}{2}=n$$

$$\frac{2}{10}+\frac{1}{10}+\frac{5}{10}=\frac{2+1+5}{10}=\frac{8}{10}=\frac{8\div 2}{10\div 2}=\frac{4}{5}$$

COLUMN FORM

$$\begin{array}{rcl} \frac{1}{5} & = & \frac{2}{10} \\ \frac{1}{10} & = & \frac{1}{10} \\ +\frac{1}{2} & = & \frac{5}{10} \\ \hline & & \frac{8}{10}=\frac{8\div 2}{10\div 2}=\frac{4}{5} \end{array}$$

Another Way of Showing Addition

The addition of fractional numbers can be shown in column form. Study the illustration at the top of this page.

If asked to add $\frac{1}{8}$ and $\frac{3}{8}$, you could proceed as shown below.

$$\begin{array}{r} \frac{1}{8} \\ +\frac{3}{8} \\ \hline \frac{4}{8}=\frac{4\div 4}{8\div 4}=\frac{1}{2} \end{array}$$

If the addends are given as fractions with unequal denominators, such as adding $\frac{3}{5}$ and $\frac{2}{3}$, proceed as shown below.

$$\begin{array}{rcl} \frac{3}{5} & = & \frac{9}{15} \\ +\frac{2}{3} & = & \frac{10}{15} \\ \hline & & \frac{19}{15}=\frac{15}{15}+\frac{4}{15}=1+\frac{4}{15}=1\frac{4}{15} \end{array}$$

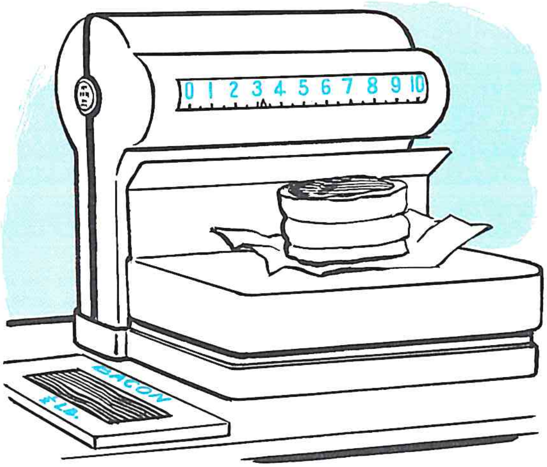
In this case, the numbers were given other names just as was done when adding fractional numbers in equation form.

Oral Explain the steps you would use in performing the following additions.

a	*b*	*c*	*d*	*e*
$\begin{array}{r}\frac{1}{5}\\+\frac{2}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{8}\\+\frac{4}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\+\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{12}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{4}\\+\frac{3}{5}\\\hline\end{array}$

Written Copy. Write each sum as a fraction or as a mixed fraction in lowest terms.

	a	*b*	*c*	*d*
1.	$\begin{array}{r}\frac{2}{7}\\+\frac{3}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{8}\\+\frac{5}{16}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\+\frac{1}{12}\\\hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\+\frac{4}{5}\\\hline\end{array}$
2.	$\begin{array}{r}\frac{5}{6}\\\frac{1}{2}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{10}\\\frac{7}{10}\\+\frac{4}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{8}\\\frac{1}{8}\\+\frac{3}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{12}{21}\\\frac{5}{21}\\+\frac{7}{21}\\\hline\end{array}$
3.	$\begin{array}{r}\frac{3}{20}\\\frac{7}{20}\\+\frac{13}{20}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\\frac{3}{5}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{18}\\\frac{4}{9}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{15}{32}\\\frac{7}{32}\\+\frac{18}{32}\\\hline\end{array}$
4.	$\begin{array}{r}\frac{3}{8}\\\frac{2}{7}\\+\frac{3}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{4}\\\frac{5}{8}\\+\frac{3}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{18}{45}\\\frac{4}{9}\\+\frac{2}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{25}{72}\\\frac{3}{8}\\+\frac{4}{9}\\\hline\end{array}$



Mrs. Thompson bought $\frac{1}{2}$ pound of bacon and a roast that weighed $3\frac{1}{4}$ pounds. How many pounds of meat did she buy?

To answer the question, add the numbers which represent the two weights. An equation for this problem is given below.

$$3\frac{1}{4}+\frac{1}{2}=n$$

Since $3\frac{1}{4}$ means $3+\frac{1}{4}$, write the equation as shown below.

$$3+\frac{1}{4}+\frac{1}{2}=n$$

The order of grouping the addends does not change the sum, so add the fractional parts first.

$$\begin{aligned} 3+\left(\tfrac{1}{4}+\tfrac{1}{2}\right)&=n \\ 3+\left(\tfrac{1}{4}+\tfrac{2}{4}\right)&=n \\ 3+\tfrac{3}{4}&=n \\ 3\tfrac{3}{4}&=n \end{aligned}$$

Mrs. Thompson bought $3\frac{3}{4}$ pounds of meat altogether.

Using Mixed Fractions in Addition

Study the following solution of $\frac{2}{3}+2\frac{3}{4}+\frac{1}{2}=n$.

Write $2\frac{3}{4}$ as $2+\frac{3}{4}$.

$$\frac{2}{3}+2\frac{3}{4}+\frac{1}{2}=\frac{2}{3}+\left(2+\frac{3}{4}\right)+\frac{1}{2}$$

Change the order of addition.

$$\frac{2}{3}+\left(2+\frac{3}{4}\right)+\frac{1}{2}=2+\left(\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\right)$$

Find a common denominator.

$$2+\left(\frac{2}{3}+\frac{3}{4}+\frac{1}{2}\right)=2+\left(\frac{8}{12}+\frac{9}{12}+\frac{6}{12}\right)$$

Add the fractional parts.

$$2+\left(\frac{8}{12}+\frac{9}{12}+\frac{6}{12}\right)=2+\frac{23}{12}$$

Rename $\frac{23}{12}$ as $\frac{12}{12}+\frac{11}{12}$.

$$2+\frac{23}{12}=2+\frac{12}{12}+\frac{11}{12}$$

Add and write as a mixed fraction.

$$2+\frac{12}{12}+\frac{11}{12}=2+1+\frac{11}{12}=3\frac{11}{12}$$

The same addition is shown below in abbreviated column form.

$$\begin{array}{rl} \frac{2}{3}= & \frac{8}{12} \\ 2\frac{3}{4}= & 2\frac{9}{12} \\ +\frac{1}{2}= & \frac{6}{12} \\ \hline & 2\frac{23}{12}=2+1+\frac{11}{12}=3\frac{11}{12} \end{array}$$

Oral Answer the questions below.

1. Is it ever necessary to rename the whole number before you add?

2. Since the grouping of addends leaves the sum unchanged, would it be correct to omit the parentheses in the solutions on this page?

Written Copy. Find each sum. Reduce the fraction in each answer to lowest terms if possible.

	a	*b*
1.	$\frac{2}{3}+6\frac{1}{3}=n$	$\frac{1}{2}+\frac{3}{4}+5\frac{2}{3}=n$
2.	$4\frac{1}{2}+\frac{3}{8}=n$	$\frac{3}{10}+9\frac{2}{5}+\frac{1}{20}=n$
3.	$7\frac{1}{3}+\frac{4}{7}=n$	$3\frac{1}{6}+\frac{3}{5}+\frac{7}{15}=n$
4.	$\frac{1}{4}+1\frac{5}{12}=n$	$\frac{3}{4}+1\frac{7}{8}+\frac{1}{6}=n$
5.	$12\frac{1}{2}+\frac{7}{10}=n$	$23\frac{1}{4}+\frac{3}{5}+\frac{5}{6}=n$
6.	$\frac{3}{7}+1\frac{5}{28}=n$	$\frac{7}{10}+2\frac{1}{5}+\frac{7}{8}=n$

Copy. Find each sum. Reduce the fraction in each answer to lowest terms if possible.

	a	*b*	*c*	*d*
7.	$\begin{array}{r}8\frac{1}{4}\\+\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}7\frac{1}{2}\\+\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{9}\\+3\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{2}\\+4\frac{3}{8}\\\hline\end{array}$
8.	$\begin{array}{r}9\frac{3}{4}\\+\frac{7}{8}\\\hline\end{array}$	$\begin{array}{r}11\frac{2}{5}\\+\frac{7}{10}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\+4\frac{5}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{6}\\+3\frac{7}{12}\\\hline\end{array}$
9.	$\begin{array}{r}5\frac{9}{10}\\\frac{2}{5}\\+\frac{1}{5}\\\hline\end{array}$	$\begin{array}{r}28\frac{1}{3}\\\frac{1}{9}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{2}\\\frac{5}{6}\\+2\frac{1}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\\frac{5}{12}\\+5\frac{2}{3}\\\hline\end{array}$
10.	$\begin{array}{r}9\frac{3}{16}\\\frac{1}{2}\\+\frac{7}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\32\frac{9}{16}\\+\frac{5}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{20}\\\frac{1}{4}\\+8\frac{3}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\\frac{5}{6}\\+3\frac{2}{5}\\\hline\end{array}$
11.	$\begin{array}{r}7\frac{1}{6}\\2\frac{1}{2}\\+\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}13\frac{1}{8}\\1\frac{3}{4}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}5\frac{1}{6}\\\frac{3}{5}\\+2\frac{3}{10}\\\hline\end{array}$	$\begin{array}{r}18\frac{3}{4}\\6\frac{7}{12}\\+\frac{5}{9}\\\hline\end{array}$

Something to do Find the answers to the following problems mentally. Write the answers only.

1. Mary studied her lesson for $\frac{1}{2}$ hour at school and for $\frac{1}{4}$ hour at home. What fractional part of an hour did she study in all?

2. Sally is 50 inches tall, Jane is 55 inches tall, and Peggy is 60 inches tall. What is the average height of the three girls?

3. Mark is reading a book that has 15 chapters. He has finished reading Chapter VIII. How many more chapters does he have to read?

4. School began at 8:45 a.m., and Billy arrived at 9:30 a.m. What part of an hour was he late? How many minutes late was he?

5. Donald spent $\frac{1}{5}$ of a dollar for candy and $\frac{3}{10}$ of a dollar for a notebook. What fractional part of a dollar did he spend? How many cents did he spend?

6. Mrs. Fletcher paid $3.20 for a 4-pound roast. What was the cost of one pound of the roast?

7. Roger has $7.00. He needs $10.00 to buy new shoe skates. If he can save 50¢ a week, how many weeks will it take him to save enough to buy the skates?

8. Which is larger, $\frac{5}{6}$ or $\frac{11}{12}$?

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Using Two or More Mixed Fractions

When the addends are given as mixed fractions, as in $3\frac{1}{5}+2\frac{3}{10}$, use these steps in the addition.

a. Express the addends as the sum of a whole number and a fractional number. $(3+\frac{1}{5}+2+\frac{3}{10})$

b. Add the fractional numbers. Express the sum as a mixed fraction if possible. $(\frac{1}{5}+\frac{3}{10}=\frac{2}{10}+\frac{3}{10}=\frac{5}{10}=\frac{1}{2})$

c. Next add the whole numbers. $(3+2=5)$

d. Then express the sum as a mixed fraction. In some cases the sum may be a whole number. $(5+\frac{1}{2}=5\frac{1}{2})$

Observe how these steps are used in the following additions.

$4\frac{2}{5}+1\frac{3}{10}=n$	$5\frac{1}{2}+2\frac{3}{4}=n$
$4+\frac{2}{5}+1+\frac{3}{10}=n$	$5+\frac{1}{2}+2+\frac{3}{4}=n$
$4+1+\frac{2}{5}+\frac{3}{10}=n$	$5+2+\frac{1}{2}+\frac{3}{4}=n$
$4+1+\frac{4}{10}+\frac{3}{10}=n$	$5+2+\frac{2}{4}+\frac{3}{4}=n$
$5+\frac{4+3}{10}=n$	$7+\frac{5}{4}=n$
$5+\frac{7}{10}=n$	$7+1+\frac{1}{4}=n$
$5\frac{7}{10}=n$	$8\frac{1}{4}=n$

The above additions can also be written in column form.

$$\begin{array}{rl} 4\frac{2}{5} & =4\frac{4}{10} \\ \underline{+1\frac{3}{10}} & =\underline{1\frac{3}{10}} \\ & 5\frac{7}{10} \end{array} \qquad \begin{array}{rl} 5\frac{1}{2} & =5\frac{2}{4} \\ \underline{+2\frac{3}{4}} & =\underline{2\frac{3}{4}} \\ & 7\frac{5}{4}=8\frac{1}{4} \end{array}$$

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Oral Answer the following questions about $3\frac{1}{2}+2\frac{4}{5}=n$.

1. Does n represent a number greater than or less than 5? Greater than or less than 6? Why?

2. How would you proceed to solve the equation for n?

Written Copy. Find each sum.

	a	b	c	d
1.	$4\frac{2}{3}$ $+9\frac{1}{6}$	$7\frac{1}{4}$ $+5\frac{1}{8}$	$6\frac{1}{10}$ $+7\frac{3}{5}$	$5\frac{1}{3}$ $+4\frac{5}{12}$
2.	$11\frac{2}{7}$ $+3\frac{1}{3}$	$6\frac{1}{2}$ $+7\frac{1}{10}$	$17\frac{2}{3}$ $+5\frac{4}{9}$	$20\frac{1}{5}$ $+9\frac{1}{3}$
3.	$4\frac{1}{2}$ $3\frac{5}{6}$ $+7\frac{2}{3}$	$14\frac{2}{5}$ $4\frac{7}{10}$ $+5\frac{4}{5}$	$4\frac{7}{8}$ 8 $+7\frac{9}{16}$	$15\frac{2}{5}$ $12\frac{1}{4}$ $+1\frac{7}{10}$
4.	12 $42\frac{1}{3}$ $+8\frac{1}{6}$	$25\frac{1}{2}$ $17\frac{1}{6}$ $+4\frac{5}{12}$	$18\frac{2}{9}$ $17\frac{1}{6}$ $+7\frac{2}{3}$	$26\frac{6}{7}$ 13 $+2\frac{2}{3}$

Copy. Solve for n.

	a	b
5.	$3\frac{3}{4}+3\frac{7}{8}=n$	$3\frac{1}{6}+2\frac{3}{4}+4\frac{1}{3}=n$
6.	$5\frac{1}{3}+4\frac{5}{12}=n$	$7\frac{5}{9}+3\frac{2}{3}+1\frac{4}{9}=n$
7.	$5\frac{5}{12}+7\frac{5}{6}=n$	$\frac{5}{7}+3\frac{2}{3}+4\frac{4}{7}=n$
8.	$10\frac{1}{4}+3\frac{7}{16}=n$	$21+\frac{7}{8}+5\frac{2}{3}=n$
9.	$7\frac{3}{4}+1\frac{5}{7}=n$	$3\frac{1}{4}+5\frac{3}{5}+\frac{1}{3}=n$

Solving Problems

Write an equation for each problem on this page. Solve the equation and give an answer to the question asked in the problem.

1. Jeff walked $\frac{3}{4}$ mile to school, $\frac{1}{2}$ mile to the library, and then $\frac{3}{4}$ mile home. How many miles did he walk altogether?

2. Judy bought 3 pieces of ribbon. One piece was $\frac{3}{4}$ yard long, another was $\frac{7}{8}$ yard long, and the third piece was $\frac{1}{2}$ yard long. How many yards of ribbon did she buy?

3. It takes $4\frac{1}{2}$ hours to drive from Boston to New York. Driving from New York to Philadelphia takes $1\frac{3}{4}$ hours. How long does it take to drive from Boston to Philadelphia by way of New York?

4. Herbert caught 3 fish. Their weights were $\frac{7}{8}$ lb., $1\frac{3}{4}$ lb., and $2\frac{1}{3}$ lb. What was the combined weight of the three fish?

5. The weather bureau reported that the three snowfalls in February measured $9\frac{1}{4}$ inches, $2\frac{3}{10}$ inches, and $3\frac{1}{2}$ inches. What was the total snowfall during February?

6. Bill's mother bought $\frac{3}{4}$ pound of mints, $\frac{1}{2}$ pound of fudge, and $2\frac{3}{4}$ pounds of hard candy. How many pounds of candy did she buy?

7. Robert helped his father plant the garden. He worked $\frac{3}{4}$ hr. on Thursday, $1\frac{1}{3}$ hr. on Friday, and $3\frac{1}{5}$ hr. on Saturday. How many hours did he work in all?

8. The temperature went up $18\frac{3}{10}°$ between 7 a.m. and 10 a.m. If the temperature at 7 a.m. was $32\frac{1}{5}°$, what was it at 10 a.m.?

9. The pupils in the band practiced $1\frac{1}{4}$ hr. on Monday and $1\frac{5}{6}$ hr. on Thursday. How many hours did they practice on the two days?

10. Marvin used a fishing rod which consisted of 3 sections. When put together, the lengths of the sections were 2 ft., $2\frac{1}{6}$ ft., and $2\frac{1}{4}$ ft. How long was the rod when it was put together?

Can you do this? In some cases two of the addends have a sum which is a whole number. Observing such sums makes the addition easier.

$$3\tfrac{1}{4}+5\tfrac{2}{5}+7\tfrac{3}{4}=n$$
$$(3\tfrac{1}{4}+7\tfrac{3}{4})+5\tfrac{2}{5}=n$$
$$11+5\tfrac{2}{5}=n$$
$$16\tfrac{2}{5}=n$$

Copy. Find each sum.

1. $3\frac{2}{7}+15\frac{4}{9}+8\frac{5}{7}=n$

2. $4\frac{4}{9}+7\frac{1}{3}+5\frac{5}{9}+\frac{7}{8}+2\frac{2}{3}=n$

Review and Practice

Part 1 Each blank in the following statements is to be replaced by a word for one of the basic operations. In each case, write the word which makes the sentence true.

1. To find the total weight of any number of articles, each having the same weight, you —.

2. To find the sum of two or more numbers, you —.

3. You know how many children are at a picnic and how many cookies you have for the group. To find each child's fair share of the cookies, you —.

4. You know how many pupils are in your class and how many are absent. To find how many pupils are present, you —.

5. You know how many feet there are in 1 mile. To find the number of inches in 1 mile you —.

Part 2 Copy. Solve each equation.

	a	*b*
1.	$9 \times 5 \times 2 = n$	$(3 \times 50) \div n = 15$
2.	$6 + 5 \times 3 = n$	$(56 + 14) \div n = 10$
3.	$8 + n + 9 = 21$	$(85 \div 5) + n = 41$
4.	$8 \times 4 \times n = 96$	$(92 - 32) \times 4 = n$
5.	$5 \times 2 - 7 = n$	$(24 \times 7) \div 12 = n$

Part 3 Copy. Find each sum, difference, or product as indicated by the operation sign.

	a	*b*	*c*
1.	356 89 +737	5280 907 +2765	98621 5872 +1379
2.	582 −356	7324 −836	47253 −6474
3.	472 891 357 823 +456	7215 825 4379 672 +2767	59261 8442 77206 4003 +7628
4.	486 ×57	7018 ×436	34218 ×254
5.	507 ×99	6504 ×708	50027 ×520

Part 4 Copy. Find each quotient and each remainder. Express the remainders as fractions.

	a	*b*
1.	$32\overline{)1248}$	$47\overline{)24152}$
2.	$37\overline{)5476}$	$52\overline{)76519}$
3.	$82\overline{)89267}$	$66\overline{)371924}$
4.	$55\overline{)34182}$	$17\overline{)154763}$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. To add fractional numbers with a common denominator, write the sum of the numerators over their common denominator. (201)

2. If a fraction has a numerator which is less than the denominator, it is a proper fraction. (202)

3. If a numerator is equal to or greater than the denominator, the fraction is improper. (202)

4. A mixed fraction indicates the sum of a whole number and a fractional number. (204)

5. To add fractional numbers with unequal denominators, rename the numbers so they have a common denominator. Then proceed as in the addition of fractional numbers with a common denominator. (206)

6. If an addend is a mixed fraction, change the mixed fraction to the sum of a whole number and a fractional number. (214)

Words to Know

1. Proper fraction (202)

2. Improper fraction (202)

3. Mixed fraction (204)

Questions to Discuss

1. What kind of a fraction is $\frac{4}{7}$? $\frac{9}{8}$, $\frac{5}{5}$? (202)

2. Three pupils gave the sum of $1\frac{3}{10}+\frac{9}{10}$ as $\frac{22}{10}$, $2\frac{2}{10}$, and $2\frac{1}{5}$. Are all of these correct? Which answer is preferred? Why? (204)

3. What is the smallest common denominator for $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{9}$? (208)

4. Does the grouping of fractional addends change the sum? (208)

5. What steps would you use when solving for n in the equation $4\frac{2}{3}+\frac{7}{8}+\frac{1}{6}=n$? (212)

Written Practice

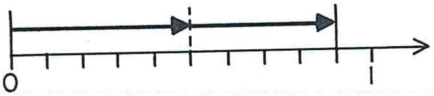
Copy. Find each sum.

	a	*b*	
1.	$\frac{3}{8}+\frac{1}{8}=n$	$\frac{5}{21}+\frac{9}{21}=n$	(203)
2.	$\frac{7}{8}+\frac{5}{8}=n$	$\frac{17}{30}+\frac{23}{30}=n$	(204)
3.	$\frac{5}{7}+\frac{1}{7}+\frac{3}{7}=n$	$\frac{5}{9}+\frac{4}{9}+\frac{7}{9}=n$	(205)
4.	$\frac{2}{3}+\frac{7}{18}=n$	$\frac{4}{7}+\frac{2}{5}=n$	(207)
5.	$\frac{2}{5}+\frac{1}{2}+\frac{2}{3}=n$	$\frac{1}{4}+\frac{2}{3}+\frac{4}{5}=n$	(208)
6.	$\frac{5}{7}+3\frac{1}{7}=n$	$\frac{4}{5}+5\frac{3}{4}=n$	(213)
7.	$6\frac{1}{2}+1\frac{3}{4}=n$	$7\frac{1}{4}+4\frac{2}{3}=n$	(214)

Self-Evaluation

Part 1 Write the equation shown on each number line below.

1.



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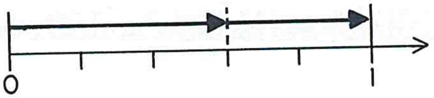

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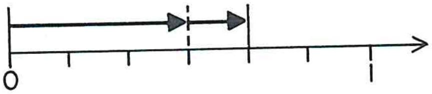

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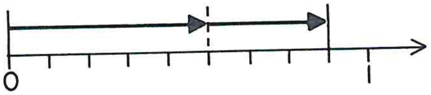


Part 2 Copy. Write each answer in lowest terms.

	a	*b*	*c*
1.	$\frac{3}{7}+\frac{2}{7}=n$	$\frac{1}{9}+\frac{2}{9}=n$	$\frac{3}{8}+\frac{5}{8}=n$
2.	$\frac{5}{12}+\frac{5}{12}=n$	$\frac{3}{5}+\frac{4}{5}=n$	$\frac{5}{16}+\frac{3}{16}=n$
3.	$\frac{7}{18}+\frac{2}{18}=n$	$\frac{4}{15}+\frac{6}{15}=n$	$\frac{3}{10}+\frac{7}{10}=n$
4.	$\frac{1}{4}+\frac{3}{8}=n$	$\frac{5}{6}+\frac{1}{12}=n$	$\frac{2}{5}+\frac{3}{10}=n$
5.	$\frac{2}{3}+\frac{1}{5}=n$	$\frac{2}{9}+\frac{1}{6}=n$	$\frac{1}{4}+\frac{1}{3}=n$

Part 3 Copy. Write each improper fraction as a mixed fraction.

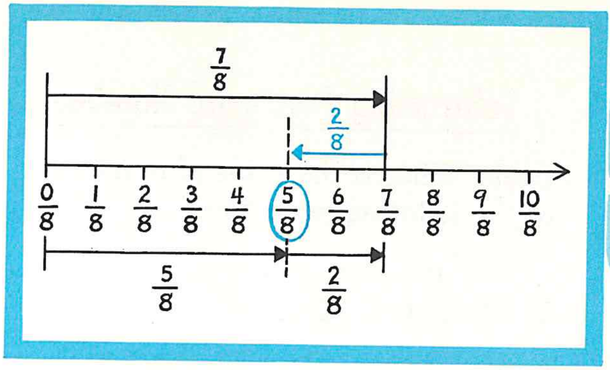
	a	*b*	*c*	*d*
1.	$\frac{5}{4}$	$\frac{8}{7}$	$\frac{7}{5}$	$\frac{9}{2}$
2.	$\frac{12}{11}$	$\frac{15}{8}$	$\frac{12}{5}$	$\frac{11}{7}$

Copy. Write each mixed fraction as an improper fraction.

	a	*b*	*c*	*d*
3.	$1\frac{2}{3}$	$2\frac{1}{5}$	$1\frac{7}{8}$	$1\frac{3}{4}$
4.	$3\frac{1}{6}$	$2\frac{4}{7}$	$4\frac{5}{6}$	$5\frac{1}{4}$

Part 4 Copy. Find each sum. Write each answer as a proper fraction in lowest terms or as a mixed fraction in lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{5}{12}$ $+\frac{1}{4}$	$\frac{1}{2}$ $+\frac{3}{10}$	$\frac{9}{16}$ $+\frac{7}{16}$	$\frac{5}{11}$ $+\frac{8}{11}$
2.	$\frac{1}{3}$ $+\frac{11}{18}$	$\frac{3}{4}$ $+\frac{3}{5}$	$\frac{7}{8}$ $+\frac{3}{4}$	$\frac{4}{5}$ $+\frac{2}{3}$
3.	$3\frac{1}{2}$ $+\frac{1}{4}$	$1\frac{2}{5}$ $+\frac{3}{10}$	$7\frac{5}{6}$ $+\frac{7}{12}$	$2\frac{1}{5}$ $+\frac{3}{4}$
4.	$2\frac{1}{2}$ $+3\frac{1}{3}$	$4\frac{1}{3}$ $+6\frac{1}{6}$	$3\frac{2}{5}$ $+5\frac{2}{3}$	$1\frac{3}{4}$ $+7\frac{5}{6}$
5.	$\frac{1}{12}$ $\frac{5}{12}$ $+\frac{3}{12}$	$\frac{9}{16}$ $\frac{3}{16}$ $+\frac{5}{16}$	$3\frac{1}{11}$ $1\frac{2}{11}$ $+\frac{7}{11}$	$4\frac{5}{6}$ $\frac{2}{3}$ $+1\frac{1}{6}$
6.	$\frac{3}{4}$ $\frac{1}{3}$ $+\frac{5}{6}$	$2\frac{1}{2}$ $3\frac{2}{5}$ $+1\frac{3}{10}$	$11\frac{1}{4}$ $3\frac{1}{8}$ $+2\frac{2}{3}$	$9\frac{2}{5}$ $3\frac{1}{4}$ $+7\frac{7}{10}$
7.	$3\frac{1}{8}$ $+\frac{2}{5}$	$1\frac{5}{6}$ $+\frac{3}{4}$	$5\frac{2}{9}$ $+2\frac{1}{4}$	$12\frac{1}{6}$ $+7\frac{3}{8}$


$\frac{7}{8}$
$\frac{2}{8}$
$\frac{0}{8}$ $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$ $\frac{9}{8}$ $\frac{10}{8}$
$\frac{5}{8}$
$\frac{2}{8}$

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Subtraction of Fractional Numbers

David needed $\frac{2}{8}$ gallon of paint for his project. He poured this amount of paint from a container which had $\frac{7}{8}$ gallon of paint in it. What fractional part of a gallon of paint remained in the container?

Since David removed $\frac{2}{8}$ gallon of paint from $\frac{7}{8}$ gallon of paint, you could subtract as indicated in the equation $\frac{7}{8}-\frac{2}{8}=n$.

We want the arithmetic of fractional numbers to be in full agreement with the arithmetic of whole numbers. Since addition and subtraction are inverse operations with whole numbers, they should be inverse operations with fractional numbers also.

To solve the equation $\frac{7}{8}-\frac{2}{8}=n$, think of $n+\frac{2}{8}=\frac{7}{8}$. According to addition of fractional numbers, n must be $\frac{5}{8}$; so $\frac{7}{8}-\frac{2}{8}=\frac{5}{8}$.

Study the number line solution at the top of this page. Subtraction is shown by arrows above the number line, and addition is shown by the arrows below the number line.

Oral Use the idea of inverse operations in giving the value of n in each subtraction below.

1. If $\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$, then $\frac{5}{7}-\frac{2}{7}=n$.
2. If $\frac{5}{9}+\frac{3}{9}=\frac{8}{9}$, then $\frac{8}{9}-\frac{3}{9}=n$.
3. If $\frac{4}{15}+\frac{8}{15}=\frac{12}{15}$, then $\frac{12}{15}-\frac{8}{15}=n$.
4. If $\frac{2}{5}+\frac{3}{5}=\frac{5}{5}$, then $\frac{5}{5}-\frac{3}{5}=n$.
5. If $\frac{7}{11}+\frac{3}{11}=\frac{10}{11}$, then $\frac{10}{11}-\frac{3}{11}=n$.

Written Draw a number line solution for each equation below.

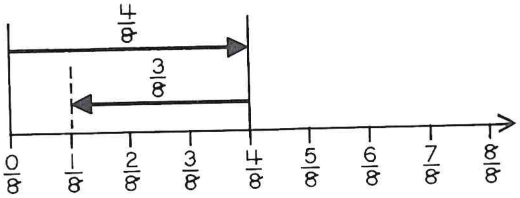
	a	*b*	*c*
1.	$\frac{3}{4}-\frac{1}{4}=n$	$\frac{5}{6}-\frac{3}{6}=n$	$\frac{4}{5}-\frac{2}{5}=n$
2.	$\frac{6}{8}-\frac{2}{8}=n$	$\frac{9}{10}-\frac{3}{10}=n$	$\frac{7}{7}-\frac{4}{7}=n$

Check the solution of each equation above by writing the inverse, or addition, equation.

Subtracting Fractional Numbers

Sue lives $\frac{4}{8}$ mile from school and Jim lives $\frac{3}{8}$ mile from school. How much farther from school does Sue live than Jim?

The equation $\frac{4}{8}-\frac{3}{8}=n$ could be used to find the answer. The number line solution below shows the answer to be $\frac{1}{8}$ mile.



Hence, $\frac{4}{8}-\frac{3}{8}=\frac{1}{8}$. This shows how to subtract fractional numbers having the same denominator.

$$\frac{4}{8}-\frac{3}{8}=\frac{4-3}{8}=\frac{1}{8}$$

To subtract fractional numbers having the same denominator, subtract the numbers named by the numerators as you would subtract whole numbers. Write this difference over their common denominator.

You subtract fractional numbers with a common denominator just as you subtract whole numbers.

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4 tens	4 eighths	$\frac{4}{8}$
−3 tens	−3 eighths	$-\frac{3}{8}$
1 ten	1 eighth	$\frac{1}{8}$

Oral What is the value of n in each of the following equations?

	a	*b*	*c*
1.	$\frac{3}{5}-\frac{1}{5}=n$	$\frac{3}{4}-\frac{2}{4}=n$	$\frac{7}{11}-\frac{4}{11}=n$
2.	$\frac{7}{12}-\frac{5}{12}=n$	$\frac{7}{8}-\frac{5}{8}=n$	$\frac{17}{20}-\frac{13}{20}=n$
3.	$\frac{7}{9}-\frac{4}{9}=n$	$\frac{4}{15}-\frac{1}{15}=n$	$\frac{10}{21}-\frac{5}{21}=n$

Written Copy. Write each difference in lowest terms.

	a	*b*	*c*
1.	$\frac{23}{25}-\frac{18}{25}=n$	$\frac{37}{56}-\frac{19}{56}=n$	$\frac{42}{70}-\frac{28}{70}=n$
2.	$\frac{25}{32}-\frac{17}{32}=n$	$\frac{17}{18}-\frac{8}{18}=n$	$\frac{11}{16}-\frac{3}{16}=n$
3.	$\frac{9}{13}-\frac{6}{13}=n$	$\frac{6}{7}-\frac{2}{7}=n$	$\frac{12}{14}-\frac{5}{14}=n$

Copy. Write each difference in lowest terms.

	a	*b*	*c*	*d*
4.	$\frac{5}{6}$ $-\frac{2}{6}$	$\frac{6}{9}$ $-\frac{5}{9}$	$\frac{9}{16}$ $-\frac{3}{16}$	$\frac{5}{8}$ $-\frac{1}{8}$
5.	$\frac{11}{12}$ $-\frac{5}{12}$	$\frac{8}{15}$ $-\frac{3}{15}$	$\frac{8}{9}$ $-\frac{5}{9}$	$\frac{3}{4}$ $-\frac{1}{4}$
6.	$\frac{7}{10}$ $-\frac{2}{10}$	$\frac{13}{16}$ $-\frac{3}{16}$	$\frac{13}{14}$ $-\frac{5}{14}$	$\frac{17}{30}$ $-\frac{9}{30}$
7.	$\frac{41}{50}$ $-\frac{19}{50}$	$\frac{29}{30}$ $-\frac{14}{30}$	$\frac{8}{15}$ $-\frac{2}{15}$	$\frac{9}{10}$ $-\frac{5}{10}$
8.	$\frac{15}{18}$ $-\frac{12}{18}$	$\frac{17}{36}$ $-\frac{5}{36}$	$\frac{13}{25}$ $-\frac{3}{25}$	$\frac{21}{24}$ $-\frac{9}{24}$

Subtracting from a Whole Number

Polly took a cake for lunch at a meeting. She brought $\frac{2}{5}$ of the cake home again. What fractional part of the cake had been eaten?

You might be able to give the answer, $\frac{3}{5}$ of the cake, without computation, but let us examine the subtraction a little closer.

You are to subtract $\frac{2}{5}$ from 1, so name 1 as a fraction so that the two fractions have a common denominator. That is, use $\frac{5}{5}$ as a name for 1, and subtract as shown below.

$$1-\frac{2}{5}=\frac{5}{5}-\frac{2}{5}=\frac{5-2}{5}=\frac{3}{5}$$

In some cases you need to subtract a fractional number from a whole number larger than 1. For instance, solve $3-\frac{4}{7}=n$. Study the solution below in equation form and in column form.

$$\begin{aligned} 3-\tfrac{4}{7}&=n \\ (2+1)-\tfrac{4}{7}&=n \\ 2+(\tfrac{7}{7}-\tfrac{4}{7})&=n \\ 2\tfrac{3}{7}&=n \end{aligned} \qquad\qquad \begin{array}{rcr} 3 & = & 2\frac{7}{7} \\ -\frac{4}{7} & = & \frac{4}{7} \\ \hline & & 2\frac{3}{7} \end{array}$$

In the above solution, the whole number 3 was named as a sum which has 1 as one of its addends. The addend 1 was then renamed as a fraction which has the same denominator as the fraction you are to subtract.

Oral Use the equation, $5-\frac{5}{8}=n$, to answer the following questions.

1. What is the first step in the solution of the equation?

2. Which numeral do you use as the denominator when writing 1 as a fraction?

3. Can you give the answer mentally? If so, what is the answer?

Written Copy. Find each difference.

	a	*b*	*c*
1.	$1-\frac{2}{7}=n$	$5-\frac{1}{3}=n$	$2-\frac{6}{13}=n$
2.	$3-\frac{14}{15}=n$	$4-\frac{5}{11}=n$	$7-\frac{9}{20}=n$
3.	$\begin{array}{r} 12 \\ -\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 13 \\ -\frac{5}{9} \\ \hline \end{array}$	$\begin{array}{r} 217 \\ -\frac{7}{30} \\ \hline \end{array}$

Solve each problem below.

4. Janet's mother used $\frac{2}{3}$ of a pie for dinner. What fractional part of the pie was left?

5. Kenneth bought 2 packages of gum. He chewed $\frac{2}{5}$ of the gum contained in 1 package. How much gum did he have left?

6. The stakes in Don's croquet set are 2 feet long. He drove one stake $\frac{3}{4}$ foot into the ground. What was the measure in feet of the part still above ground?

Mixed Fractions and Proper Fractions

Study the steps in each of the following subtractions.

A	B
$2\frac{7}{10}-\frac{3}{10}=n$	$4\frac{1}{6}-\frac{5}{6}=n$
$(2+\frac{7}{10})-\frac{3}{10}=n$	$(4+\frac{1}{6})-\frac{5}{6}=n$
$2+(\frac{7}{10}-\frac{3}{10})=n$	$(3+\frac{7}{6})-\frac{5}{6}=n$
$2+\frac{7-3}{10}=n$	$3+(\frac{7}{6}-\frac{5}{6})=n$
$2+\frac{4}{10}=n$	$3+\frac{7-5}{6}=n$
$2\frac{4}{10}$ or $2\frac{2}{5}=n$	$3+\frac{2}{6}=n$
	$3\frac{2}{6}$ or $3\frac{1}{3}=n$

In A, $2\frac{7}{10}$ means $2+\frac{7}{10}$. The equation is then written $(2+\frac{7}{10})-\frac{3}{10}=n$. Subtract the fractional numbers, add the difference to the whole number, and reduce to lowest terms if possible.

In B, it is impossible to subtract $\frac{5}{6}$ from $\frac{1}{6}$. Think of 4 as $3+1$, or $3+\frac{6}{6}$. Then $4\frac{1}{6}$ can be written $(3+\frac{6}{6})+\frac{1}{6}$ or $3+(\frac{6}{6}+\frac{1}{6})$ or $3+\frac{7}{6}$. Then subtract as you did in subtraction A.

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Subtractions A and B can also be written in abbreviated form as shown below.

$$\begin{array}{r} 2\frac{7}{10} \\ -\frac{3}{10} \\ \hline 2\frac{4}{10}=2\frac{2}{5} \end{array} \qquad\qquad \begin{array}{r} 4\frac{1}{6}=3\frac{7}{6} \\ -\frac{5}{6}=\frac{5}{6} \\ \hline 3\frac{2}{6}=3\frac{1}{3} \end{array}$$

Oral Answer the following questions about subtraction B.

1. Why was $4\frac{1}{6}$ written $4+\frac{1}{6}$?

2. Why was $4+\frac{1}{6}$ written $3+\frac{7}{6}$?

3. What was done in changing $3\frac{2}{6}$ to $3\frac{1}{3}$?

Written Copy. Write each difference in lowest terms.

	a	*b*
1.	$4\frac{4}{7}-\frac{3}{7}=n$	$5\frac{11}{12}-\frac{5}{12}=n$
2.	$3\frac{3}{7}-\frac{3}{7}=n$	$12\frac{2}{15}-\frac{12}{15}=n$
3.	$2\frac{1}{8}-\frac{5}{8}=n$	$5\frac{2}{7}-\frac{4}{7}=n$

Copy. Write each difference in lowest terms.

	a	*b*	*c*	*d*
4.	$\begin{array}{r} 2\frac{3}{4} \\ -\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{3}{11} \\ -\frac{3}{11} \\ \hline \end{array}$	$\begin{array}{r} 14\frac{8}{13} \\ -\frac{5}{13} \\ \hline \end{array}$	$\begin{array}{r} 27\frac{5}{9} \\ -\frac{2}{9} \\ \hline \end{array}$
5.	$\begin{array}{r} 7\frac{2}{5} \\ -\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{1}{4} \\ -\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 27\frac{2}{9} \\ -\frac{5}{9} \\ \hline \end{array}$	$\begin{array}{r} 31\frac{3}{11} \\ -\frac{8}{11} \\ \hline \end{array}$
6.	$\begin{array}{r} 9\frac{19}{20} \\ -\frac{11}{20} \\ \hline \end{array}$	$\begin{array}{r} 6\frac{7}{15} \\ -\frac{10}{15} \\ \hline \end{array}$	$\begin{array}{r} 52\frac{3}{14} \\ -\frac{1}{14} \\ \hline \end{array}$	$\begin{array}{r} 45\frac{5}{18} \\ -\frac{13}{18} \\ \hline \end{array}$
7.	$\begin{array}{r} 6\frac{5}{9} \\ -\frac{8}{9} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{5}{8} \\ -\frac{6}{8} \\ \hline \end{array}$	$\begin{array}{r} 25\frac{4}{10} \\ -\frac{9}{10} \\ \hline \end{array}$	$\begin{array}{r} 41\frac{4}{11} \\ -\frac{8}{11} \\ \hline \end{array}$
8.	$\begin{array}{r} 7\frac{3}{7} \\ -\frac{6}{7} \\ \hline \end{array}$	$\begin{array}{r} 1\frac{4}{15} \\ -\frac{11}{15} \\ \hline \end{array}$	$\begin{array}{r} 41\frac{30}{31} \\ -\frac{15}{31} \\ \hline \end{array}$	$\begin{array}{r} 79\frac{3}{17} \\ -\frac{12}{17} \\ \hline \end{array}$

Using Fractions in Solving Problems

Read each problem carefully. Decide which operation to use. Write a suitable equation. Solve the equation and write your answer for the problem.

1. Bob spent $\frac{5}{6}$ hour mowing the lawn and $\frac{1}{6}$ hour sweeping the sidewalks. How much more time did he spend mowing the lawn than sweeping the sidewalks?

2. A measuring cup was filled with paint to the $\frac{3}{8}$ mark. Enough paint thinner was added to bring the level of the mixture to the $\frac{5}{8}$ mark. How much paint thinner was added?

3. At the start of a trip David noticed the gasoline gauge on his father's car read $\frac{7}{8}$ full. Later David noticed it read $\frac{3}{8}$ full. What fractional part of a tank of gasoline had been used?

4. Rita brought $\frac{3}{4}$ dozen cookies and Lois brought $\frac{2}{3}$ dozen cookies to the picnic. How many dozen cookies did the two girls bring?

5. James weighs $62\frac{1}{4}$ pounds and Donald weighs $65\frac{1}{2}$ pounds. What is their combined weight?

6. Mrs. Clark bought 3 dozen eggs and used $\frac{2}{3}$ dozen of them in preparing dinner. How many dozen eggs did she have left?

7. Charles caught two fish last Saturday. Their weights were $2\frac{1}{8}$ pounds and $\frac{7}{8}$ pound. What was the difference in their weights?

8. Read problem 7 again. Find the combined weight of the fish.

9. Mrs. Brown bought 2 dozen apples, $\frac{1}{3}$ dozen lemons, and $\frac{3}{4}$ dozen grapefruit. How many pieces of fruit did she buy? How many dozen?

10. The first 6 months of last year Ronald grew $1\frac{7}{8}$ inches. The next 6 months he grew $1\frac{1}{8}$ inches. How many inches more did he grow during the first 6 months?

A quick review Find each sum or difference by performing the indicated operation.

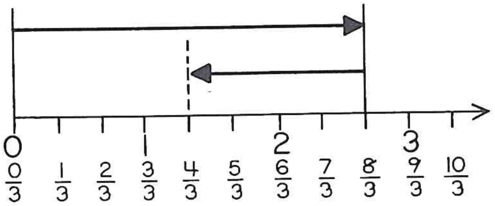
	a	*b*	*c*	*d*
1.	$\begin{array}{r}\frac{2}{3}\\ +\frac{1}{4}\\ \hline\end{array}$	$\begin{array}{r}\frac{3}{7}\\ +\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}3\frac{3}{5}\\ +\frac{4}{5}\\ \hline\end{array}$	$\begin{array}{r}5\frac{3}{8}\\ +1\frac{2}{3}\\ \hline\end{array}$
2.	$\begin{array}{r}\frac{7}{8}\\ -\frac{5}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{7}{12}\\ -\frac{5}{12}\\ \hline\end{array}$	$\begin{array}{r}7\\ -\frac{2}{9}\\ \hline\end{array}$	$\begin{array}{r}4\frac{7}{11}\\ -2\frac{4}{11}\\ \hline\end{array}$
3.	$\begin{array}{r}\frac{4}{7}\\ \frac{2}{7}\\ +\frac{5}{7}\\ \hline\end{array}$	$\begin{array}{r}\frac{5}{8}\\ 3\frac{1}{4}\\ +\frac{1}{2}\\ \hline\end{array}$	$\begin{array}{r}3\frac{2}{5}\\ 1\frac{3}{4}\\ +\frac{7}{10}\\ \hline\end{array}$	$\begin{array}{r}12\frac{1}{2}\\ 3\frac{1}{6}\\ +\frac{2}{3}\\ \hline\end{array}$
4.	$\begin{array}{r}\frac{1}{3}\\ \frac{5}{6}\\ +\frac{7}{9}\\ \hline\end{array}$	$\begin{array}{r}2\frac{7}{9}\\ 4\frac{1}{6}\\ +\frac{11}{18}\\ \hline\end{array}$	$\begin{array}{r}9\frac{2}{3}\\ 5\frac{1}{4}\\ +\frac{3}{5}\\ \hline\end{array}$	$\begin{array}{r}31\frac{7}{8}\\ 7\frac{2}{3}\\ +\frac{3}{4}\\ \hline\end{array}$

Mixed Fractions in Subtraction

The cake recipe that Lois is using calls for $2\frac{2}{3}$ cups of flour. She has only $1\frac{1}{3}$ cup of flour. How much more flour does she need?

You are to find how much greater one number is than another, so you subtract $1\frac{1}{3}$ from $2\frac{2}{3}$. The equation is $2\frac{2}{3}-1\frac{1}{3}=n$.

On the number line below is a solution to this problem.



$$2\tfrac{2}{3}-1\tfrac{1}{3}=\tfrac{8}{3}-\tfrac{4}{3}=\tfrac{8-4}{3}=\tfrac{4}{3}=1+\tfrac{1}{3}=1\tfrac{1}{3}$$

The number line has given the correct solution. You can streamline the process by simply grouping the numerals as shown below.

$$\begin{aligned} 2\tfrac{2}{3}-1\tfrac{1}{3}&=n\\ (2-1)+(\tfrac{2}{3}-\tfrac{1}{3})&=n\\ 1+\tfrac{2-1}{3}&=n\\ 1+\tfrac{1}{3}&=n\\ 1\tfrac{1}{3}&=n \end{aligned}$$

This solution tells you how to complete a subtraction with mixed fractions: Subtract the fractional numbers, subtract the whole numbers, add these results, reduce to lowest terms if possible.

Study the following subtractions.

$$\begin{aligned} 7\tfrac{5}{8}-4&=n & 5\tfrac{7}{8}-3\tfrac{3}{8}&=n\\ (7+\tfrac{5}{8})-4&=n & (5-3)+(\tfrac{7}{8}-\tfrac{3}{8})&=n\\ (7-4)+\tfrac{5}{8}&=n & 2+\tfrac{4}{8}&=n\\ 3+\tfrac{5}{8}&=n & 2+\tfrac{1}{2}&=n\\ 3\tfrac{5}{8}&=n & 2\tfrac{1}{2}&=n \end{aligned}$$

The above solutions are usually written in shorter form as shown below.

$$\begin{array}{r} 7\tfrac{5}{8}\\ -4\\ \hline 3\tfrac{5}{8} \end{array} \qquad \begin{array}{r} 5\tfrac{7}{8}\\ -3\tfrac{3}{8}\\ \hline 2\tfrac{4}{8} \end{array}=2\tfrac{1}{2}$$

The same reasoning and steps are used in the short form of subtraction as are used in the equation form. It is best to give the answers in lowest terms.

Notice how the numbers are renamed in the subtractions below.

A	B
$\begin{array}{r} 6 = 5\frac{3}{3} \\ \underline{-3\frac{1}{3} = 3\frac{1}{3}} \\ 2\frac{2}{3} \end{array}$	$\begin{array}{r} 4\frac{1}{4} = 3\frac{5}{4} \\ \underline{-2\frac{3}{4} = 2\frac{3}{4}} \\ 1\frac{2}{4} = 1\frac{1}{2} \end{array}$

To subtract the fractional numbers in A, name 6 as $5+1$ or $5+\frac{3}{3}$ or $5\frac{3}{3}$. Then subtract fractional numbers and subtract whole numbers.

You cannot subtract $\frac{3}{4}$ from $\frac{1}{4}$ in B, so think of $4\frac{1}{4}$ as $3+1+\frac{1}{4}$ or $3+\frac{4}{4}+\frac{1}{4}$ or $3\frac{5}{4}$. Then you subtract fractional numbers and subtract whole numbers.

Subtraction B is shown below in equation form. The solution appears longer because no short cuts were taken.

$$\begin{aligned} 4\tfrac{1}{4}-2\tfrac{3}{4}&=n \\ 3+(1+\tfrac{1}{4})-(2+\tfrac{3}{4})&=n \\ (3+\tfrac{5}{4})-(2+\tfrac{3}{4})&=n \\ (3-2)+(\tfrac{5}{4}-\tfrac{3}{4})&=n \\ 1+\tfrac{2}{4}&=n \\ 1+\tfrac{1}{2}&=n \\ 1\tfrac{1}{2}&=n \end{aligned}$$

Oral Answer the questions below.

1. Can you subtract the fractional numbers in $5\frac{2}{3}-3\frac{1}{3}$ without changing the form of a whole number?

2. What steps would you use in solving $4-2\frac{3}{11}=n$?

3. What steps would you use in solving $7\frac{1}{5}-2\frac{3}{5}=n$?

4. Is the answer to question 3 more or less than 5? Why?

Written Copy. Write each difference in lowest terms.

	a	*b*
1.	$4\frac{1}{5}-3\frac{1}{5}=n$	$8\frac{6}{7}-4\frac{4}{7}=n$
2.	$9\frac{2}{5}-3\frac{1}{5}=n$	$7\frac{1}{12}-2\frac{5}{12}=n$
3.	$5\frac{2}{5}-2\frac{2}{5}=n$	$7-3\frac{3}{8}=n$

Copy. Find each difference. Give each answer in lowest terms.

	a	*b*	*c*	*d*
4.	$\begin{array}{r} 8\frac{7}{8} \\ \underline{-4\frac{5}{8}} \end{array}$	$\begin{array}{r} 9\frac{3}{16} \\ \underline{-1\frac{1}{16}} \end{array}$	$\begin{array}{r} 6\frac{11}{12} \\ \underline{-3\frac{7}{12}} \end{array}$	$\begin{array}{r} 8\frac{2}{3} \\ \underline{-2} \end{array}$
5.	$\begin{array}{r} 16\frac{9}{16} \\ \underline{-8\frac{5}{16}} \end{array}$	$\begin{array}{r} 12\frac{4}{15} \\ \underline{-8\frac{1}{15}} \end{array}$	$\begin{array}{r} 14 \\ \underline{-6\frac{3}{8}} \end{array}$	$\begin{array}{r} 21\frac{13}{15} \\ \underline{-12\frac{3}{15}} \end{array}$
6.	$\begin{array}{r} 6\frac{2}{7} \\ \underline{-5\frac{4}{7}} \end{array}$	$\begin{array}{r} 3\frac{2}{9} \\ \underline{-2\frac{5}{9}} \end{array}$	$\begin{array}{r} 9\frac{3}{8} \\ \underline{-4\frac{7}{8}} \end{array}$	$\begin{array}{r} 11\frac{1}{5} \\ \underline{-6\frac{3}{5}} \end{array}$
7.	$\begin{array}{r} 8\frac{2}{7} \\ \underline{-4\frac{6}{7}} \end{array}$	$\begin{array}{r} 9\frac{1}{5} \\ \underline{-8\frac{3}{5}} \end{array}$	$\begin{array}{r} 15\frac{4}{11} \\ \underline{-6\frac{5}{11}} \end{array}$	$\begin{array}{r} 32\frac{3}{10} \\ \underline{-27\frac{7}{10}} \end{array}$
8.	$\begin{array}{r} 9\frac{2}{3} \\ \underline{-7\frac{2}{3}} \end{array}$	$\begin{array}{r} 5\frac{7}{16} \\ \underline{-5\frac{3}{16}} \end{array}$	$\begin{array}{r} 7\frac{2}{9} \\ \underline{-\frac{5}{9}} \end{array}$	$\begin{array}{r} 36\frac{5}{7} \\ \underline{-28\frac{2}{7}} \end{array}$
9.	$\begin{array}{r} 17\frac{4}{9} \\ \underline{-8\frac{1}{9}} \end{array}$	$\begin{array}{r} 5\frac{1}{6} \\ \underline{-3\frac{5}{6}} \end{array}$	$\begin{array}{r} 52\frac{3}{4} \\ \underline{-46\frac{1}{4}} \end{array}$	$\begin{array}{r} 86\frac{2}{6} \\ \underline{-57\frac{5}{6}} \end{array}$

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Subtracting Fractional Numbers

Jane lives $\frac{5}{6}$ mile from school and Sally lives $\frac{2}{3}$ mile from school. How much farther from school does Jane live than Sally?

To find the difference between $\frac{5}{6}$ and $\frac{2}{3}$ solve the equation $\frac{5}{6}-\frac{2}{3}=n$. Since the denominators are not equal, change one, or both, of the fractions so that you get fractions with a common denominator.

Since 6 is divisible by 3, 6 is the smallest common denominator. The equation is shown below.

$$\frac{5}{6}-\frac{2}{3}=\frac{5}{6}-\frac{4}{6}=\frac{5-4}{6}=\frac{1}{6}=n$$

Jane lives $\frac{1}{6}$ mile farther from school than Sally.

In the equation $\frac{3}{5}-\frac{1}{4}=n$, the denominators are not equal, so rewrite the fractions so they have a common denominator. Since 20 is the smallest number divisible by both 4 and 5, write both fractions as twentieths.

Study the following completed subtraction in equation form and also in column form.

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$$\frac{3}{5}-\frac{1}{4}=n \qquad \frac{12}{20}-\frac{5}{20}=n \qquad \frac{7}{20}=n$$

$$\begin{array}{r} \frac{3}{5}=\frac{12}{20} \\ -\frac{1}{4}=\frac{5}{20} \\ \hline \frac{7}{20} \end{array}$$

Oral What is the smallest common denominator for each of these subtractions?

	a	*b*	*c*
1.	$\frac{5}{8}-\frac{1}{4}=n$	$\frac{7}{12}-\frac{1}{3}=n$	$\frac{3}{6}-\frac{1}{5}=n$
2.	$\begin{array}{r}\frac{7}{8}\\-\frac{2}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{7}\\-\frac{1}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{9}{10}\\-\frac{2}{3}\\\hline\end{array}$

3. Explain each step in the following subtraction.

$$\frac{11}{12}-\frac{2}{8}=\frac{22}{24}-\frac{6}{24}=\frac{22-6}{24}=\frac{16}{24}=\frac{2}{3}$$

Written Copy. Write each difference in lowest terms.

	a	*b*	*c*
1.	$\frac{7}{8}-\frac{3}{4}=n$	$\frac{3}{5}-\frac{3}{10}=n$	$\frac{5}{8}-\frac{1}{2}=n$
2.	$\frac{2}{3}-\frac{1}{6}=n$	$\frac{5}{12}-\frac{1}{4}=n$	$\frac{3}{4}-\frac{5}{8}=n$
3.	$\frac{2}{3}-\frac{1}{9}=n$	$\frac{9}{10}-\frac{3}{5}=n$	$\frac{1}{3}-\frac{1}{6}=n$
4.	$\frac{3}{5}-\frac{1}{2}=n$	$\frac{3}{4}-\frac{1}{6}=n$	$\frac{2}{3}-\frac{4}{9}=n$
5.	$\frac{4}{5}-\frac{1}{4}=n$	$\frac{5}{7}-\frac{1}{4}=n$	$\frac{7}{9}-\frac{2}{5}=n$

Copy. Write each difference in lowest terms.

	a	*b*	*c*	*d*
6.	$\begin{array}{r}\frac{7}{8}\\-\frac{7}{16}\\\hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\-\frac{1}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\-\frac{1}{9}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{15}\\-\frac{2}{5}\\\hline\end{array}$
7.	$\begin{array}{r}\frac{7}{10}\\-\frac{2}{15}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{7}\\-\frac{3}{14}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\-\frac{5}{12}\\\hline\end{array}$	$\begin{array}{r}\frac{4}{5}\\-\frac{3}{7}\\\hline\end{array}$

Mixed Fractions in Subtraction

It is often necessary to subtract a fractional number from a number greater than 1. Examples of such subtractions are $9\frac{1}{2}-\frac{1}{4}=n$ and $4\frac{1}{5}-\frac{7}{10}=n$.

In each of the above equations, the denominators are not equal. Rename the numbers so that they have a common denominator and subtract as you did when there was a common denominator.

Study the following subtractions.

$$\begin{array}{rr}
9\frac{1}{2}-\frac{1}{4}=n & 4\frac{1}{5}-\frac{7}{10}=n \\
(9+\frac{1}{2})-\frac{1}{4}=n & 3+(1+\frac{1}{5})-\frac{7}{10}=n \\
9+(\frac{1}{2}-\frac{1}{4})=n & 3+(\frac{5}{5}+\frac{1}{5})-\frac{7}{10}=n \\
9+(\frac{2}{4}-\frac{1}{4})=n & 3+(\frac{6}{5}-\frac{7}{10})=n \\
9+\frac{2-1}{4}=n & 3+(\frac{12}{10}-\frac{7}{10})=n \\
9+\frac{1}{4}=n & 3+\frac{12-7}{10}=n \\
9\frac{1}{4}=n & 3+\frac{5}{10}=n \\
 & 3+\frac{1}{2}\text{ or }3\frac{1}{2}=n
\end{array}$$

The above subtractions can also be shown in column form.

$$\begin{array}{rcr}
9\frac{1}{2}=9\frac{2}{4} & \qquad & 4\frac{1}{5}=4\frac{2}{10}=3\frac{12}{10} \\
\underline{-\frac{1}{4}}=\underline{\frac{1}{4}} & & \underline{-\frac{7}{10}}=\underline{\frac{7}{10}}=\underline{\frac{7}{10}} \\
9\frac{1}{4} & & 3\frac{5}{10}=3\frac{1}{2}
\end{array}$$

Can you explain the steps used in the subtractions shown above?

Oral Explain the steps you would use in finding the difference in each of the following subtractions.

	a	*b*	*c*
1.	$3\frac{1}{2}-\frac{1}{3}=n$	$4\frac{5}{8}-\frac{5}{8}=n$	$2\frac{1}{3}-\frac{3}{4}=n$
2.	$\begin{array}{r}9\frac{3}{5}\\ \underline{-\frac{1}{2}}\end{array}$	$\begin{array}{r}12\frac{1}{4}\\ \underline{-\frac{2}{3}}\end{array}$	$\begin{array}{r}25\frac{3}{7}\\ \underline{-\frac{1}{3}}\end{array}$

Written Copy. Write each difference in lowest terms.

	a	*b*	*c*
1.	$1\frac{2}{3}-\frac{5}{6}=n$	$8\frac{5}{6}-\frac{1}{12}=n$	$3\frac{1}{2}-\frac{1}{8}=n$
2.	$9\frac{1}{6}-\frac{2}{3}=n$	$6\frac{3}{4}-\frac{1}{6}=n$	$11\frac{7}{16}-\frac{3}{4}=n$
3.	$5\frac{2}{7}-\frac{1}{2}=n$	$4\frac{3}{7}-\frac{2}{5}=n$	$12\frac{5}{6}-\frac{2}{3}=n$
4.	$7\frac{2}{5}-\frac{1}{8}=n$	$3\frac{2}{10}-\frac{7}{15}=n$	$19\frac{3}{7}-\frac{1}{4}=n$

Copy. Write each difference in lowest terms.

	a	*b*	*c*	*d*
5.	$\begin{array}{r}6\frac{2}{3}\\ \underline{-\frac{5}{9}}\end{array}$	$\begin{array}{r}4\frac{5}{8}\\ \underline{-\frac{1}{4}}\end{array}$	$\begin{array}{r}3\frac{3}{4}\\ \underline{-\frac{3}{8}}\end{array}$	$\begin{array}{r}6\frac{7}{8}\\ \underline{-\frac{3}{4}}\end{array}$
6.	$\begin{array}{r}8\frac{1}{5}\\ \underline{-\frac{3}{10}}\end{array}$	$\begin{array}{r}7\frac{3}{10}\\ \underline{-\frac{4}{5}}\end{array}$	$\begin{array}{r}1\frac{5}{18}\\ \underline{-\frac{1}{6}}\end{array}$	$\begin{array}{r}3\frac{2}{5}\\ \underline{-\frac{3}{4}}\end{array}$
7.	$\begin{array}{r}7\frac{3}{7}\\ \underline{-\frac{1}{2}}\end{array}$	$\begin{array}{r}6\frac{3}{4}\\ \underline{-\frac{11}{12}}\end{array}$	$\begin{array}{r}5\frac{1}{6}\\ \underline{-\frac{2}{3}}\end{array}$	$\begin{array}{r}17\frac{3}{5}\\ \underline{-\frac{2}{6}}\end{array}$
8.	$\begin{array}{r}3\frac{1}{2}\\ \underline{-\frac{7}{10}}\end{array}$	$\begin{array}{r}2\frac{1}{4}\\ \underline{-\frac{5}{12}}\end{array}$	$\begin{array}{r}21\frac{2}{3}\\ \underline{-\frac{4}{5}}\end{array}$	$\begin{array}{r}31\frac{5}{9}\\ \underline{-\frac{1}{4}}\end{array}$

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More Subtraction with Mixed Fractions

Judy caught a fish that weighed $3\frac{1}{4}$ pounds. Her brother caught one that weighed $1\frac{1}{2}$ pounds. How much heavier was Judy's fish than her brother's?

You can find the answer by solving the equation $3\frac{1}{4}-1\frac{1}{2}=n$. Both of the mixed fractions have fractional parts with denominators which are not equal. Rename one, or both, of the fractional numbers so that they have a common denominator.

You can proceed as follows.

$$\begin{aligned}3\tfrac{1}{4}-1\tfrac{1}{2}&=n\\2+(1+\tfrac{1}{4})-(1+\tfrac{1}{2})&=n\\(2+\tfrac{5}{4})-(1+\tfrac{1}{2})&=n\\(2-1)+(\tfrac{5}{4}-\tfrac{1}{2})&=n\\1+(\tfrac{5}{4}-\tfrac{2}{4})&=n\\1+\tfrac{5-2}{4}&=n\\1+\tfrac{3}{4}\text{ or }1\tfrac{3}{4}&=n\end{aligned}$$

$$\begin{array}{r}3\frac{1}{4}=2\frac{5}{4}\\ \underline{-1\frac{1}{2}}=\underline{1\frac{2}{4}}\\ 1\frac{3}{4}\end{array}$$

Judy's fish weighed $1\frac{3}{4}$ pounds more than the one her brother caught.

In any subtraction of this kind:

a. Rename the minuend as the sum of a whole number and an improper fraction.

b. Rename one or both of the fractions so that they will have a common denominator.

c. Subtract as you did before.

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Oral Answer the following questions about the subtractions which are given below.

a	*b*	*c*	*d*
$7\frac{1}{4}$ $-4\frac{3}{4}$	$6\frac{7}{12}$ $-3\frac{1}{6}$	$9\frac{1}{2}$ $-7\frac{2}{3}$	$14\frac{7}{10}$ $-9\frac{4}{15}$

1. In which subtractions must you rename a whole number as a fraction so that subtraction is possible?

2. In which subtractions must you rename one or both fractions so they will have a common denominator?

3. In which subtractions must you do both?

Written Copy. Write each difference in lowest terms.

	a	*b*	*c*	*d*
1.	$3\frac{1}{6}$ $-1\frac{1}{3}$	$7\frac{2}{3}$ $-2\frac{5}{6}$	$9\frac{1}{8}$ $-6\frac{3}{4}$	$6\frac{5}{12}$ $-1\frac{1}{3}$
2.	$9\frac{4}{9}$ $-8\frac{2}{3}$	$4\frac{1}{6}$ $-3\frac{11}{12}$	$7\frac{4}{15}$ $-5\frac{3}{5}$	$6\frac{1}{2}$ $-4\frac{7}{8}$
3.	$9\frac{7}{10}$ $-3\frac{3}{5}$	$5\frac{6}{7}$ $-2\frac{1}{3}$	$4\frac{5}{9}$ $-2\frac{1}{2}$	$14\frac{5}{8}$ $-3\frac{2}{5}$
4.	$11\frac{5}{24}$ $-6\frac{1}{6}$	$9\frac{7}{18}$ $-4\frac{1}{3}$	$12\frac{5}{6}$ $-7\frac{2}{7}$	$27\frac{3}{8}$ $-9\frac{1}{6}$
5.	$10\frac{1}{2}$ $-7\frac{2}{9}$	$8\frac{7}{15}$ $-3\frac{1}{5}$	$14\frac{4}{5}$ $-7\frac{1}{6}$	$42\frac{1}{8}$ $-26\frac{3}{5}$

Review and Practice

Part 1 Copy. Find each sum, difference, or product by performing the indicated operation.

	a	*b*	*c*
1.	3567 755 4009 +2076	4289 7629 745 +3000	6285 9988 8877 +6594
2.	5001 −4988	8057 −928	3654 −2095
3.	56 ×73	786 ×45	986 ×98
4.	7642 ×5	5684 ×86	2996 ×243

Part 2 Copy. Find each quotient and each remainder.

	a	*b*	*c*
1.	$8\overline{)496}$	$3\overline{)902}$	$7\overline{)5892}$
2.	$21\overline{)790}$	$86\overline{)536}$	$74\overline{)25761}$
3.	$32\overline{)3108}$	$45\overline{)3962}$	$37\overline{)86214}$

Part 3 Solve the equations below.

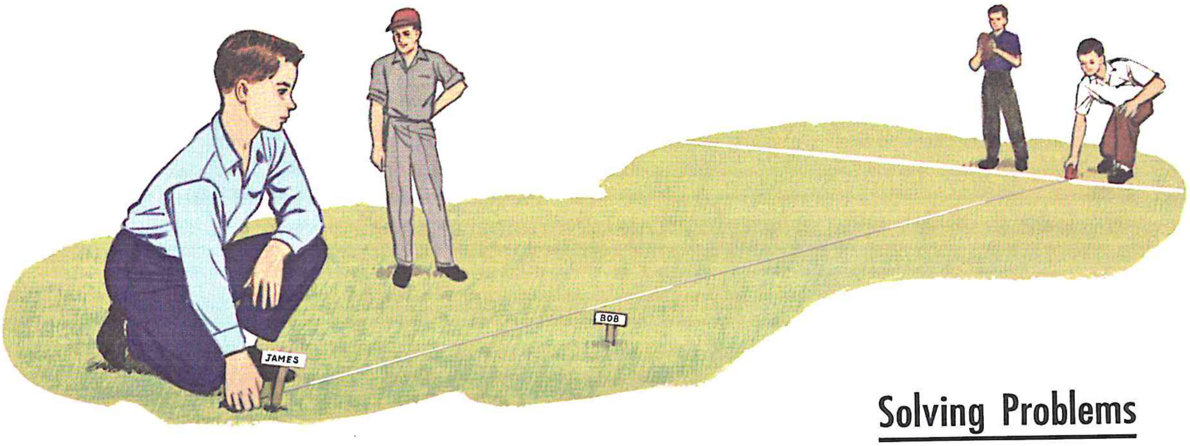
	a	*b*
1.	$(42 \div 6) \times 8 = n$	$(5 \times 7) + 9 = n$
2.	$n - (24 \div 3) = 20$	$(40 \div 8) + n = 13$
3.	$(56 - 36) \div n = 5$	$n \times (7 - 4) = 18$

Part 4 Copy. Find each sum or difference by performing the indicated operation. Give each answer in lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{7}{12}$ $+\frac{2}{12}$	$\frac{9}{15}$ $+\frac{1}{15}$	$\frac{1}{4}$ $+\frac{1}{10}$	$\frac{7}{15}$ $+\frac{7}{10}$
2.	$5\frac{1}{4}$ $+\frac{1}{6}$	$\frac{1}{6}$ $+3\frac{1}{9}$	$3\frac{5}{12}$ $+2\frac{3}{4}$	$2\frac{1}{4}$ $+\frac{7}{9}$
3.	$\frac{9}{10}$ $-\frac{3}{10}$	$5\frac{3}{7}$ -5	$7\frac{2}{9}$ $-\frac{7}{9}$	$4\frac{2}{15}$ $-2\frac{1}{10}$
4.	$6\frac{11}{12}$ $-5\frac{3}{4}$	$7\frac{2}{5}$ $-1\frac{7}{15}$	$4\frac{5}{8}$ $-2\frac{1}{3}$	$8\frac{3}{4}$ $-5\frac{1}{8}$
5.	$2\frac{1}{8}$ $5\frac{1}{3}$ $+\frac{3}{4}$	$6\frac{1}{2}$ $1\frac{1}{4}$ $+2\frac{2}{3}$	$8\frac{1}{4}$ $3\frac{3}{5}$ $+7\frac{5}{6}$	$11\frac{1}{7}$ $3\frac{1}{2}$ $+5\frac{9}{14}$
6.	$3\frac{5}{8}$ $4\frac{1}{2}$ $+2\frac{2}{3}$	$7\frac{2}{15}$ $1\frac{1}{3}$ $+6\frac{2}{5}$	$15\frac{1}{2}$ $4\frac{1}{4}$ $+2\frac{2}{5}$	$5\frac{1}{10}$ $7\frac{4}{5}$ $+3\frac{4}{15}$

Can you do this? Find a replacement for n so that each of the following statements becomes true.

	a	*b*
1.	$(\frac{3}{5}+\frac{1}{5})-\frac{2}{5}=n$	$(\frac{1}{2}-\frac{1}{4})+\frac{3}{4}=n$
2.	$(\frac{2}{3}+\frac{3}{4})-\frac{1}{2}=n$	$\frac{7}{8}-(\frac{1}{4}+\frac{1}{2})=n$
3.	$(\frac{1}{6}+\frac{1}{4})+n=1$	$(2\frac{1}{2}+\frac{2}{3})-n=2\frac{2}{3}$



Solving Problems

Read each problem carefully. Decide which operation to use. Write a suitable equation. Solve the equation and write your answer to the problem in lowest terms.

1. James threw the football $54\frac{1}{3}$ feet and Bob threw it $47\frac{3}{4}$ feet. How many feet farther did James throw the football than Bob?

2. The Girl Scouts went on a Saturday hike. They walked $3\frac{1}{4}$ miles to a woodland park. They took a different route home, walking $2\frac{5}{8}$ miles. How much longer was the trip to the park than the trip back?

3. Read problem 2 again. What was the total distance that the girls walked on the hike?

4. June is $4\frac{1}{4}$ feet tall and her younger brother is $3\frac{5}{6}$ feet tall. June is what fractional part of a foot taller than her brother?

5. Mr. Becker bought 4 gallons of paint and used $2\frac{7}{8}$ gallons of it on his garage. How much paint did he have left?

6. Dick mowed $\frac{1}{4}$ of the lawn, his father mowed $\frac{1}{3}$ of it, and his brother mowed $\frac{1}{5}$ of it. What fractional part of the lawn did they mow?

7. Mrs. Barber bought $3\frac{1}{2}$ pounds of ground beef, $4\frac{1}{4}$ pounds of steak, and a $5\frac{1}{8}$-pound roast. How many pounds of meat did she buy?

8. The normal temperature of the human body is $98\frac{6}{10}^\circ$ F. Normal room temperature should be about 68° F. What is the difference between these two temperature readings?

9. Robert saved 4 dollars from his allowance and earned $2\frac{1}{2}$ dollars. Then he spent $3\frac{2}{5}$ dollars. How many dollars did he have left?

10. Mary worked $\frac{3}{5}$ of her arithmetic assignment during study period and $\frac{1}{6}$ of it after school. What fractional part of the assignment remains to be worked?

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. The arithmetic of fractional numbers should be in full agreement with the arithmetic of whole numbers. (219)

2. When subtracting fractional numbers having the same denominator, subtract the numbers named by the numerators and write the result over the common denominator. (220)

3. To subtract a fractional number from a whole number, name the minuend by a mixed fraction. (221)

4. To subtract with mixed fractions: Subtract the fractional numbers, subtract the whole numbers, add the results. (224)

5. When fractions or mixed fractions with denominators which are not equal are used in a subtraction, rename as many of the numbers as necessary so that they have a common denominator before doing the subtraction. (228)

Questions to Discuss

1. How would you rename 4 in the subtraction, $4-\frac{5}{8}=n$? (221)

2. Explain how to solve the equation, $28\frac{1}{4}-\frac{3}{4}=n$. (222)

3. How do you find the smallest common denominator for the fractions $\frac{2}{5}$ and $\frac{3}{4}$? (226)

4. Explain the steps used to solve $7\frac{1}{5}-3\frac{3}{4}=n$. (228)

Written Practice

Copy. Find each difference.

	a	*b*	
1.	$\frac{7}{9}-\frac{4}{9}=n$	$\frac{17}{24}-\frac{9}{24}=n$	(220)
2.	$1-\frac{3}{5}=n$	$5-\frac{2}{11}=n$	(221)
3.	$7\frac{8}{9}-\frac{5}{9}=n$	$36\frac{1}{12}-\frac{7}{12}=n$	(222)
4.	$4\frac{5}{7}-2\frac{3}{7}=n$	$11\frac{5}{12}-7\frac{7}{12}=n$	(225)
5.	$\frac{4}{5}-\frac{3}{10}=n$	$\frac{11}{18}-\frac{1}{2}=n$	(226)
6.	$3\frac{1}{2}-\frac{2}{5}=n$	$17\frac{2}{5}-\frac{3}{4}=n$	(227)
7.	$9\frac{3}{5}-5\frac{1}{4}=n$	$12\frac{1}{5}-8\frac{5}{6}=n$	(228)

Copy. Find each difference.

	a	*b*	*c*	
8.	$\begin{array}{r}\frac{5}{8}\\ -\frac{3}{8}\\ \hline\end{array}$	$\begin{array}{r}\frac{15}{21}\\ -\frac{2}{21}\\ \hline\end{array}$	$\begin{array}{r}\frac{7}{9}\\ -\frac{1}{9}\\ \hline\end{array}$	(220)
9.	$\begin{array}{r}5\frac{7}{8}\\ -\frac{3}{8}\\ \hline\end{array}$	$\begin{array}{r}7\frac{1}{6}\\ -4\frac{5}{6}\\ \hline\end{array}$	$\begin{array}{r}41\frac{5}{11}\\ -35\frac{7}{11}\\ \hline\end{array}$	(222)
10.	$\begin{array}{r}6\frac{7}{16}\\ -3\frac{5}{16}\\ \hline\end{array}$	$\begin{array}{r}18\frac{1}{10}\\ -7\frac{7}{10}\\ \hline\end{array}$	$\begin{array}{r}19\frac{25}{32}\\ -9\frac{17}{32}\\ \hline\end{array}$	(225)

Self-Evaluation

Part 1 Copy. Find each sum or difference by performing the indicated operation. Give all answers in lowest terms.

	a	*b*
1.	$\frac{5}{8}-\frac{2}{8}=n$	$3\frac{7}{8}-\frac{3}{8}=n$
2.	$\frac{5}{6}+\frac{2}{6}=n$	$7\frac{1}{4}-\frac{3}{4}=n$
3.	$8-4\frac{5}{9}=n$	$6\frac{3}{8}-\frac{1}{4}=n$
4.	$\frac{3}{4}+\frac{2}{5}=n$	$8\frac{1}{4}+6\frac{3}{5}=n$
5.	$7\frac{1}{5}-\frac{2}{3}=n$	$6\frac{1}{4}-5\frac{7}{8}=n$
6.	$3\frac{1}{2}-2\frac{1}{2}=n$	$2\frac{7}{8}-1\frac{5}{6}=n$
7.	$3\frac{6}{8}-3\frac{3}{4}=n$	$9\frac{7}{8}+5\frac{1}{3}=n$

Part 2 Copy. Find each sum or difference by performing the indicated operation. Give all answers in lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{3}{7}$ $+\frac{2}{7}$	$\frac{8}{9}$ $-\frac{5}{9}$	$7\frac{5}{7}$ $-4\frac{2}{7}$	$15\frac{7}{8}$ $-8\frac{3}{8}$
2.	$\frac{5}{12}$ $\frac{1}{12}$ $+\frac{7}{12}$	$\frac{2}{3}$ $\frac{1}{4}$ $+\frac{1}{6}$	$26\frac{2}{3}$ $\frac{5}{8}$ $+3\frac{1}{4}$	17 $5\frac{1}{2}$ $+8\frac{3}{7}$
3.	8 $-\frac{2}{5}$	$7\frac{7}{7}$ $-\frac{4}{7}$	$14\frac{5}{6}$ $-5\frac{3}{4}$	$31\frac{2}{5}$ $-17\frac{5}{6}$
4.	$9\frac{3}{11}$ -5	$3\frac{4}{5}$ $+1\frac{5}{7}$	$17\frac{1}{6}$ $-11\frac{2}{9}$	$47\frac{3}{4}$ $+52\frac{5}{9}$

Part 3 Solve each of the following problems.

1. In the town where Betty lives, 16 blocks are equivalent to 1 mile. Betty lives 10 blocks from school. What fractional part of a mile does she live from school?

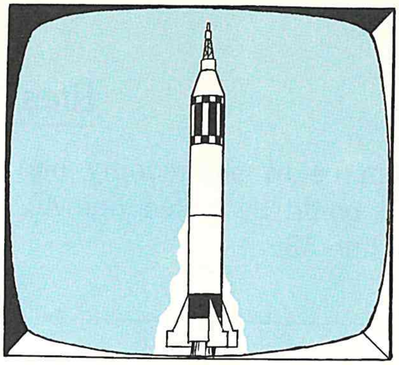
2. David ran the 50-yard dash in $9\frac{3}{10}$ seconds, and Kenneth ran it in $7\frac{3}{5}$ seconds. How much less time did Kenneth take to run the dash than David?

3. Mrs. Bower's flower garden has the shape of a triangle. The measures of the 3 sides are $20\frac{1}{2}$ feet, $11\frac{3}{4}$ feet, and 15 feet. How many feet of fence are needed to enclose the flower garden?

4. Ray had 50 feet of rope. He used $34\frac{1}{2}$ feet of the rope to make a swing. How many feet of rope were not used for the swing?

5. Marilyn poured $2\frac{7}{8}$ cups of water into a pan and heated it. After boiling the water for a while, it measured $2\frac{1}{4}$ cups. What fractional part of a cup of water was changed into steam?

6. A $1\frac{3}{16}$-pound slice was cut from an $8\frac{3}{4}$-pound ham. What was the weight of the ham after the slice was removed?



Meaning of Geometry

You are living in the space age. Imagine that you are planning a trip to the moon in a space ship. To reach the moon, you must know its direction and location in space.

Space and its properties, such as location, are studied in a part of mathematics called **geometry.**

A location on the earth could be marked by a stake in the ground. A location on the floor could be marked by a paint spot on the floor. A location in the atmosphere is not so easily marked.

To aid you in thinking about locations, you can draw pictures to represent them.

Dots on your paper or on the chalkboard mark locations. A large dot covers many, many locations. The smaller the dot, the more accurately it marks the location.

The *exact location* that a dot marks is called a **point.**

There are points (locations) on the earth, in the earth, in the sky, and everywhere in space. All of these points are included in what we call space. *Space is the set of all points* or exact locations.

An exact location has no dimension, so a point has no size. A point is part of our thinking. A point is very important in the study of geometry.

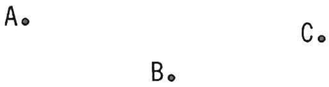
Oral Answer the questions below.

1. The inside of a bottle contains more points than anyone can count. If the bottle is moved, does the inside of the bottle contain the same points as it did before?

2. Two children stretch a piece of string tightly between them. Does the string go through 1 point, through 2 points, or through more points than anyone can count?

Points are usually named or labeled with capital letters of the alphabet, such as A, B, and C.

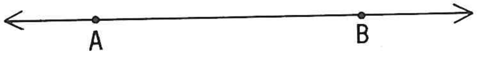
The dots below mark a set of three points. You can refer to them as point A, point B, and point C.



Mark 2 points on a piece of paper. Label them A and B. Fold the paper so that the crease passes through points A and B. Lay the paper flat and trace along the crease.

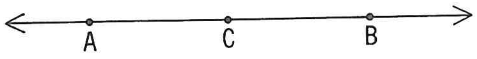
If the sheet of paper had been larger, the length of the crease would have been longer. Imagine that the crease goes on and on without end in both of its directions. This is one way of thinking about what is called the **line** AB.

You cannot draw a picture of the whole line AB because it never ends, so picture it as shown below.



A and B mark two points on the line and the arrows indicate that the line extends indefinitely.

You can name a line by telling two points of the line. For example, the diagram below shows only one line, but it could be called line AC, line CB, or line AB.



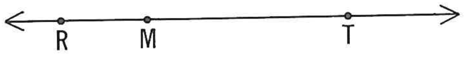
You can name the two points in either order, so the line above could also be called line CA, line BC, or line BA.

Oral Answer the questions below.

1. Does a line have any end?

2. Can a line be measured?

3. Below is a picture of line RT. Give three names for the line.



Written Do the following exercises and answer each question.

1. Mark points C and D. Draw line CD. Do other lines pass through both of these points? How many?

2. Mark point K. Draw 3 lines through point K. Could another line pass through K? How many lines may pass through one point?

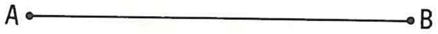
3. Mark points A, B, and C. Draw all the lines that pass through any 2 of these points. How many are there? There can be 2 answers.

Line Segments

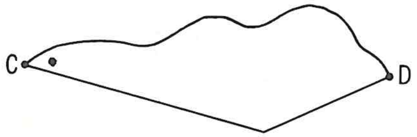
Many of the important geometric figures contain only parts of lines rather than whole lines.

Mark points A and B on a sheet of paper. Fold the paper so that the crease passes through A and B. Lay the paper flat and trace along the crease from A to B.

Your drawing should be similar to that shown below and is a way of illustrating what is called the **line segment** AB.

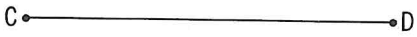


Mark points C and D on a sheet of paper. There are many ways of going from C to D. Two ways are shown below.



Each path passes through many points; therefore each path is a set of points.

The shortest possible path between the points C and D is that part of the line CD which includes points C and D and all the points between them. This is a way of thinking about a line segment.



Line segment CD can also be called line segment DC.

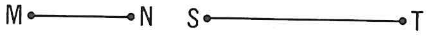
Oral Answer the questions below.

1. Could you mark another point on line segment AB shown on this page? Could you mark all the points of line segment AB? Why not?

2. How many endpoints does a line segment have? Does a line have?

3. Could you measure the length of a line segment?

4. Name all the line segments you see pictured below.

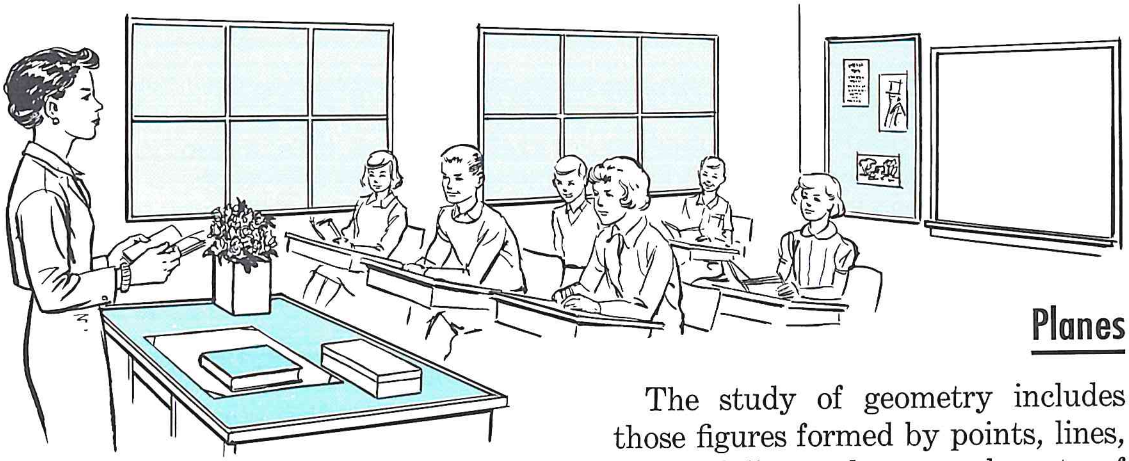


Written Do the following exercises and answer each question.

1. Mark points A and B. Draw line segment AB. Could you draw more line segments having A and B as endpoints? How many?

2. Mark points A, B, and C so that they are not on the same line. Draw all the line segments having 2 of these points as endpoints. How many are there?

3. Mark points M and N. Draw line segment MN. Now draw line MN. Can every line segment be extended to form a line?



Planes

The study of geometry includes those figures formed by points, lines, parts of lines, planes, and parts of planes.

In the picture above there are several objects which have a flat surface. The wall, the chalkboard, a window pane, the cover of a book, and the table top all suggest flat smooth surfaces.

Imagine that you could make the top of your desk larger and larger forever. Think of a desk top so large that you could never reach any of its edges. This kind of surface is an example of a **plane.**

In geometry, think of a plane as a set of points such as that suggested by the enlarged desk top.

Each object mentioned in the first paragraph represents only *part of a plane* because each object has a boundary.

A sheet of paper lying flat on your desk also represents only part of a plane. Think of a line drawn on the paper as a line *in* that plane.

Oral Answer the questions below.

1. What are 3 other objects in the picture above which represent parts of planes?

2. Does space contain more than one plane? (Hint: Your room has 4 walls.)

Written Do the following exercises and write each answer.

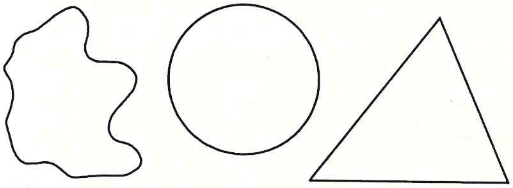
1. Draw a picture of a line on your paper. Can you draw more than one line on the paper? Does a plane contain 1 line, 2 lines, or more lines than you can count?

2. Mark points A and B on your paper. Draw line AB. Points A and B are in the plane (of your paper). Is every point of line AB in the plane of your paper? If 2 points of a line are in a plane, is the line in the plane? Is this true for a line segment?

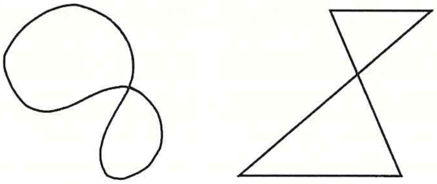
Simple Closed Figures

Each figure shown below has all of its points in a plane. Each of these figures is called a **simple closed figure.** These figures are also called *plane figures.*

You can start at any point on a simple closed figure and trace around the complete figure without having the figure intersect or cross itself.



Below are drawings which do not represent simple closed figures. Why not?



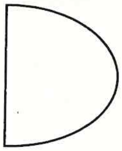
Only special kinds of simple closed figures will be discussed in this chapter.

Many of the objects you use everyday represent simple closed figures. If you trace around a coin, a book, an ink blot, or a piece of a jigsaw puzzle, you draw a picture of a simple closed figure.

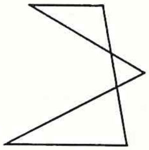
Oral Answer the questions below.

1. Which of the following figures are simple closed figures?

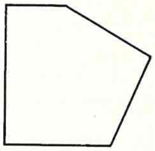
a



b

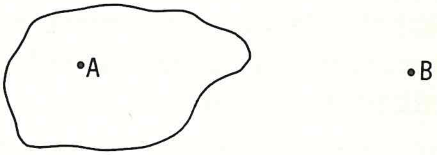


c



2. Do the boundaries of most states on a map of the United States represent a simple closed figure?

3. Is the point A inside or outside of the simple closed figure below? Is point B inside or outside?

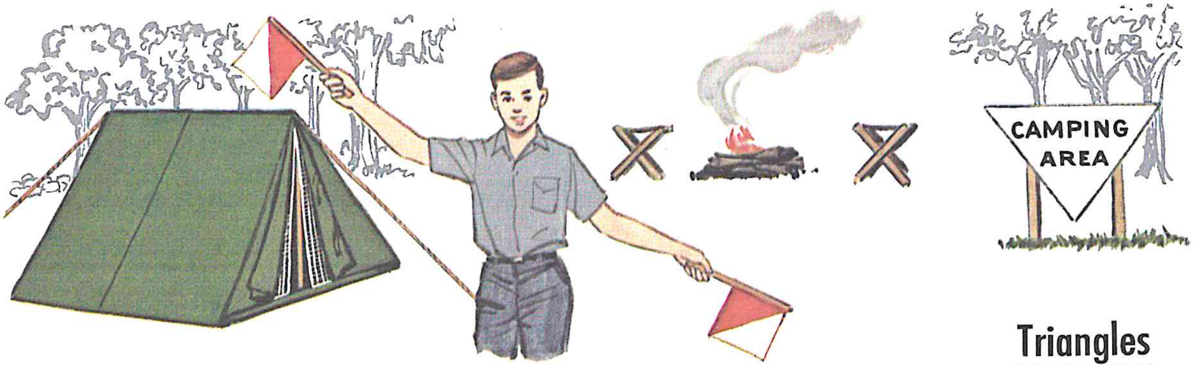


Written Do the following exercises and write each answer.

1. Draw a simple closed figure. Mark point A inside the figure, and mark point B outside the figure. Draw line segment AB. Does line segment AB cross the figure?

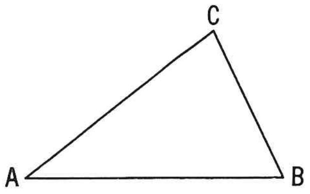
2. Draw a simple closed figure by using 3 line segments.

3. Draw a simple closed figure using 4 line segments.



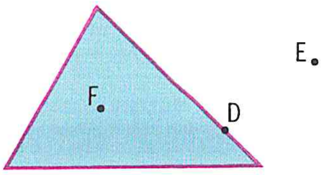
Triangles

The figure below is a simple closed figure formed by 3 line segments. It is called a **triangle.**



Line segments AB, BC, and AC are called the sides of the triangle. How many sides does a triangle have? The prefix *tri* means *three.*

Point F is inside the triangle below. Point D is a point on the triangle, and point E is outside the triangle.



In the figure above, the inside of the triangle is shown in blue and the outside of the triangle is left white. The triangle means *only* the set of points pictured in red.

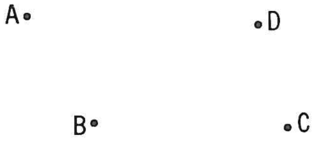
Oral Answer the questions below.

1. What are 4 objects shown in the picture at the top of the page that have the shape of a triangle?

2. What are 3 objects in your classroom that have the shape of a triangle?

Written Do the following exercises and write each answer.

1. Mark 4 points on your paper so that they appear similar to those below. Name them as shown.

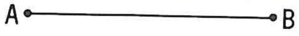


Then draw line segments AB, CD, AC, and BD. How many triangles are drawn in the resulting figure?

2. Draw a triangle. Draw a line segment which has one endpoint inside the triangle and the other endpoint outside. Does the line segment cross the triangle? Do you think this would always happen?

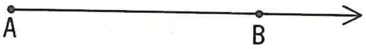
Rays

Fold a piece of paper. Lay it flat and mark points A and B on the crease. Use A and B as endpoints and draw line segment AB. Your drawing could look like the one below.



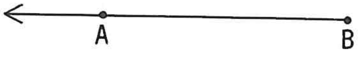
Now extend the line segment beyond B, leaving A as an endpoint. If your paper were larger, you could have drawn farther beyond B. Imagine that you could draw on and on forever along the crease beyond B.

This suggests our idea of **ray** AB. You cannot draw all of ray AB because it starts at A, goes through B, and on and on without end. Below is a way of showing ray AB. What does the arrow mean?



Point A is called the *endpoint* of ray AB. The letter naming the endpoint is always given first.

What is meant by ray BA? The letter B is given first, so it marks the endpoint of the ray. A picture of ray BA is given below.



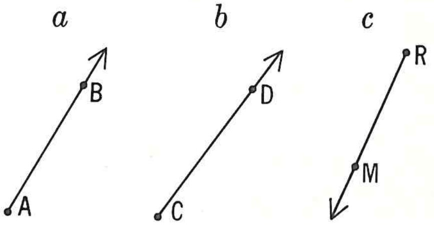
Ray BA starts at B, goes through A and on indefinitely.

Oral Answer the questions below.

1. How many endpoints has a ray?

2. Could you measure the length of a ray? Why or why not?

3. What are the names of the rays shown below?



Written Do the following exercises and answer each question.

1. Mark point R. Draw 2 rays having R as the endpoint. Could you draw more than 2 rays having R as an endpoint? How many rays could have R as an endpoint?

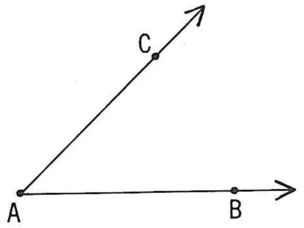
2. Mark points C and D. Draw line segment CD. Now draw ray CD. Is a line segment part of a ray?

3. Mark points T and V. Draw ray TV. Now draw line TV. Is a ray part of a line?

4. Mark points A and B. Draw line segment AB. Mark point C anywhere between A and B. Draw ray CB and ray CA. Do the two rays together represent line AB?

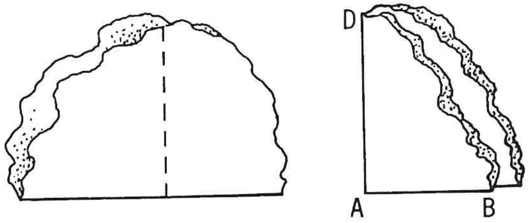
Angles

Another figure of geometry is formed by drawing 2 rays which have the same endpoint or **vertex.** It is called an **angle.** A picture of an angle is shown below.



This angle is formed by rays AB and AC. What point is the endpoint of both rays? Ray AB and ray AC are called the *sides* of the angle. Point A is called the *vertex* of the angle.

You can use a piece of paper to make a model of an important angle. Fold a piece of paper. Now fold it again by making sure that the edges of the crease line up exactly, as shown below.



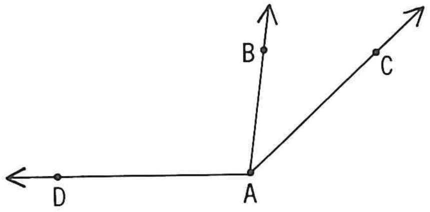
The angle suggested by the edges AD and AB is called a **right angle.** A right angle is suggested by a corner of this page.

Oral Answer the questions below.

1. What are three objects in your room that suggest angles?

2. What are three objects in your room that suggest right angles?

3. How many angles are shown in the drawing below? Tell which rays form each angle. Remember that *any* 2 rays that have the same endpoint form an angle.



Written Do the following exercises and answer each question.

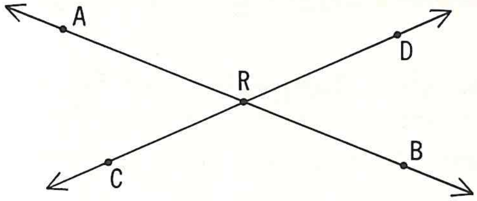
1. Draw a triangle. Name the endpoints of the sides A, B, and C. Extend two sides to form ray AB and ray AC. Do these two rays form an angle? How many angles does any triangle have?

2. Fold a piece of paper as shown in this lesson to make a model of a right angle.

3. Mark point R on your paper. Use your model of a right angle to draw a picture of a right angle which has R as the vertex.

Intersecting Lines and Parallel Lines

Two lines in a plane might intersect, or cross, as shown below.

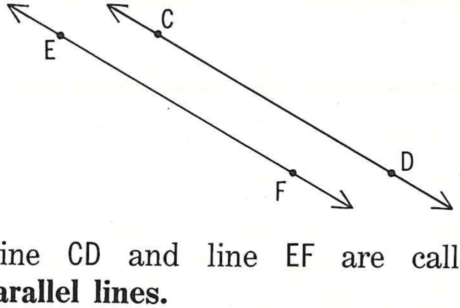


Line AB and line CD are called **intersecting lines.**

When two streets cross, they are said to intersect. What do you mean by a street intersection? When two lines intersect, their **intersection** is the point which belongs to, or is common to, both of the lines. In the drawing above, point R is called *the intersection.*

Intersecting lines are illustrated by the letter x and by the sides of a triangle.

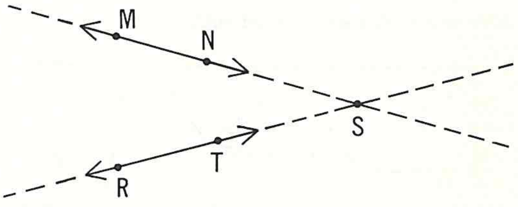
Two lines in a plane may never intersect no matter how far extended, as shown below.



Line CD and line EF are called **parallel lines.**

Parallel lines are illustrated by the edges at the top and bottom of this page, and by the two edges of your ruler.

Since a picture shows only part of a line, the marks in a drawing may not cross but still represent intersecting lines. The dashes in the drawing below show that you should think about the whole line.

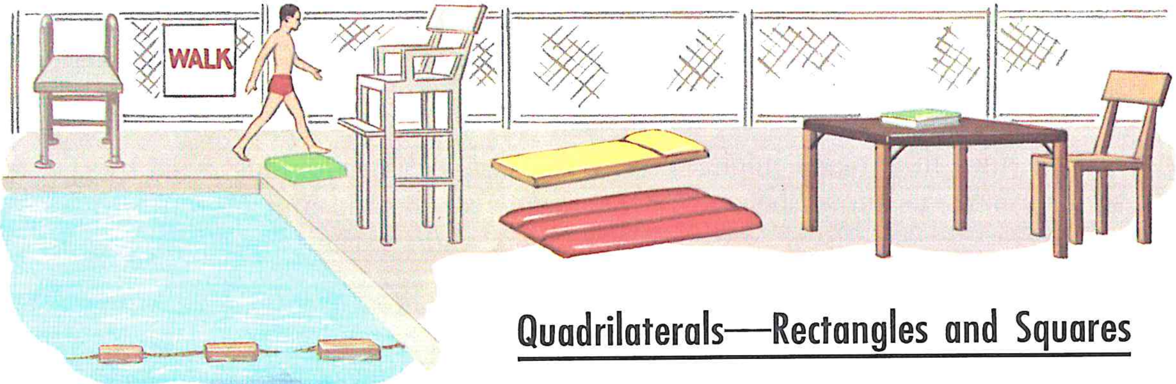


Lines MN and RT intersect. Their intersection is point S.

Oral Name 3 objects in your classroom which illustrate each figure below.

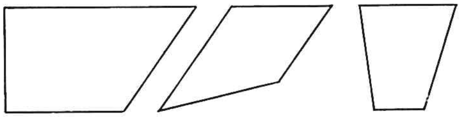
1. Intersecting lines
2. Parallel lines
3. An angle
4. A right angle

Written Draw a picture of two parallel lines. Now draw a line which intersects one of the parallel lines. Does it intersect the other line also?



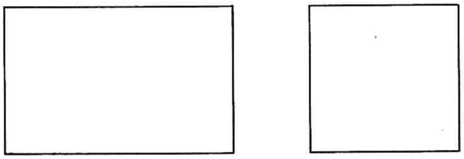
Quadrilaterals—Rectangles and Squares

Each figure below is called a **quadrilateral.** *Quad* means *four* and *lateral* refers to a side, so *quadrilateral* indicates 4 sides.



Some quadrilaterals are so often used that they have been given special names.

A **rectangle** is a quadrilateral in which all of the angles are right angles. Pictures of two rectangles are shown below.



A **square** is a rectangle whose sides all have the same measure. This definition means that the four angles of a square are right angles, and that the four sides of a square are the same length.

Oral Answer the questions below.

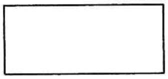
1. What are three objects shown in the picture at the top of this page which illustrate a rectangle? A square?

2. Do you think that the opposite sides of a rectangle are parallel?

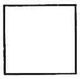
3. Does the cover of this book have the shape of a rectangle? Of a square?

4. Which figure, or figures, below illustrates a quadrilateral? A rectangle? A square?

a



b



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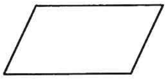


Written Do the following exercises on your paper.

1. Make a model of a right angle by folding a piece of paper. Use this model and a ruler to draw a square with sides 3 inches long.

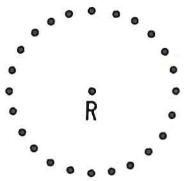
2. Draw a rectangle so that two of its intersecting sides measure 3 inches and 2 inches.

Circles

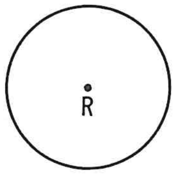
If you trace around a coin, such as a nickel or a quarter, you draw a picture of a *circle.* Many objects have the shape of a circle and are described as **circular.**

A circle is a simple closed figure whose points are located in a special way. Mark point R on your paper. Now mark several points that are 1 inch from R. Be sure to mark these points in different directions from R. Your drawing should be similar to that shown in A below.

A



B

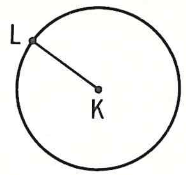


If you mark more points 1 inch from R, your drawing will look more like drawing B above.

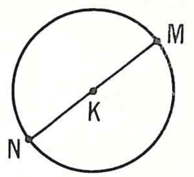
A **circle** is the set of points in a plane which are all the same distance from some fixed point in the plane.

In drawing B, point R is called the **center** of the circle. Point R is *not* a point on the circle. It is inside the circle. As with any simple closed figure, there are points inside of the circle, points on the circle, and points outside the circle.

C



D



In drawing C, K marks the center of the circle. L marks any point on the circle. Line segment KL is called a **radius** of the circle.

In drawing D, K marks the center of the circle. M and N mark points on the circle so that line segment MN passes through point K, the center. Line segment MN is called a **diameter** of the circle.

Oral Answer the questions below.

1. Does a circle have more than 1 radius? More than 1 diameter?

2. Is every point of a radius inside the circle?

3. How many points of a diameter are points on the circle?

4. If one radius of a circle is 2 inches long, how long is any other radius of the circle?

5. If a radius of a circle is 1 inch long, how long is a diameter?

6. Could two circles have the same point as their center?

Review and Practice

Part 1 Copy. Find each answer by performing the indicated operation.

	a	*b*	*c*
1.	$\begin{array}{r} 3896 \\ 28 \\ +572 \\ \hline \end{array}$	$\begin{array}{r} 9076 \\ 3804 \\ +439 \\ \hline \end{array}$	$\begin{array}{r} 5944 \\ 8755 \\ +3966 \\ \hline \end{array}$
2.	$\begin{array}{r} 3768 \\ -959 \\ \hline \end{array}$	$\begin{array}{r} 576 \\ \times 87 \\ \hline \end{array}$	$\begin{array}{r} 7003 \\ -457 \\ \hline \end{array}$
3.	$\begin{array}{r} 23\frac{1}{3} \\ +57\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} 57\frac{1}{3} \\ -14\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 21\frac{3}{10} \\ +8\frac{4}{5} \\ \hline \end{array}$
4.	$\begin{array}{r} 12\frac{1}{12} \\ -8\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 18\frac{3}{5} \\ +13\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} 41\frac{1}{6} \\ +22\frac{4}{9} \\ \hline \end{array}$
5.	$\begin{array}{r} 34\frac{9}{10} \\ -12\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 51\frac{2}{3} \\ -24\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} 71\frac{2}{7} \\ +29\frac{1}{2} \\ \hline \end{array}$
6.	$\begin{array}{r} 113\frac{5}{24} \\ +11\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} 214\frac{1}{2} \\ +76\frac{4}{9} \\ \hline \end{array}$	$\begin{array}{r} 33\frac{7}{8} \\ -15\frac{2}{5} \\ \hline \end{array}$

Part 2 Copy. Find each quotient and each remainder.

	a	*b*
1.	$32\overline{)476}$	$84\overline{)572}$
2.	$15\overline{)1426}$	$67\overline{)9231}$
3.	$83\overline{)4679}$	$54\overline{)36548}$
4.	$29\overline{)7006}$	$38\overline{)21004}$
5.	$58\overline{)5472}$	$73\overline{)64250}$

Part 3 Copy. Solve each of the following equations.

	a	*b*
1.	$\frac{2}{3}+\frac{1}{2}=n$	$(12\div 3)\times 8=n$
2.	$\frac{5}{7}-\frac{1}{2}=n$	$(7-7)\div 5=n$
3.	$3\frac{1}{5}+\frac{1}{3}=n$	$7+(15\times 3)=n$
4.	$2\frac{2}{7}-\frac{1}{2}=n$	$(3\times 8)+n=31$
5.	$7\frac{1}{4}-3\frac{3}{5}=n$	$n-(30\div 6)=11$
6.	$4\frac{2}{3}+6\frac{5}{9}=n$	$\frac{1}{3}+\frac{2}{5}+\frac{7}{15}=n$
7.	$5\frac{1}{6}-3\frac{7}{9}=n$	$3\frac{1}{2}+\frac{1}{4}+7\frac{2}{3}=n$

Part 4 Solve each of the following problems.

1. John weighs $68\frac{3}{4}$ pounds. His grandfather weighs $175\frac{1}{2}$ pounds. How much less than his grandfather does John weigh?

2. Mrs. Smith baked $2\frac{1}{2}$ dozen oatmeal cookies, $3\frac{1}{4}$ dozen sugar cookies, and $4\frac{1}{6}$ dozen chocolate chip cookies. How many dozen cookies did she bake altogether?

3. On four spelling tests Sarah made scores of 98, 86, 91, and 89. What was her average score?

4. Paul bought $\frac{3}{4}$ pound of candy that sold for 98¢ a pound. How much did he spend for candy?

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Through 2 points passes one and only one line. (234)

2. A line segment is the shortest path between 2 points. (235)

3. If any 2 points of a line are in a plane, then every point of the line is in the plane. (236)

4. A ray is part of a line. (239)

5. Two lines in a plane either intersect or are parallel. (241)

Words to Know

1. Geometry, point, space (233)

2. Line (234)

3. Line segment (235)

4. Plane (236)

5. Simple closed figure (237)

6. Triangle (238)

7. Ray (239)

8. Angle, right angle (240)

9. Intersecting lines, intersection, parallel lines (241)

10. Rectangle, square (242)

11. Radius, diameter (243)

Questions to Discuss

1. How many points must you name in order to name a line? To name a line segment? (234, 235)

2. How many endpoints does a line have? A line segment? A ray? (234, 235, 239)

3. Can you measure the length of a line? Of a line segment? Of a ray? (234, 235, 239)

4. How many angles has a triangle? A rectangle? A square? Any quadrilateral? (238, 242)

5. Is the center of a circle a point on the circle or a point inside the circle? (243)

6. If a diameter of a circle is 10 inches long, how long is a radius of that circle? (243)

Written Practice

1. Mark 4 points. Label them with the letters A, B, C, and D. Draw line AB. Draw line segment CD. (234, 235)

2. Draw two rays having point T as an endpoint. (239, 240)

3. Draw a rectangle whose intersecting sides are of length 1 inch and $2\frac{1}{2}$ inches. (242)

Self-Evaluation

Part 1 Write the answer to each of the following questions.

1. How many endpoints has a line? A line segment? A ray?

2. What do the arrows on the picture of a line mean?

3. How many angles has a triangle? A rectangle? A square?

4. How many sides has a triangle? A rectangle? A square?

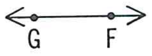
5. If a diameter of a circle has a length of 14 feet, what is the length of a radius?

6. Tell two times when the hands of a clock form a right angle.

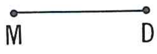
Part 2 Write the answer for each of the following exercises.

1. Name each of the following by telling whether it is a line, a line segment, or a ray, and by using the letters given.

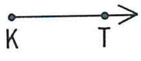
a



b

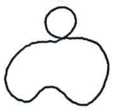


c

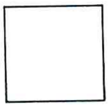


2. Tell which of the following are simple closed figures.

a



b



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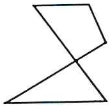
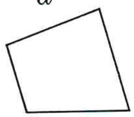
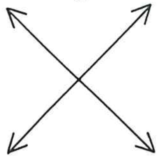


d

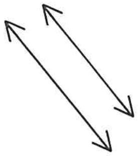


3. Tell whether each drawing below represents intersecting lines or parallel lines.

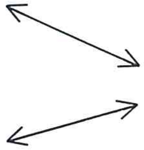
a



b

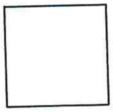


c

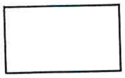


4. Tell which drawing below represents a triangle, a circle, a rectangle, or a square.

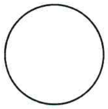
a



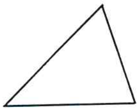
b



c



d

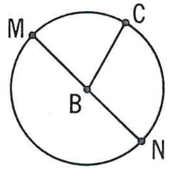


5. Using this circle, name the following parts.

a. A radius

b. A diameter

c. The center

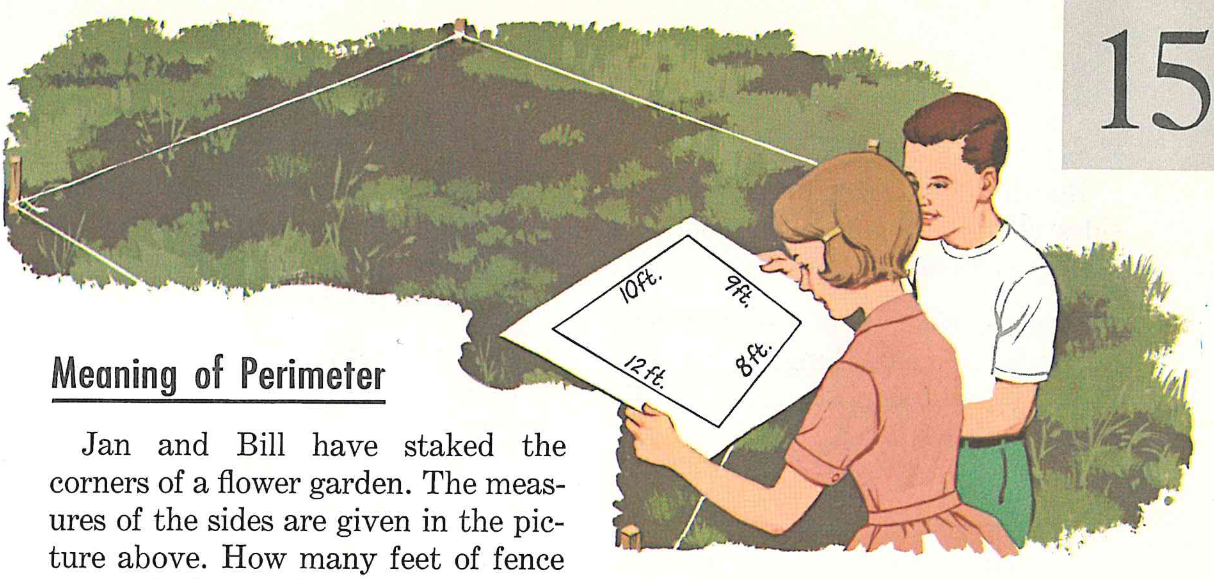


Part 3 Do each of the following exercises on your paper.

1. Mark points A, B, and C so that they are not on the same line. Draw line segments AB, BC, and AC. What kind of figure is formed?

2. Mark points D, E, and F so that they are not on the same line. Draw rays DE and DF. What kind of figure is formed?

15



Meaning of Perimeter

Jan and Bill have staked the corners of a flower garden. The measures of the sides are given in the picture above. How many feet of fence are needed to enclose the garden?

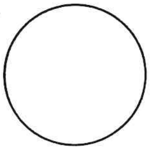
You could use an equation to find the sum of the lengths of the sides. $12+10+9+8=n$. Since $n=39$, they need 39 feet of fence to enclose the garden.

In this problem, the measure of 39 feet is called the *perimeter*. *Peri* means around. *Meter* comes from *metron* which means measure.

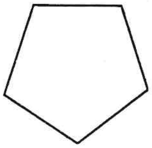
Perimeter means the *measure around* or the *distance around* a simple closed figure.

Below are drawings of simple closed figures. Does every simple closed figure have a perimeter?

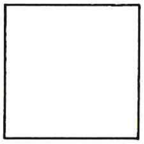
A



B



C



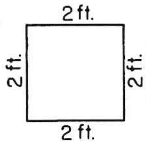
For which of the figures, A, B, or C, would it be easiest to find the perimeter? Why? The most difficult to find the perimeter? Why?

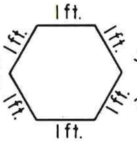
In this chapter you will learn how measurement applies to some simple closed figures formed by line segments.

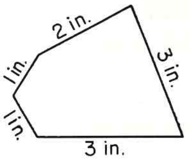
Oral Tell the perimeter of each figure below.

a *b* *c*

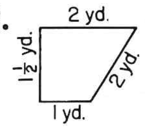
1.

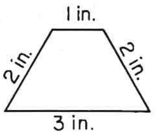


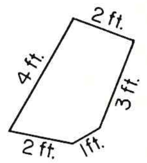




2.



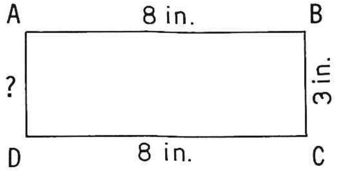




Perimeter of a Rectangle

Recall that each pair of opposite sides of a rectangle are parallel and are equal in length.

Suppose side AB in the rectangle below is 8 inches long. Since side DC is opposite side AB, then side DC is also 8 inches long.



If side BC measures 3 inches, how long is side AD?

The *length* of a rectangle is the measure of the longer sides. The *width* of a rectangle is the measure of the shorter sides. The length and the width of a rectangle are called its **dimensions.**

To find the perimeter of the rectangle above, add the measures of the sides. $8+3+8+3=22$. The perimeter is 22 inches.

Since you can group the addends in any order, the perimeter can also be found as shown below.

$$8+3+8+3=22$$
$$(8+3)+(8+3)=22$$
$$2\times(8+3)=22$$
$$2\times11=22$$

Since the measure of the length is 8 and the measure of the width is 3, the equation $2\times(8+3)=22$ tells how to find the perimeter.

The *perimeter of a rectangle* is equal to *2 times the sum of the length and the width.*

Let us abbreviate this statement by using only the first letter of the name of each measure to represent the *number* of that measure. That is, from the words perimeter, length, and width let us take p, l, and w to represent the *number* of each measure.

$$p=2\times(l+w)$$

This is called a **formula.** A *formula* states a general fact or rule.

Finding the perimeter requires the addition of measures. When finding the sum of two or more measures, they must be in the same unit.

Suppose a rectangle has a length of 2 yards and a width of 4 feet. What is its perimeter in feet?

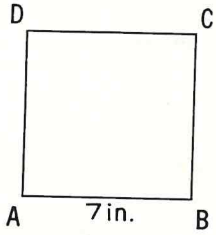
Express 2 yards as 6 feet, and use the formula to find the perimeter.

$$p=2\times(l+w)$$
$$p=2\times(6+4)$$
$$p=2\times10$$
$$p=20$$

The perimeter measures 20 feet.

A square is a special kind of rectangle. You probably remember that a square is a rectangle which has all 4 sides of the same length. In other words, a square is a rectangle in which the length and the width are equal in measure.

Suppose side AB in the square below measures 7 inches. How long is side BC? Side CD? Side DA?



The perimeter of the square could be found by adding the measures of the 4 sides.

$$p=7+7+7+7=28$$

The perimeter is 28 inches.

If s stands for the measure of a side of the square, and p for the perimeter, then the formula below tells how to find the perimeter of a square.

$$p=s+s+s+s$$

or

$$p=4\times s$$

The *perimeter of a square* is equal to *4 times the measure of a side* of the square.

Oral Answer these questions.

1. What is the formula for finding the perimeter of a rectangle? What measure does each letter in the formula stand for?

2. What is the formula for finding the perimeter of a square? What does each letter in the formula stand for?

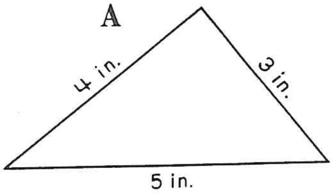
3. The dimensions of a rectangle are 3 yards and 6 feet. What is its length? Its width? How would you give the dimensions if you were to find its perimeter in feet? Its perimeter in yards?

Written Copy. Find the perimeter of each rectangle as indicated.

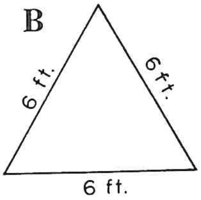
	Length	*Width*	*Perimeter*
1.	7 ft.	5 ft.	__ft.
2.	82 ft.	17 ft.	__ft.
3.	3 ft.	24 in.	__ft.
4.	17 ft.	2 yd.	__ft.
5.	7 yd.	21 ft.	__yd.
6.	96 in.	3 ft.	__in.
7.	$5\frac{1}{2}$ ft.	$3\frac{1}{2}$ ft.	__ft.
8.	19 in.	1 ft.	__in.
9.	72 in.	36 in.	__yd.
10.	9 ft.	9 ft.	__ft.

MORE PRACTICE PAGE 316

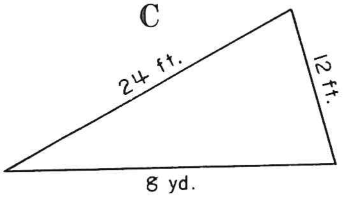
A



B



C



Perimeter of a Triangle

The perimeter of triangle A above is found by adding the measures of the three sides.

$$p=4+5+3=12$$

The perimeter is 12 inches. What is its perimeter in feet?

Triangle B is a special kind of triangle. All of its sides have the same measure. It is called an **equilateral triangle.** *Equi* means equal and *lateral* means side.

An *equilateral triangle* is a triangle with sides of equal measure. What is the perimeter of equilateral triangle B in feet?

Oral Answer the following questions about triangle C.

1. What measures would you use for the sides in finding the perimeter in yards?

2. What is its perimeter in yards?

3. What measures would you use for the sides in finding the perimeter in feet?

4. What is its perimeter in feet?

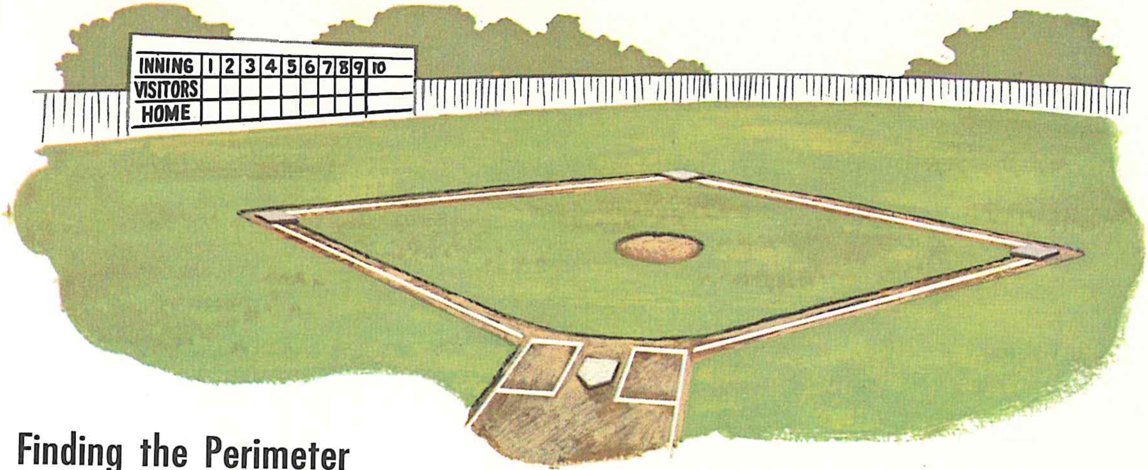
MORE PRACTICE PAGE 316

Written Copy. Find the perimeter of each triangle as indicated below.

	Measures of Sides	*Perimeter*
1.	3 in., 7 in., 6 in.	__in.
2.	8 ft., 10 ft., 12 ft.	__ft.
3.	2 yd., 5 ft., 5 ft.	__ft.
4.	18 in., 2 ft., 31 in.	__in.
5.	$3\frac{1}{2}$ in., $5\frac{1}{4}$ in., $6\frac{1}{4}$ in.	__in.
6.	12 ft., 7 yd., 8 yd.	__yd.
7.	$1\frac{1}{2}$ ft., 2 ft., 28 in.	__in.
8.	30 in., 3 ft., 42 in.	__ft.
9.	6 ft., $7\frac{1}{2}$ ft., 2 yd.	__yd.
10.	2 yd., 5 ft., 41 in.	__in.

A quick review Each equation below could result from using a formula for finding the perimeter. Solve each equation.

	a	*b*
1.	$p=2\times(6+5)$	$22=2\times(7+w)$
2.	$64=4\times s$	$38=2\times(l+8)$



Finding the Perimeter

As you study each problem on this page, decide whether you are to find the perimeter of a rectangle, of a square, or of a triangle. Use the correct formula in solving each of the following problems.

1. A softball diamond has the shape of a square with each side 60 feet long. What is its perimeter?

2. Mr. Black's office floor has the shape of a rectangle. Its dimensions are 18 feet and 12 feet. What is the perimeter of the floor?

3. A baseball diamond has the shape of a square with each side 90 feet long. How many feet longer is the perimeter of a baseball diamond than that of a softball diamond?

4. A closet door is $2\frac{1}{2}$ feet wide and $6\frac{1}{2}$ feet long. What is the perimeter of the door in feet?

5. The front of a desk drawer has a length of 14 inches and a width of 6 inches. What is the perimeter of the drawer front in inches?

6. The top of Mr. Smith's desk is 30 in. wide and 5 ft. long. He plans to put a plastic strip all the way around the edge of the top. How many feet of plastic strip will he need?

7. Mr. Jones has a lot in the shape of a triangle. The sides measure 75 feet, 40 yards, and 150 feet. How many feet of fence are needed to enclose the lot?

8. Janice rode her bicycle around the block. Her brother said that the block has the shape of a square with each side 375 feet long. About how far did Janice ride her bicycle in going around the block?

Can you do this? The perimeter of a square is 24 feet. What is the length of a side of the square? (Hint: Make a sketch to help you.)

Review of Fractions

Part 1 Copy. Find each sum or difference by performing the indicated operation. Give all answers in lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{4}{11}$ $+\frac{5}{11}$	$\frac{7}{8}$ $+\frac{3}{8}$	$\frac{7}{9}$ $-\frac{2}{9}$	$5\frac{1}{3}$ $-2\frac{2}{3}$
2.	$\frac{4}{5}$ $-\frac{1}{2}$	$\frac{5}{6}$ $+\frac{3}{4}$	$8\frac{1}{5}$ $+7\frac{3}{6}$	$14\frac{1}{4}$ $-9\frac{5}{8}$
3.	$\frac{3}{7}$ $\frac{1}{7}$ $+\frac{5}{7}$	$12\frac{5}{8}$ $7\frac{1}{8}$ $+3\frac{1}{4}$	$7\frac{2}{3}$ $5\frac{5}{6}$ $+7\frac{4}{9}$	$48\frac{2}{5}$ $12\frac{2}{3}$ $+7\frac{1}{4}$
4.	$17\frac{5}{8}$ $-3\frac{5}{6}$	$31\frac{5}{6}$ $+8\frac{7}{9}$	$611\frac{1}{7}$ $-78\frac{5}{8}$	$115\frac{1}{3}$ $+92\frac{4}{7}$

Part 2 Copy. Solve each equation. Write each answer in lowest terms.

	a	*b*
1.	$n=\frac{1}{4}+\frac{3}{8}+\frac{1}{2}$	$n=2\frac{1}{4}+1\frac{2}{3}+\frac{5}{6}$
2.	$n=(\frac{7}{10}+\frac{2}{5})-\frac{1}{2}$	$n=\frac{5}{8}+(\frac{3}{4}-\frac{1}{6})$
3.	$n=(\frac{7}{12}-\frac{1}{6})+\frac{3}{4}$	$n=8\frac{2}{3}-1\frac{7}{15}$
4.	$n=\frac{7}{9}+(\frac{5}{6}+1\frac{1}{3})$	$n=76-21\frac{5}{7}$

Part 3 Rearrange the fractions in each set so that they name numbers from smallest to largest.

1. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}, \frac{9}{16}, \frac{13}{32}$

2. $\frac{7}{2}, \frac{20}{6}, \frac{41}{12}, \frac{8}{3}, \frac{13}{4}$

3. $\frac{14}{9}, 1\frac{1}{3}, 1\frac{1}{6}, \frac{25}{18}, \frac{3}{2}$

4. $\frac{4}{5}, \frac{7}{10}, \frac{15}{20}, \frac{3}{4}, \frac{1}{2}$

5. $6\frac{1}{8}, \frac{51}{12}, 7\frac{1}{6}, \frac{13}{2}, \frac{27}{4}$

6. $\frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \frac{12}{6}$

Part 4 Solve each of the following problems.

1. According to official rules, a softball may have a maximum weight of $6\frac{3}{4}$ oz. and a baseball may have a maximum weight of $5\frac{1}{4}$ oz. What is the difference between these two weights?

2. In baseball the pitching distance is $60\frac{1}{2}$ feet. In softball the pitching distance is 46 feet. How much farther must you pitch a baseball than a softball?

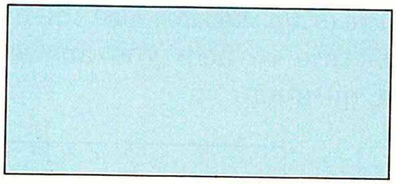
3. On Monday $\frac{1}{6}$ of the 24 pupils in the class were absent and $\frac{1}{4}$ of those present went to the library. How many pupils remained in the classroom?

4. Sally drew a picture of a simple closed figure on her paper. Its sides measured $3\frac{1}{2}$ inches, $4\frac{1}{4}$ inches, $2\frac{7}{8}$ inches, $1\frac{1}{2}$ inches, and $3\frac{5}{16}$ inches. What was its perimeter?

5. John spent $1\frac{3}{4}$ hours skating on Saturday and 90 minutes on Sunday. How many minutes did he skate during the 2 days? How many hours?

Meaning of Area

That part of a plane enclosed by a simple closed figure is called its **interior.** The interior of the rectangle below is blue.



How can you measure that part of a plane enclosed by a simple closed figure? Linear measure is not suitable since it refers only to length. The interior of a rectangle has both length and width.

The length of a line segment is usually measured by means of some convenient standard unit of length. For example, to measure the length of this book you use a line segment, probably 1 inch long, as the unit of measure.

Likewise, to measure an interval of time you use some time interval as the unit of measure.

It seems reasonable to measure a part of a plane by using some part of a plane as the unit of measure.

The picture at the top of the page shows Peggy filling a page in a stamp book. How many stamps are needed to fill the page?

You could count how many stamps are needed. She needs 35 stamps to fill the page.

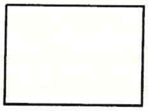
According to geometry, you are asked how many times the smaller part of a plane (the stamp) is contained in the larger part of a plane (the page).

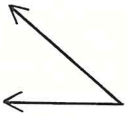
The measure of part of a plane or part of a surface is called **area.**

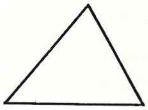
Oral Which of the following figures of geometry enclose an area?

a *b* *c*

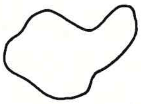
1.

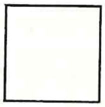


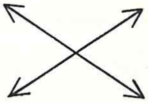




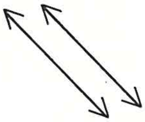
2.

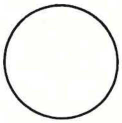


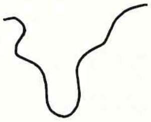




3.



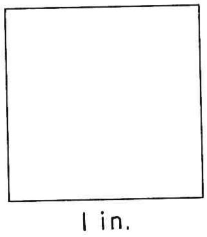




Finding the Area of a Rectangle

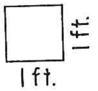
In order to measure the area of any part of a plane, you need some suitable unit of measure. You use several units of linear measure, such as inch or foot. There are several units of area measure also.

The figure below is a square with sides of length 1 inch. The square alone is called an *inch square.* The area enclosed by the square is called a **square inch** (sq. in.) and is a unit for measuring the area of a simple closed figure.

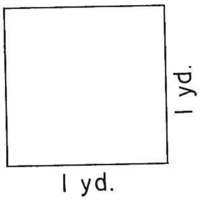


Suppose the side of each square below has the length indicated.

A



B

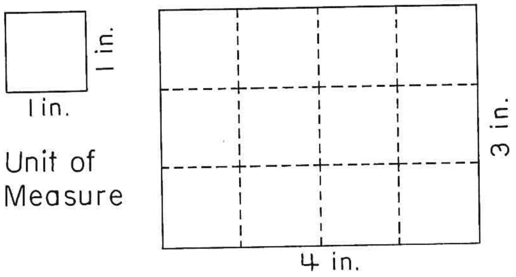


What name would you use for the unit of area measure in A? For the unit of area measure in B?

A **square foot** (sq. ft.) and a **square yard** (sq. yd.) are common units of area measure.

If a rectangle has a length of 4 inches and a width of 3 inches, what is its area?

The *area of a rectangle* means the *measure of its interior.* Use the following picture to help you answer the above question.



Counting will tell you that it takes 12 units of 1 square inch each to cover the interior of the rectangle. The area of the rectangle is 12 square inches.

How many square-inch units are in each row? How many rows are there?

Notice that the area is equal to 4×3 or 12 square inches.

The *area measure* of a rectangle is equal to the *product of its length and its width.*

If A stands for area, the above statement becomes this formula.

$$A = l \times w$$

Suppose that a rectangle has a length of 15 feet and a width of 8 feet. Find its area measure in square feet.

The length and the width of the rectangle are given, so you can replace l and w in the formula by the numerals which they represent. Then solve the resulting equation, as shown below.

$$A = l \times w$$
$$A = 15 \times 8$$
$$A = 120$$

The area measure is 120 square feet. Why is the answer given in square feet?

How many square inches are contained in 1 square foot? How many square feet are contained in 1 square yard? How many square inches are contained in 1 square yard?

Draw pictures of these figures to help you answer the above questions if necessary.

Your answers should agree with the statements in the following table of square measure. Remember that the = sign, as used in the table, means *measures the same as*.

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.

Oral Which unit of area measure, sq. in., sq. ft., or sq. yd., is most suitable for measuring the area of each object named below?

1. Your classroom floor
2. A page in your book
3. The school playground
4. The chalkboard
5. A football field
6. A postage stamp

Written Copy. Find the area of each rectangle as indicated.

	Length	*Width*	*Area*
1.	5 ft.	3 ft.	___sq. ft.
2.	9 in.	5 in.	___sq. in.
3.	15 in.	12 in.	___sq. in.
4.	32 yd.	20 yd.	___sq. yd.
5.	13 yd.	8 yd.	___sq. yd.
6.	38 ft.	27 ft.	___sq. ft.
7.	14 in.	14 in.	___sq. in.
8.	21 yd.	12 yd.	___sq. yd.
9.	112 ft.	86 ft.	___sq. ft.
10.	60 in.	47 in.	___sq. in.
11.	512 ft.	373 ft.	___sq. ft.
12.	439 yd.	176 yd.	___sq. yd.

More About the Area of a Rectangle

The statement and the formula for the area of a rectangle are true only when the length and the width are given in the *same* unit of measure.

For example, the dimensions of a rectangle are 5 feet and 2 yards. What is its area in square feet? Before using the formula, express both dimensions in feet. Why?

Since 2 yd. = 6 ft., the rectangle has dimensions of 5 feet and 6 feet.

The area is found as follows.

$$A = l \times w$$
$$A = 6 \times 5$$
$$A = 30$$

The area is 30 square feet.

As another example, suppose a rectangle has a length of 3 feet and a width of 7 inches. What is its area in square inches?

Express both dimensions in inches. Since 3 ft. = 36 in., the rectangle has dimensions of 36 inches and 7 inches.

The area is found as follows.

$$A = l \times w$$
$$A = 36 \times 7$$
$$A = 252$$

The area is 252 square inches. Why square inches?

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Oral How would you express the dimensions of each of the following rectangles in order to find the area in the units indicated?

	Length	*Width*	*Area in*
1.	17 yd.	9 ft.	sq. yd.
2.	2 ft.	13 in.	sq. in.
3.	1 yd.	19 in.	sq. in.
4.	18 ft.	5 yd.	sq. ft.

Written Copy. Find the area of each rectangle as indicated.

	Length	*Width*	*Area*
1.	23 in.	1 ft.	__sq. in.
2.	15 ft.	3 yd.	__sq. yd.
3.	60 in.	3 ft.	__sq. ft.
4.	23 ft.	4 yd.	__sq. ft.
5.	2 yd.	30 in.	__sq. in.
6.	72 in.	1 yd.	__sq. yd.
7.	17 in.	8 in.	__sq. in.
8.	8 ft.	53 in.	__sq. in.
9.	16 ft.	11 ft.	__sq. ft.
10.	36 ft.	5 yd.	__sq. yd.
11.	47 yd.	37 yd.	__sq. yd.

Finding Perimeter and Area

Solve the following problems concerning area and perimeter.

1. Mr. Ross put ceiling tile in his recreation room. Each tile has the shape of a square with sides 1 foot long. The room is 24 feet long and 13 feet wide. How many tiles did he use?

2. A sheet of art paper measures 15 inches by 12 inches. Find its area measure in square inches.

3. Mrs. Williams has a rug 18 ft. long and 12 ft. wide in her living room. How many square feet of floor area does the rug cover? What is the perimeter of the rug?

4. A football field is 300 feet long and 160 feet wide. How many square feet of playing area does the field cover?

5. The top of a desk has dimensions of 54 inches and 30 inches. A desk pad measures 18 inches by 24 inches. What is the area measure of the desk top? What is the area measure of the pad? How many square inches of desk top are not covered by the pad?

6. A playground measures 470 feet by 550 feet. What is the measure of its perimeter in feet? What is the measure of its area?

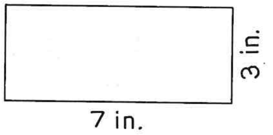
7. Mr. Jackson has a driveway which is 14 yds. long and 9 ft. wide. What is its area measure in square yards? In square feet?

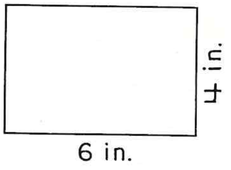
8. Mr. Adams fertilized his lawn by spreading 1 lb. of fertilizer on each 100 sq. ft. of lawn area. His lawn is 120 ft. long and 60 ft. wide. Did he use more or less than 75 lb. of fertilizer?

9. James and his father bought a sheet of plywood 4 ft. wide and 8 ft. long. What is its perimeter? What is its area?

10. How many square inches are needed to cover 1 sq. yd. of surface?

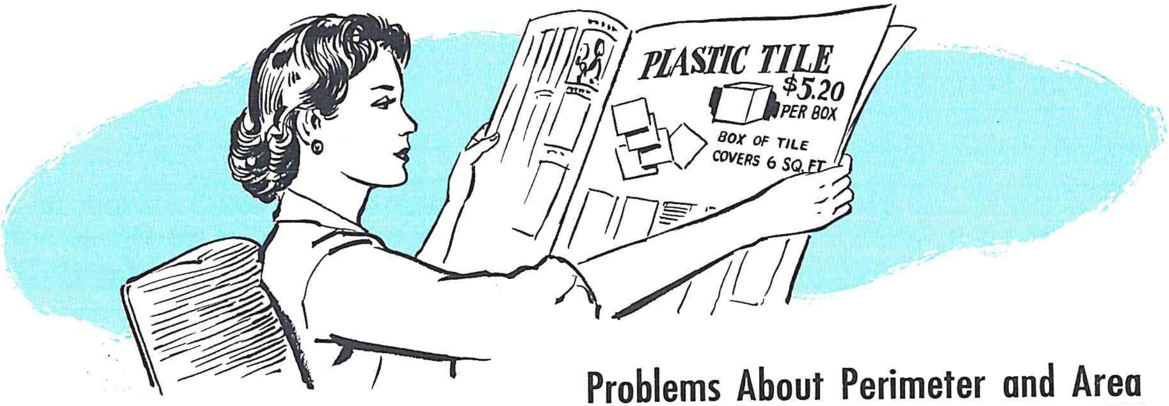
Something to do Find the perimeter and find the area of each rectangle shown below.





Can two rectangles have the same perimeter but different areas?

Find the perimeter and the area of a square whose side measures 4 inches. Can the same number represent both the perimeter and the area of a rectangle?



Problems About Perimeter and Area

Mrs. Green wants to cover a 2 ft. by 6 ft. rectangular part of her kitchen wall with plastic tile. According to the advertisement above, how much will it cost?

$$A = l \times w$$
$$A = 6 \times 2 = 12$$

The area to be covered contains 12 sq. ft. and each box of tiles covers 6 sq. ft. Since $12 \div 6 = 2$, two boxes of tiles are needed.

At $5.20 per box, 2 boxes of tiles will cost $10.40.

The above problem is typical of the kind of problems on this page. Solve each of these problems.

1. A screen window was 2 feet wide and 4 feet long. Fiberglas screen sold for 12¢ a square foot. What did it cost to put fiberglas screen on the window?

2. Mr. King built a table with a rectangular top 5 ft. long and 3 ft. wide. He covered the top with plastic which sold for 65¢ a square foot. How much did it cost to cover the top with plastic?

3. A rectangular roof measures 45 feet by 24 feet. A bundle of asphalt shingles covers 50 square feet of roof. How many bundles of shingles are needed to cover the roof?

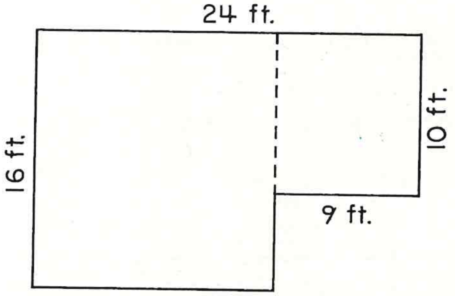
4. A carpenter put weather stripping around 6 windows. Each window was $2\frac{1}{3}$ ft. wide and $3\frac{2}{3}$ ft. long. How many feet of weather stripping did he use for all the windows?

5. Mr. Jackson painted his rectangular basement floor. The floor was 36 ft. long and 25 ft. wide. Each gallon of paint covered 300 square feet of surface. How many gallons of paint did he use?

6. A carpenter fitted baseboard around a living room which was 24 ft. long and 14 ft. wide. How many feet of baseboard did he use if the door openings account for 8 feet of the measure of the perimeter?

Areas of Other Closed Figures

The figure below shows the shape and dimensions of a recreation room. How many square feet of floor area are there in the room?



The floor does not have the shape of a rectangle. The dotted line shows how you can think of the floor as being formed by two rectangles.

The area of the floor is then found by adding the areas of the two rectangles.

The larger rectangle measures 15 feet by 16 feet. Its area is 15×16 or 240 square feet. The smaller rectangle measures 9 feet by 10 feet and has an area of 9×10 or 90 square feet.

The area of the floor is 240+90 or 330 square feet.

Oral Answer the following questions about the problem above.

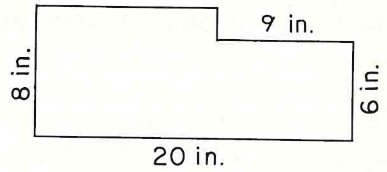
1. What is another way of dividing the room into two rectangles?

2. What would be the area of each rectangle?

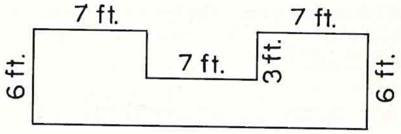
3. What is the area of the floor?

Written Find the area of each figure pictured below. Think of each figure as 2 or more rectangles.

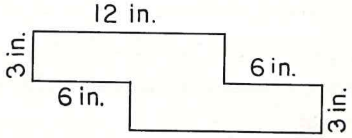
1.



2.



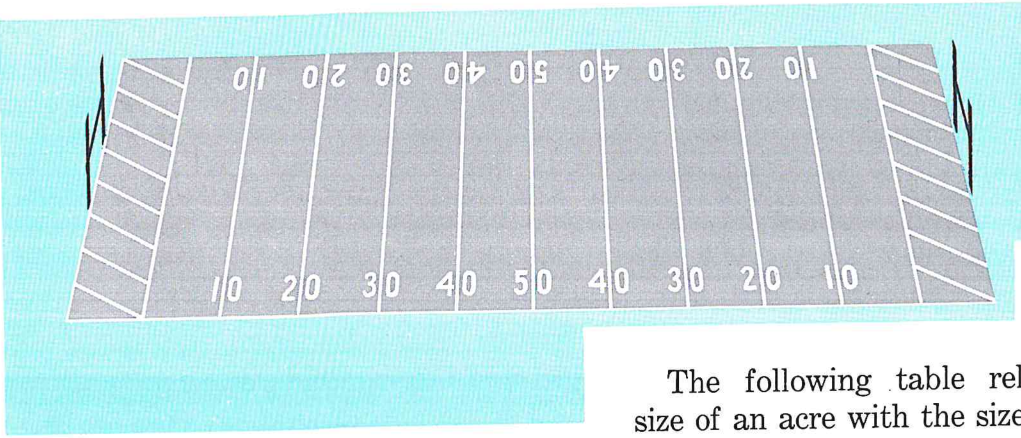
3.



Solve these problems.

4. Paul bought a sheet of plywood 4 ft. wide and 8 ft. long. He used a piece 2 ft. wide and 3 ft. long from this sheet. How many square feet of plywood remained?

5. Mr. Jensen painted one wall of his living room. The wall was 14 ft. long and 8 ft. high with a 4 ft. by 6 ft. window. How many square feet of wall did he paint?



Acre

Land area is usually measured in **acres.**

To get an idea of the size of 1 acre, look at the football field pictured above. It is 300 feet long and 160 feet wide. What is its area in square feet?

An acre is equivalent to 43,560 sq. ft. and the playing area of a football field measures 48,000 sq. ft.; so the playing area of a football field is approximately 1 acre. In fact, it is very close to $1\frac{1}{10}$ acre.

The length of a city block varies from city to city, but let us consider a block as 440 feet in length. Then a square block contains about $4\frac{1}{2}$ acres.

If you were to picture an acre as a square, each side of the square would have a length between 208 feet and 209 feet.

A building lot which is 60 feet by 120 feet contains approximately $\frac{1}{6}$ of an acre.

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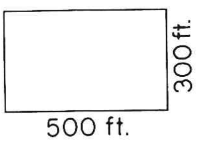
The following table relates the size of an acre with the size of other units of area measure.

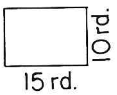
43,560 sq. ft. = 1 acre
160 sq. rd. = 1 acre
640 acres = 1 sq. mi.

Oral For each figure below, tell whether it represents more than or less than 1 acre.

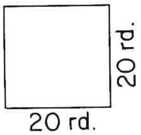
a



b



c



Written Solve each problem.

1. The area of Mr. Mann's farm is $\frac{1}{4}$ of a square mile. How many acres does he have?

2. A builder offers $\frac{1}{2}$-acre lots for sale. How many square feet are contained in each lot?

3. A square mile of land is also called a section. How many acres are contained in a section?

Problem Solving

The problems on this page require the use of the basic operations with geometric figures or measurement.

Be sure that you understand each problem, know what you are asked to find, and plan a method of solution before you begin to write.

1. Yellowstone National Park is shaped like a rectangle. It is about 62 miles long and about 56 miles wide. What is its perimeter? What is its area in square miles?

2. A truck used a gallon of gasoline for each 8 miles it traveled. How many gallons of gasoline did the truck use on a trip of 2144 miles?

3. Jan left for school at 8:30 a.m. and returned home at 2:45 p.m. How many hours was she away from home?

4. Mr. Brown bought a farm which had an area of $\frac{1}{4}$ square mile. He paid $250 per acre for the land. How much did he pay for the farm?

5. Gene and his father bought a sheet of plywood to make a cabinet. The sheet of plywood was 8 feet long and 4 feet wide and cost 22¢ a square foot. How much did they pay for the sheet of plywood?

6. A painter put floor sealer on the basketball playing floor. Each gallon of floor sealer covered 700 sq. ft. of surface. The basketball playing floor was 84 feet by 50 feet. How many gallons of floor sealer were needed?

7. The sides of a triangular plot of ground measure 60 feet, 72 feet, and 45 feet. What is its perimeter in feet? In yards?

8. Mrs. Jensen had wall-to-wall carpeting laid in her living room. The room had the shape of a rectangle 24 feet long and 15 feet wide. The carpet cost $12.00 per square yard. How much did it cost to carpet the room? What is the measure of the perimeter of the room?

9. What is the perimeter of a square if each side measures 4 feet? What is its area in square feet? Is it possible for the same number to represent both the perimeter and the area of a rectangle?

Can you do this? The perimeter of a rectangle is 18 inches and its width is 3 inches. What is the length of the rectangle?

Use the formula for finding the perimeter of a rectangle, $p=2\times(l+w)$, and replace those letters for which the measurements are given. Then solve the resulting equation.

Review and Practice

You should be able to do all the exercises on this page without referring to other pages in the book.

Part 1 Copy. Find the correct answer for each exercise by performing the indicated operation.

	a	*b*	*c*
1.	$\begin{array}{r}218\\ \times 607\\ \hline\end{array}$	$\begin{array}{r}833\\ -476\\ \hline\end{array}$	$\begin{array}{r}575\\ \times 349\\ \hline\end{array}$
2.	$\begin{array}{r}5291\\ \times 705\\ \hline\end{array}$	$\begin{array}{r}7648\\ -839\\ \hline\end{array}$	$\begin{array}{r}3276\\ -507\\ \hline\end{array}$
3.	$\begin{array}{r}4762\\ 381\\ 47\\ +586\\ \hline\end{array}$	$\begin{array}{r}3280\\ 7705\\ 87\\ +363\\ \hline\end{array}$	$\begin{array}{r}8088\\ 7621\\ 5755\\ +4329\\ \hline\end{array}$
4.	$\begin{array}{r}213\frac{1}{2}\\ +78\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}717\frac{1}{5}\\ -426\frac{3}{10}\\ \hline\end{array}$	$\begin{array}{r}581\\ -88\frac{5}{8}\\ \hline\end{array}$

Part 2 Copy. Solve each equation.

	a	*b*
1.	$2891 \div 7 = n$	$6 + (64 \div 8) = n$
2.	$1134 \div 27 = n$	$(6 \times 13) + n = 80$
3.	$n + 7 = 26$	$n - (42 - 12) = 25$
4.	$72 - n = 57$	$39538 \div 53 = n$
5.	$608 \times 205 = n$	$(36 \div 3) - n = 7$
6.	$5 \times n = 45$	$40281 \div 463 = n$
7.	$52 \div n = 13$	$12 \times (115 - 36) = n$

Part 3 Copy. Write a numeral in each of the blanks so that the statements below become true. (= means *measures the same as.*)

1. 78 qt. = ___gal.___qt.
2. 200 hrs. = ___da.___hr.
3. 1030 min. = ___hr.___min.
4. 500 oz. = ___lb.___oz.
5. 71 pk. = ___bu.___pk.
6. $\frac{1}{2}$ mi. = ___ft.
7. 4 sq. yd. = ___sq. ft.
8. 5 sq. mi. = ___acres
9. $\frac{3}{4}$ yd. = ___in.
10. 3 sq. ft. = ___sq. in.
11. 43 pt. = ___gal.___qt.___pt.

Part 4 Copy. Find the correct answer, in lowest terms, for each exercise by performing the indicated operation.

	a	*b*	*c*
1.	$\begin{array}{r}47\frac{2}{3}\\ +17\frac{1}{4}\\ \hline\end{array}$	$\begin{array}{r}176\frac{1}{2}\\ -87\frac{5}{6}\\ \hline\end{array}$	$\begin{array}{r}307\frac{3}{4}\\ -28\frac{2}{5}\\ \hline\end{array}$
2.	$\begin{array}{r}71\frac{1}{2}\\ 18\frac{2}{3}\\ +9\frac{3}{4}\\ \hline\end{array}$	$\begin{array}{r}103\frac{1}{6}\\ 215\frac{1}{2}\\ +37\frac{2}{3}\\ \hline\end{array}$	$\begin{array}{r}411\frac{7}{8}\\ 372\frac{1}{3}\\ +59\frac{5}{6}\\ \hline\end{array}$

Checkup Time

The numerals in (   ) tell the pages where you can turn for help.

Important Ideas

1. The measure of the perimeter means the measure around a simple closed figure. (247)

2. The perimeter of a rectangle is equal to 2 times the sum of the measures of its length and its width. $p=2\times(l+w)$ (248)

3. When finding the perimeter of any figure, express all of its dimensions in the same unit of measure before computing. (248)

4. The area of a rectangle means the measure of the interior of the rectangle. (254)

5. The area of a rectangle is equal to the product of the measures of its length and its width. (254)

6. When finding the area of a rectangle, express all of its dimensions in the same unit. (256)

Words to Know

1. Perimeter (247)
2. Dimensions (248)
3. Formula (248)
4. Equilateral triangle (250)
5. Area (253)
6. Interior of a rectangle (253)
7. Square inch, square foot, square yard (254)
8. Acre (260)

Questions to Discuss

1. How can you find the perimeter of any simple closed figure formed by line segments? (247)

2. What is meant by the formula, $p=2\times(l+w)$? (248)

Written Practice

Copy. Find the perimeter or the area of each rectangle with the dimensions indicated below.

	Length	*Width*	*Perimeter*	
1.	8 in.	5 in.	__in.	(248)
2.	12 in.	12 in.	__in.	(249)
3.	15 ft.	3 yd.	__yd.	(249)

	Length	*Width*	*Area*	
4.	7 in.	6 in.	__sq. in.	(254)
5.	60 in.	3 ft.	__sq. ft.	(255)

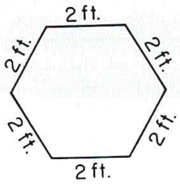
6. The sides of a triangle measure 11 in., 8 in., and 9 in. What is its perimeter in inches? (250)

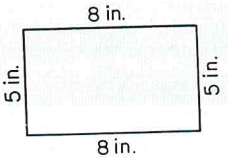
Self-Evaluation

Part 1 Show your work and write your answer for finding the perimeter of each of the following figures.

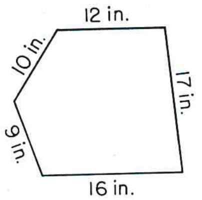
a *b*

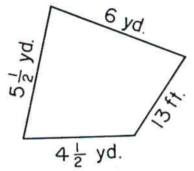
1.





2.





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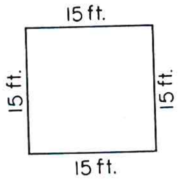

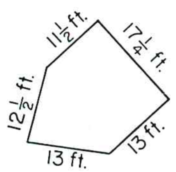

Part 2 Copy. Find the perimeter of each rectangle as indicated.

	Length	*Width*	*Perimeter*
1.	27 in.	19 in.	___in.
2.	$7\frac{1}{2}$ ft.	$5\frac{3}{4}$ ft.	___ft.
3.	$13\frac{1}{2}$ yd.	$8\frac{1}{4}$ yd.	___yd.
4.	$1\frac{1}{2}$ ft.	13 in.	___in.
5.	3 yd.	7 ft.	___ft.
6.	4 yd.	9 ft.	___yd.

Part 3 Copy. Find the area of each rectangle as indicated.

	Length	*Width*	*Area*
1.	15 in.	12 in.	___sq. in.
2.	21 ft.	14 ft.	___sq. ft.
3.	48 ft.	8 yd.	___sq. yd.
4.	4 ft.	20 in.	___sq. in.
5.	32 yd.	15 yd.	___sq. yd.

Part 4 Solve each of the following problems.

1. Joe and his father fenced a pen for their pony. The pen had the shape of a rectangle which was 75 yd. long and 40 yd. wide. How many yards of fence did they use?

2. If each box of floor tile covers 27 sq. ft. of floor area, how many boxes of floor tile are needed for a rectangular room which measures 18 feet by 12 feet?

3. Mr. Rose's lawn had the shape of a rectangle which was 120 feet long and 60 feet wide. He paid 2¢ per square yard to have it fertilized. How much did it cost him to fertilize his lawn?

4. A rectangle had an area of 98 sq. in. and a width of 7 inches. What was its length?

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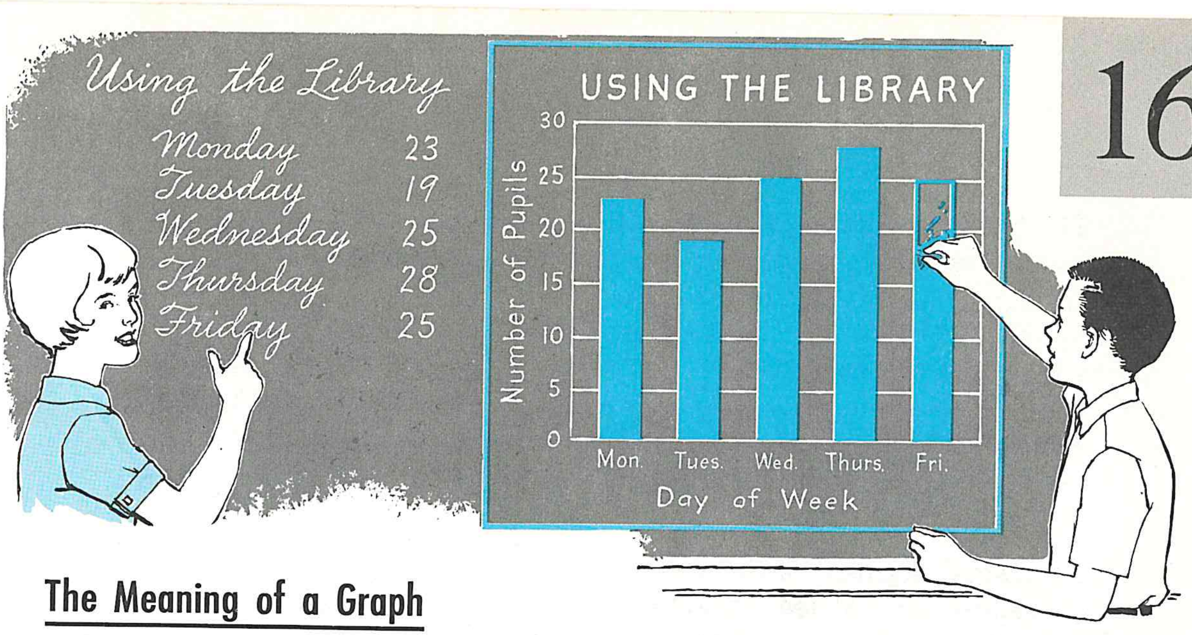


The Meaning of a Graph

Most people find it easier to get information from pictures than from written material. A photograph or a drawing may contain a great amount of information.

The fifth-grade pupils kept a record of the number of pupils who used the library each day. The record for one week is shown at the top of this page. The girl has listed the number of pupils who used the library each day. The boy has drawn a *graph* to show this information.

A **graph** is a drawing which pictures numerical relationships. It is used to give facts in visual form so that they will be easily and quickly understood.

The *title* of a graph is usually above the graph. It should tell briefly what the graph is about.

What is the title of the graph shown above?

The horizontal number line at the bottom of a graph, and the vertical number line at the left of a graph, should be labeled according to what is represented on them.

What is represented on the vertical number line at the left of the graph? What is represented on the horizontal line?

Oral Answer the following questions about the graph above.

1. On which day did the greatest number of pupils use the library?

2. On which day did the least number of pupils use the library?

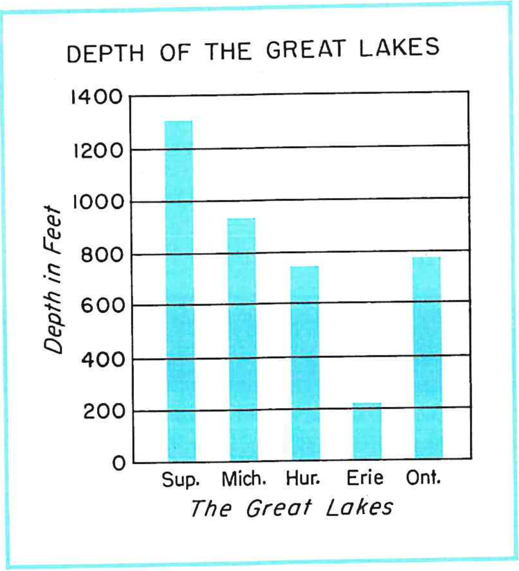
3. On which 2 days did the same number of pupils use the library?

The maximum depths of the Great Lakes are as follows: Lake Superior, 1302 ft.; Lake Michigan, 923 ft.; Lake Huron, 750 ft.; Lake Erie, 210 ft.; Lake Ontario, 778 ft.

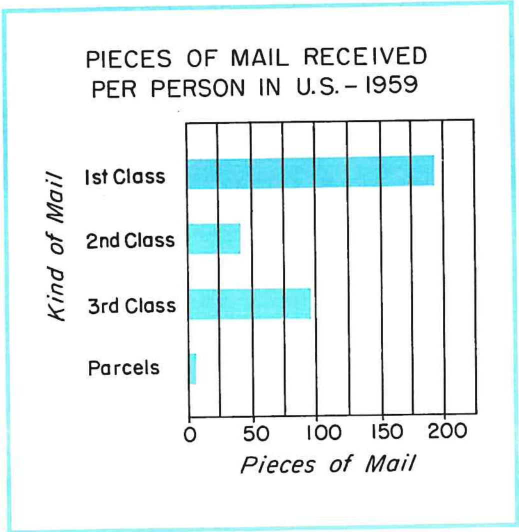
The **bar graph** below shows the same information. Why do you think it is called a *bar graph?*

Each small segment of the vertical scale represents 200 feet, beginning with 0 feet and ending with 1400 feet. Why is it unnecessary to go beyond 1400 feet?

The bar which represents the depth of Lake Superior ends about halfway between the marks for 1200 ft. and 1400 ft. Why?



What is the following bar graph about? What is its title?



In this graph the bars are horizontal rather than vertical. This is simply another way of constructing a bar graph.

What does each small segment of the horizontal scale represent? What does the top bar represent? What does the bottom bar represent?

Approximately how many pieces of 3rd-class mail did each person receive during 1959?

Can you obtain an exact number of pieces of mail from reading the graph? What is the advantage of showing information on a graph?

Oral Answer the following questions about the graph showing the depths of the Great Lakes.

1. Which of the Great Lakes has the greatest maximum depth?

2. Which two lakes have nearly the same depth?

3. Which of the lakes has the least maximum depth?

4. Lake Ontario is about how many times as deep as Lake Erie?

5. The depth of Lake Huron is about what fractional part of the depth of Lake Superior?

6. What is your guess as to the average depth of all 5 lakes?

Answer the following questions about the graph showing the pieces of mail received.

7. Each person received about how many pieces of 1st-class mail? Of 2nd-class mail?

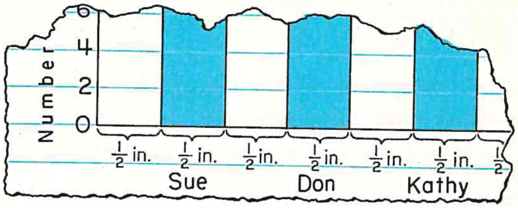
8. Each person received about how many times as much 1st-class mail as 3rd-class mail? As much 1st-class mail as 2nd-class mail?

9. About how many parcels did each person receive?

10. Did each person receive more than or less than 350 pieces of mail during the year?

Written Draw the graphs as explained below.

1. On a piece of ruled paper, draw scales as shown below.



Mark off ½-inch segments on the horizontal line and use every other segment as the bottom of one of the bars. Let the distance between two ruled lines represent 2 books.

Using these scales, graph the number of books read by the following students: Sue, 7 books; Don, 5 books; Kathy, 9 books; Ken, 4 books; and Peg, 6 books.

2. Use a horizontal scale as in problem 1, but let the distance between 2 ruled lines represent 5,000 square miles. Draw a graph of the information below.

Areas of the 6 Largest Lakes
in North America

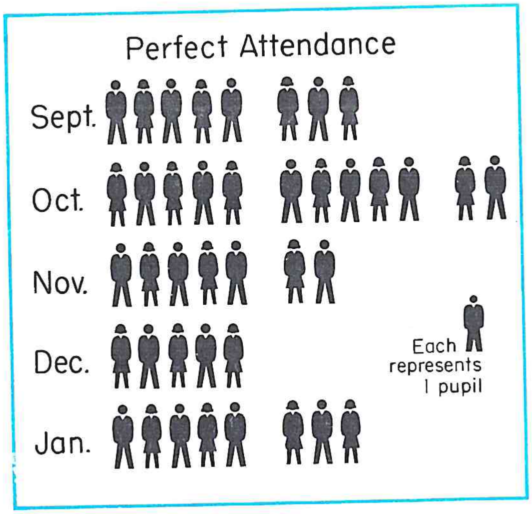
Superior	31,820 sq. mi.
Huron	23,010 sq. mi.
Michigan	22,400 sq. mi.
Great Bear	12,000 sq. mi.
Great Slave	11,170 sq. mi.
Erie	9,930 sq. mi.

Pictographs

Industry has found that a **picture graph,** or **pictograph,** catches the reader's eye and makes it easy for him to grasp the idea presented.

Instead of bars to represent the quantities, a pictograph has pictures, or symbols, of an object. Each picture or symbol stands for a certain number of things.

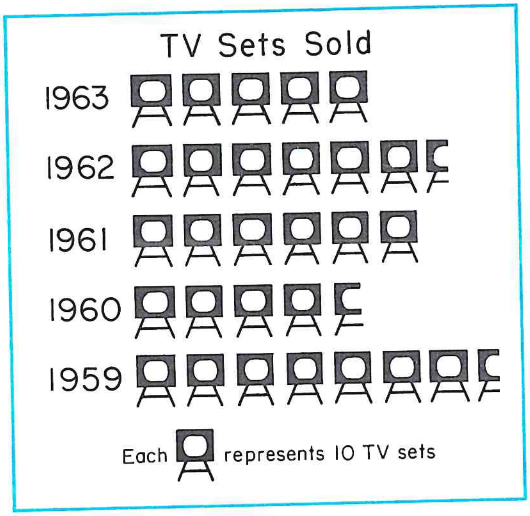
Below is a pictograph showing how many pupils had perfect attendance for the months indicated.



The symbols are grouped into sets of 5 so that it is easier to read the graph. Every pictograph should have a statement telling *how many* things and *what kind* of things each symbol represents.

Most pictographs represent large quantities and each symbol represents more than 1 thing.

The number of TV sets sold at Andy's Store from 1959 to 1963 is given in the pictograph below.



The actual sales for 1959 were 75 sets; 1960, 47 sets; 1961, 60 sets; 1962, 65 sets; and 1963, 50 sets. Note that each symbol represents 10 TV sets.

In 1962, 65 TV sets were sold. On the graph, 6 symbols and part of the seventh symbol are shown in the second row. What does the part of the symbol represent? For what other years was it necessary to use part of a symbol?

The following suggestions will be helpful in making a pictograph.

a. Select a suitable symbol. Let the symbol represent a convenient number of things.

b. Decide on a vertical scale. Leave room between the rows.

c. Arrange the correct number of symbols neatly in each row.

d. Write the title above the graph. Write a statement telling what each symbol represents.

Oral Read the following paragraph and answer the questions below.

The number of houses in Lawndale in 1950 was 650; in 1955, 725 houses; and in 1960, 1075 houses.

1. What is a suitable title for the graph of this information?

2. What would you use as a symbol, or picture, on the graph? Does your choice of a symbol alter the meaning of the graph?

3. What number of houses would you let each symbol represent?

4. If each symbol represented 100 houses, how many symbols would be needed to show the number of houses in each year?

5. How would you label the vertical scale on the graph?

Written Draw pictographs as indicated in each problem.

1. Charles listed the hours he spent doing his daily activities. Use a small clock face to represent 1 hour and draw a pictograph of his information.

Sleeping	$10\frac{1}{2}$ hrs.
Eating	3 hrs.
At school	6 hrs.
At play	$2\frac{1}{2}$ hrs.
At work	2 hrs.

2. Use the information given in problem 1 to draw a bar graph. Use the vertical scale to represent the number of hours and the horizontal scale to represent the activities.

3. The number of automobile sales in the United States from 1954 to 1959 is estimated below. Let a small drawing of an automobile represent 1 million automobiles and draw a pictograph of this information.

1954	$5\frac{7}{10}$ million
1955	$7\frac{3}{4}$ million
1956	$5\frac{3}{4}$ million
1957	6 million
1958	$4\frac{2}{5}$ million
1959	$5\frac{7}{10}$ million

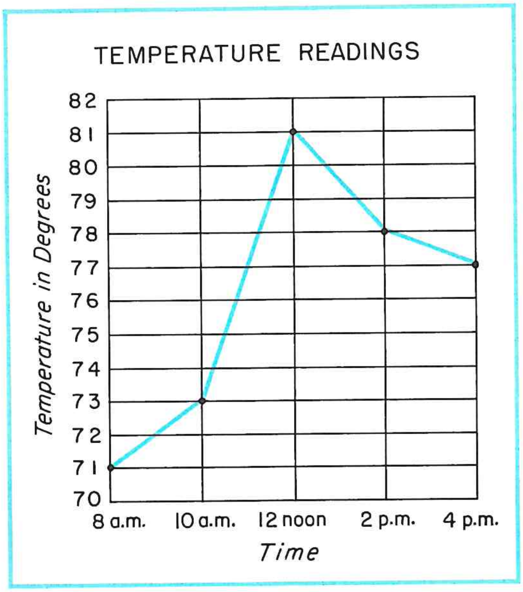
Something to do Find a bar graph and a pictograph in a newspaper or a magazine. Bring them to school for class discussion.

Line Graphs

To make quick comparisons of a number of quantities, you could use a bar graph or a pictograph. To see more accurately how one quantity changes with respect to another quantity, you will find a **line graph** most useful.

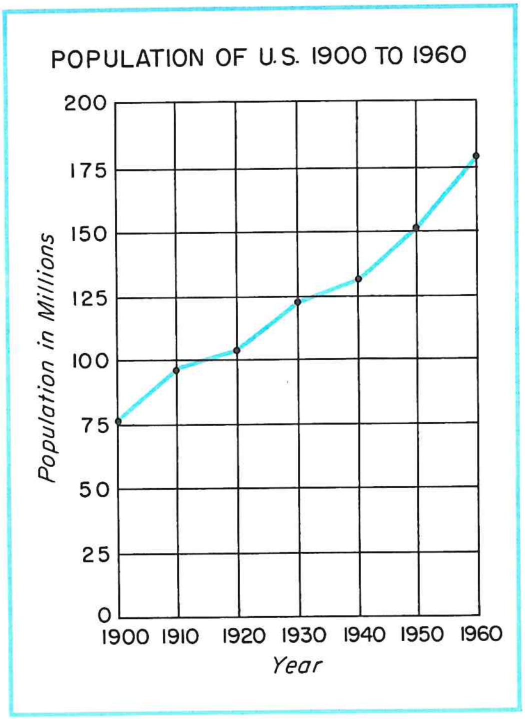
Line graphs are very important in mathematics. You can think of the two scales as parts of number lines.

The line graph below shows the temperature readings for different hours during a day.



The blue line shows how much and how fast the temperature changed with respect to time.

In the line graph below, notice that each small segment of the vertical scale represents 25 million people, and that each small segment of the horizontal scale represents a period of 10 years.



Oral Answer the following questions about the graph showing the temperature readings.

1. At what time was the temperature the highest?

2. During what 2-hour period did the temperature rise the fastest?

3. During what 2-hour period did the temperature fall most rapidly?

4. During what 2-hour period did the temperature change the least?

5. What is your estimate of the temperature at 9 a.m.? At 11 a.m.? At 1 p.m.?

6. Why is it false to say that the temperature at 10 a.m. was 3 times what it was at 8 a.m.?

Answer the following questions about the graph of the United States population.

7. During what 10-year period did the population increase the most?

8. During what 10-year period did the population increase the least?

9. Did the population decrease during any 10-year period?

10. In what year was the population double the population of 1900?

11. What is your estimate of the population in 1915? The population in 1925?

12. What is your estimate of the year in which the population reached 100 million people? The year it reached 175 million people?

13. In what year was the population approximately one-half of the population in 1960? In 1950?

Written Draw a line graph of the information given in each problem. Follow the directions given.

1. James kept the following record of his scores on achievement tests.

Test Number	Score
1	79
2	92
3	86
4	87
5	97
6	90

Let the horizontal scale show the test numbers. Let the vertical scale represent his test scores by starting with 70, then 75, then 80, and so on until you reach 100.

2. The following table tells the number of TV stations in the United States for the years given.

Year	TV Stations
1954	410
1955	437
1956	474
1957	501
1958	514

Let the horizontal scale represent the years and the vertical scale represent the number of TV stations.

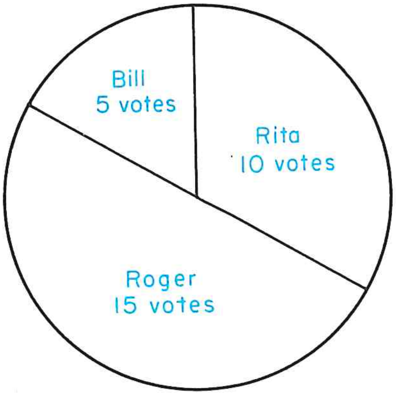
Begin the vertical scale with 400 and proceed with intervals of 20 until you reach 520.

Circle Graphs

A **circle graph** may be used when you consider several parts of one quantity. A circle graph makes it easy to compare a part with any other part or with the whole.

The following circle graph shows how 30 fifth-grade pupils cast their votes for the three candidates for room chairman.

ELECTION RESULTS

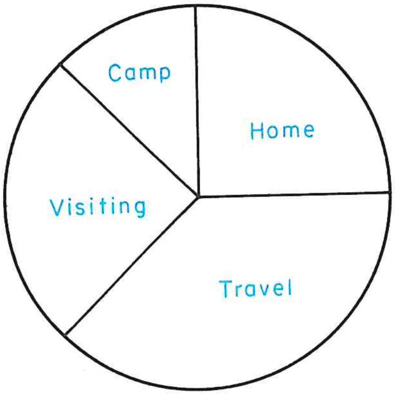


What fractional part of the total number of votes did Roger receive? Did Rita receive? Did Bill receive?

Roger received $\frac{1}{2}$ of the votes, so $\frac{1}{2}$ of the interior of the circle is labeled for him. Rita received $\frac{1}{3}$ of the votes, so $\frac{1}{3}$ of the interior is labeled for her. Bill received $\frac{1}{6}$ of the votes, so $\frac{1}{6}$ of the interior is labeled for him.

Oral Answer the questions about the graph showing how Jane spent her 40-day vacation.

JANE'S VACATION



1. How did she spend the largest part of her vacation?

2. About what part of her vacation was spent at home?

3. About how many days of her vacation were spent visiting? At camp? At home?

4. The time spent at camp was about what fractional part of the time spent at home? Of the time spent traveling?

Something to do Find a line graph and a circle graph in a newspaper or magazine. Cut them out and bring them to school for discussion.

Problem Solving

The following suggestions should enable you to solve the problems on this page by yourself.

a. Read the problem carefully. Know what is given and what you are asked to find.

b. Decide which operation or operations to use and the order in which you will use them.

c. Write a suitable equation to describe the problem.

d. Solve the equation. Use the solution in answering the question asked in the problem.

1. David's father had a cement walk laid in front of the house. The walk was 4 feet wide and 60 feet long. At 40¢ a square foot what did the walk cost?

2. A wholesale grocer bought a 2165-pound load of potatoes. He put them into bags holding 25 pounds each. How many bags were completely filled? What fractional part of the next bag was filled?

3. Marilyn's garden had the shape of a square with sides 11 feet long. The planting instructions say that each flower needs 1 square foot of area. How many flowers can she plant in her garden?

4. Mr. Swanson wanted to mail some important papers in a sealed envelope. The papers weighed 1 pound and 2 ounces. At $.04 an ounce how much postage must he pay?

5. Alice paid 10¢ to ride the bus a distance of 3 miles. At this rate what would it cost to ride the bus a distance of 51 miles?

6. A transport plane flew 5850 miles in 18 hours. What was its average speed in miles per hour?

7. Bill read that his heart should beat about 72 times each minute. How many times should his heart beat in $\frac{3}{4}$ hour?

8. The atmosphere pushes against the earth's surface with a pressure of about 15 pounds per square inch. How many pounds of pressure are there on each square foot of the earth's surface?

9. An 8-ounce bag of potato chips sold for 25¢ and a 1-pound bag sold for 48¢. How much would you save by buying 1 pound of potato chips in the 1-pound bag?

10. Mr. Gray had 8 rows of potatoes in his garden. He harvested 26 bushels from the 8 rows. What was the average number of bushels harvested from each row?

Review and Practice with Fractions

Part 1 Copy. Find each sum or difference by performing the indicated operation.

	a	*b*	*c*	*d*
1.	$\frac{3}{5}$ $+\frac{7}{10}$	$\frac{5}{8}$ $-\frac{3}{16}$	$5\frac{11}{12}$ $+7\frac{2}{3}$	$17\frac{7}{8}$ $-6\frac{1}{4}$
2.	$\frac{4}{5}$ $-\frac{7}{10}$	$3\frac{1}{3}$ $-1\frac{7}{12}$	$10\frac{3}{4}$ $-5\frac{7}{12}$	$47\frac{2}{5}$ $+9\frac{3}{4}$
3.	$\frac{3}{4}$ $\frac{5}{8}$ $+\frac{1}{2}$	$2\frac{5}{16}$ $1\frac{1}{2}$ $+\frac{3}{4}$	$7\frac{1}{6}$ $4\frac{2}{3}$ $+\frac{7}{12}$	$32\frac{7}{10}$ $16\frac{3}{5}$ $+8\frac{1}{2}$
4.	$5\frac{2}{3}$ $-\frac{11}{12}$	$4\frac{5}{12}$ $+1\frac{3}{5}$	$15\frac{3}{4}$ $-3\frac{2}{5}$	$20\frac{7}{16}$ $-9\frac{7}{8}$
5.	$\frac{5}{8}$ $+\frac{1}{5}$	$8\frac{5}{6}$ $-1\frac{7}{8}$	$7\frac{3}{5}$ $+\frac{5}{6}$	$42\frac{2}{3}$ $-18\frac{4}{7}$

Part 2 Copy. Solve each equation for n.

1. $7\frac{1}{2}-(2\frac{1}{4}+\frac{3}{8})=n$
2. $(5\frac{3}{4}-3\frac{2}{3})+1\frac{5}{6}=n$
3. $(8\frac{5}{6}+1\frac{2}{3})-7\frac{1}{2}=n$
4. $(9\frac{5}{6}-\frac{3}{4})+n=9\frac{3}{4}$
5. $(36+7\frac{3}{5})-2\frac{7}{10}=n$
6. $(75\frac{3}{8}-54)+7\frac{5}{6}=n$
7. $(112+18\frac{1}{7})-n=92\frac{1}{3}$

Part 3 Solve each of the following problems. Use a suitable equation in each solution.

1. A triangular lot had dimensions of $58\frac{1}{2}$ feet, $130\frac{3}{4}$ feet, and $145\frac{5}{6}$ feet. What was the perimeter of the lot in feet?

2. At 32¢ a gallon, how much would $\frac{5}{8}$ gallon of gasoline cost?

3. John helped his father $\frac{1}{4}$ hour before school and 20 minutes after school. What fractional part of an hour did John spend helping his father that day? How many minutes was this?

4. One gallon of paint was supposed to cover 180 square feet of surface area. A porch floor was 10 feet wide and 12 feet long. Were 3 quarts of this paint enough to paint the porch floor?

5. An experimental jet plane can travel 2448 miles per hour at full power. If it flew for 20 minutes at full power, how many miles did it travel?

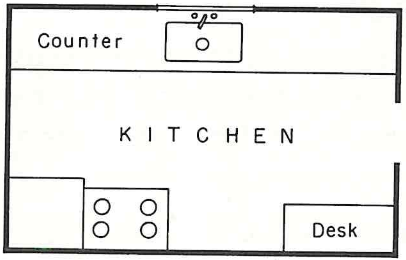
6. Bill's delivery truck can carry a load of 2 tons. He loaded $\frac{4}{5}$ ton of groceries, $\frac{1}{2}$ ton of paint, and $\frac{1}{4}$ ton of oil. What fractional part of a ton could he still load? How many pounds is this?

Scale Drawings

The construction of a dam across a river or the manufacture of a small part of a wrist watch begins as an idea in some person's mind. His idea is then put on paper in the form of a **scale drawing.**

The term *scale drawing* is used because the size of the drawing is usually smaller or larger than the size of the object. A scale is used to tell how the drawing has been reduced or enlarged.

Below is a scale drawing of the floor plan of a kitchen.



Scale: $\frac{1}{8}$ in. = 1 ft.

Below the scale drawing, the scale is given. To say $\frac{1}{8}$ in. = 1 ft. means that a length of $\frac{1}{8}$ in. on the drawing represents a length of 1 ft. on the kitchen floor.

Measure the length of the kitchen in the scale drawing. Now measure the width. The length is 2 in. and the width is $1\frac{1}{4}$ in. on the drawing.

What are the dimensions of the kitchen floor? Since $2 = \frac{16}{8}$, the length of 2 in. on the drawing represents a length of 16 ft. for the kitchen. Since $1\frac{1}{4} = \frac{10}{8}$, the length of $1\frac{1}{4}$ in. on the drawing represents a length of 10 ft. for the kitchen.

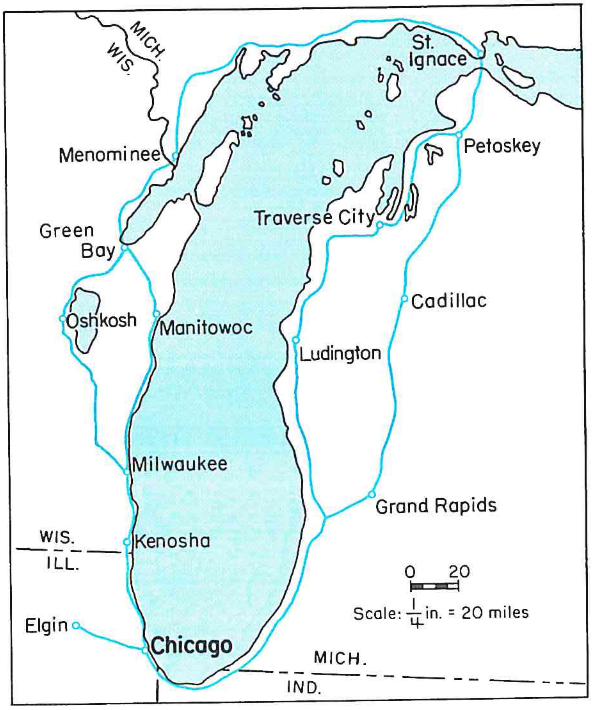
Oral Would a scale drawing of each object named below be *larger than* or *smaller than* the actual object?

1. An airplane
2. A microscopic animal
3. A flashlight bulb
4. The Statue of Liberty
5. A nerve cell

Written Measure the necessary parts of the scale drawing on this page and write the answers to the following questions.

1. How wide will the opening be for the kitchen door?
2. What will be the measure of the width of the counter?
3. What will be the measure of the length of the desk?
4. What will be the measure of the perimeter of the kitchen?
5. What will be the measure of the width of the kitchen window?

A Map Is a Scale Drawing



A *map* is a picture of a part of the earth's surface. A map must be drawn to scale in order to be accurate and useful. The scale of a map shows how much of the actual earth's surface is represented by a given unit measure on the map.

The scale on the map above is given as $\frac{1}{4}$ inch = 20 miles, which means that a distance of $\frac{1}{4}$ inch on the map represents a distance of 20 miles on the earth's surface.

According to this scale, what numerals should replace the question marks in the following table?

Inches on Map	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	?	$1\frac{1}{4}$	?
Miles on Earth	20	?	?	80	?	120

A map is drawn on a plane surface, but the map represents a part of the earth's surface which is not a plane. For this reason, a map gives only approximate distances when read as a scale drawing.

The distances given on a road map are not obtained from scale measuring but from actual road measuring with a specially equipped car.

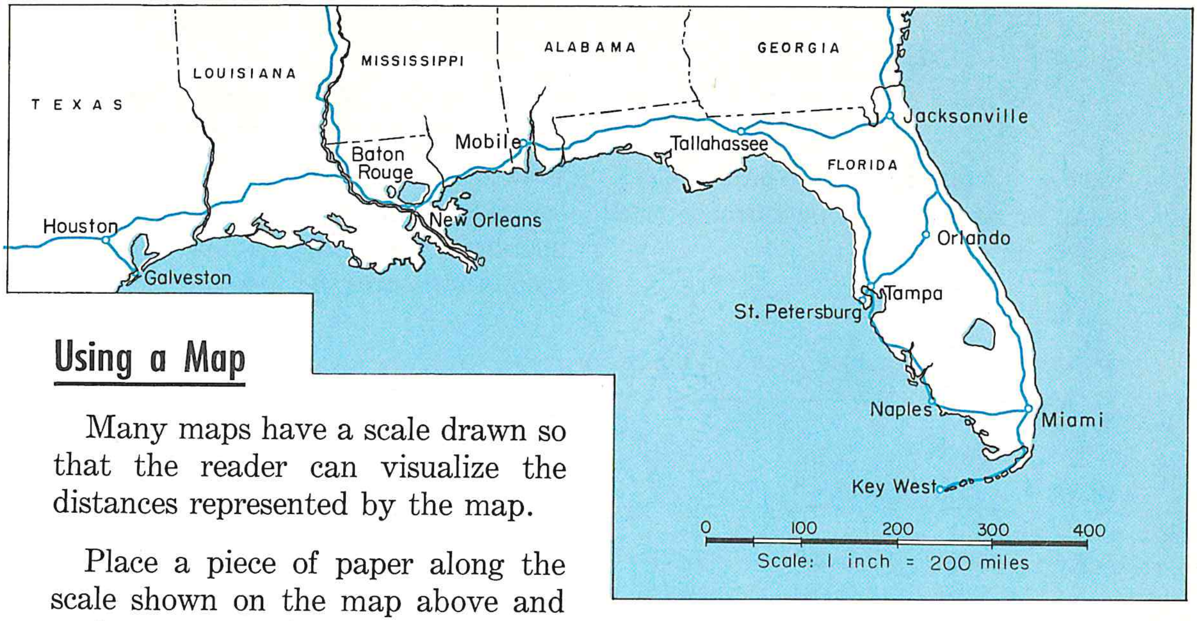
Oral Use the map to give approximate answers for these questions.

1. On the map Elgin and Chicago are shown $\frac{3}{8}$ in. apart. How many miles apart are these two cities?

2. If the distance between two cities is 180 miles, what distance should be between their locations on the map?

Written Make the necessary measurements on the map. Tell the approximate distance in miles between the following cities.

1. Chicago and Traverse City
2. Manitowoc and Ludington
3. Chicago and St. Ignace
4. Grand Rapids and Traverse City
5. Menominee and Grand Rapids
6. Oshkosh and Green Bay
7. Petoskey and Cadillac



Using a Map

Many maps have a scale drawn so that the reader can visualize the distances represented by the map.

Place a piece of paper along the scale shown on the map above and make a copy of the scale. Check your scale to be sure that the marks for successive 100-mile intervals are $\frac{1}{2}$ inch apart.

Use the scale you have made to find the approximate answers for the following problems.

1. Mr. Moore and his family drove from Mobile to Tallahassee. How many miles did they drive?

2. What is the total distance by air from Houston to New Orleans and return?

3. Mrs. Esser drove from Baton Rouge to New Orleans and then drove to Mobile. How many miles did she drive altogether?

4. A family went by yacht from St. Petersburg to Key West. How many miles did they travel?

5. Andrew had an airline ticket for the trip from Miami to Houston which included stops at Tampa and New Orleans. How many miles did he travel on this trip?

6. A bus made 3 round trips between Houston and Galveston on Sunday. The bus traveled an average of 10 miles on each gallon of fuel. How many gallons of fuel did the bus use on Sunday?

7. A navy ship took 48 hours to travel from Key West to Galveston. What was its average speed?

8. How much farther is it from Tallahassee to Tampa than it is from Tallahassee to Jacksonville?

9. How many miles longer is the trip from Naples to Key West by highway instead of by water?

Review and Practice

Part 1 Copy. Find each sum, difference, or product by performing the indicated operation.

	a	*b*	*c*
1.	5146 +2738	8002 −4375	6035 −4723
2.	7286 ×7	5607 ×36	356 ×718
3.	4268 159 3746 +518	5647 2578 4533 +7386	7875 426 15 +6359
4.	5726 −4937	3070 −708	2600 ×78

Part 2 Copy. Complete each division and check the result.

	a	*b*
1.	$17\overline{)833}$	$60\overline{)7500}$
2.	$91\overline{)4459}$	$24\overline{)7560}$
3.	$46\overline{)2944}$	$57\overline{)35454}$
4.	$76\overline{)5232}$	$72\overline{)28368}$
5.	$31\overline{)1581}$	$45\overline{)16380}$
6.	$63\overline{)6147}$	$15\overline{)12089}$
7.	$63\overline{)9702}$	$47\overline{)70030}$

Part 3 Copy. Find a replacement for each blank so that the sentences become true. (Remember that = means *measures the same as.*)

	a	*b*
1.	1 mi. = ___ ft.	1 sq. ft. = ___ sq. in.
2.	7 ft. = ___ in.	1 sq. yd. = ___ sq. ft.
3.	$2\frac{1}{2}$ ft. = ___ in.	$\frac{1}{2}$ sq. ft. = ___ sq. in.
4.	$3\frac{1}{2}$ gal. = ___ qt.	3 sq. yd. = ___ sq. ft.
5.	$1\frac{1}{4}$ hr. = ___ min.	18 da. = ___ wk.
6.	$\frac{5}{6}$ yd. = ___ in.	54 sq. ft. = ___ sq. yd.
7.	$\frac{1}{2}$ mi. = ___ ft.	$2\frac{2}{3}$ sq. yd. = ___ sq. ft.
8.	$3\frac{1}{4}$ bu. = ___ pk.	10 sq. ft. = ___ sq. in.
9.	$2\frac{1}{2}$ hr. = ___ min.	72 sq. in. = ___ sq. ft.
10.	36 hr. = ___ da.	3000 lb. = ___ T.

Part 4 Copy. Solve for n.

	a	*b*
1.	$(5\times4)\div2=n$	$36\times(17-9)=n$
2.	$(7-7)\times5=n$	$72\div(27+9)=n$
3.	$3+(6\times14)=n$	$(84\div12)+n=14$
4.	$(9\times3)\div3=n$	$n+(92\div4)=25$
5.	$n-(7\times5)=0$	$(72\div9)\div2=n$
6.	$(3+5)\times n=30$	$(48-18)\div n=6$

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. A graph is a drawing which shows numerical relationships. (265)

2. The title of a graph tells what the graph is about. (265)

3. In order for a graph to be meaningful the title should be given and both scales should be labeled. (265)

4. A symbol on a pictograph may represent 1 or several objects. (268)

5. A line graph shows how one quantity changes in relationship to some other quantity. (270)

6. Scale drawings are useful in construction, manufacturing, and travel. (275)

Words to Know

1. A graph (265)

2. Bar graph (266)

3. Pictograph (268)

4. Picture graph (268)

5. Line graph (270)

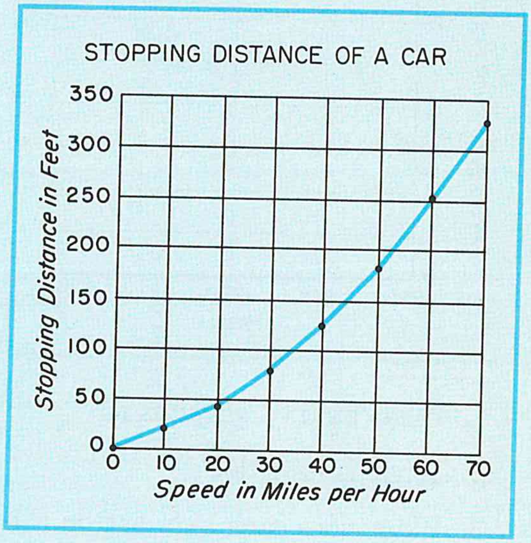
6. Circle graph (272)

7. Scale drawing (275)

8. A map (276)

Questions to Discuss

The graph below relates the stopping of a car with good brakes, riding on good pavement, to its speed.



1. What is its title? (265)

2. What kind of graph is it? (270)

3. What does each segment of the vertical scale represent? (270)

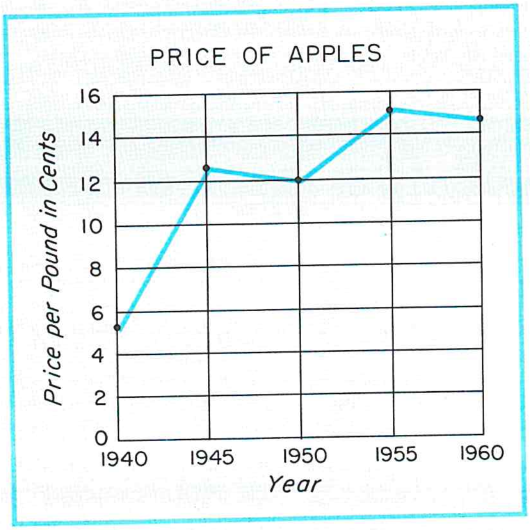
Written Practice

1. Use the information given in the graph above to construct a bar graph. (266, 267)

2. What is meant by "Scale: $\frac{1}{2}$ inch = 60 miles?" (275, 276)

Self-Evaluation

Part 1 Study the graph. Write the answers to the questions below.



1. What kind of graph is it?

2. What is its title?

3. What does each segment on the vertical scale represent?

4. What does each segment of the horizontal scale represent?

5. In which year was the price of apples the highest? The lowest?

6. In which 5-year period did the price increase the most?

7. What is your estimate of the price of apples in 1942?

8. In what year was the price of apples about 14¢ per pound?

Part 2 Make a bar graph of the information given in the following table. Use these suggestions.

Let the horizontal scale represent the names of the days and the vertical scale represent the test scores.

Remember to write a title above the graph and to label both the vertical and the horizontal scales.

Stan's Test Scores

Monday	15
Tuesday	12
Wednesday	18
Thursday	14
Friday	20

Part 3 Suppose you were using a map which was drawn to a scale of $\frac{1}{2}$ inch = 100 miles.

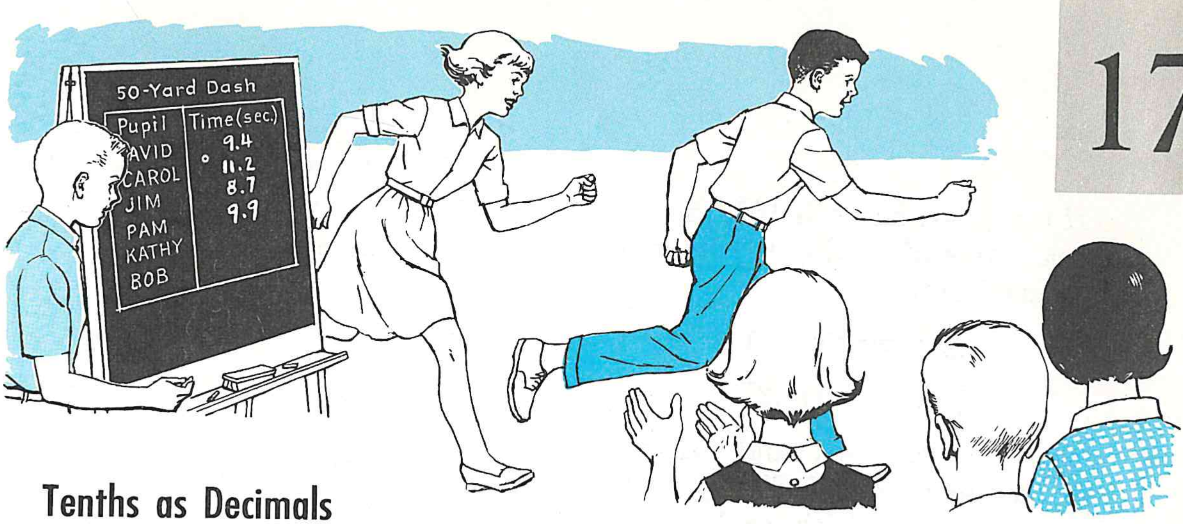
Write the answers to the following questions.

1. How many miles would be represented by a line segment 1 inch long on the map? A line segment $\frac{1}{4}$ inch long on the map? A line segment $1\frac{1}{4}$ inches long on the map?

2. If two towns are 300 miles apart, how far apart should they be shown on the map?

3. What does it mean to say "Scale: $\frac{1}{2}$ in. = 1 mi."?

17



Tenths as Decimals

You have learned that our decimal system is based on the number 10. The ten numerals, 0 through 9, have enabled you to express any whole number and any fractional number.

A special kind of fraction, called a decimal, has advantages over the ordinary fraction.

If the denominator is 10, write the fraction as shown below.

$\frac{1}{10}$ is written .1

$\frac{7}{10}$ is written .7

The new symbols, .1 and .7, are called **decimal numerals** or **decimals.** The dot in front of the 1 and the 7 is called **the decimal point.**

The symbols, $\frac{1}{10}$ and .1, are both read *one tenth*. The symbols, $\frac{7}{10}$ and .7, are both read *seven tenths*. One digit to the *right* of the decimal point denotes a fraction whose denominator is 10.

Mixed fractions can also be expressed in decimal form.

$2\frac{1}{10}$ is written 2.1

$3\frac{7}{10}$ is written 3.7

The symbol 2.1 means *two and one tenth*. What does 3.7 mean?

The numerals to the *left* of the decimal point name the *whole* number. The numerals to the *right* of the decimal point name a *fractional* number less than 1.

Oral In the picture above, how long did it take each pupil to run the 50-yard dash?

Written Write each of the following numbers in decimal form.

	a	*b*	*c*	*d*	*e*
1.	$\frac{5}{10}$	$\frac{3}{10}$	$\frac{9}{10}$	$\frac{0}{10}$	$\frac{4}{10}$
2.	$1\frac{2}{10}$	$1\frac{7}{10}$	$5\frac{3}{10}$	$7\frac{0}{10}$	$\frac{8}{10}$
3.	$2\frac{9}{10}$	$11\frac{1}{10}$	$10\frac{3}{10}$	$\frac{23}{10}$	$\frac{45}{10}$

Hundredths as Decimals

If the denominator is 100, there is an easy way of writing the fraction in decimal form.

$\frac{1}{100}$ is written .01

$\frac{27}{100}$ is written .27

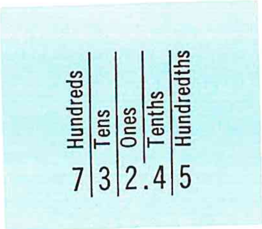
The symbols, $\frac{1}{100}$ and .01, are both read *one hundredth.* The symbols, $\frac{27}{100}$ and .27, are both read *twenty-seven hundredths.*

The numeral 7.13 means *seven and thirteen hundredths* or 7+.13. The decimal point is read as *and* in decimals naming numbers larger than 1.

When writing fractions as decimals, you use one place to the right of the decimal point when the denominator is 10, and you use two places to the right of the decimal point when the denominator is 100.

The place value of tenths and hundredths in the decimal system is shown below.

MORE PRACTICE PAGE 317



The decimal point is always placed between the ones and the tenths.

Oral Read each of the following decimal numerals.

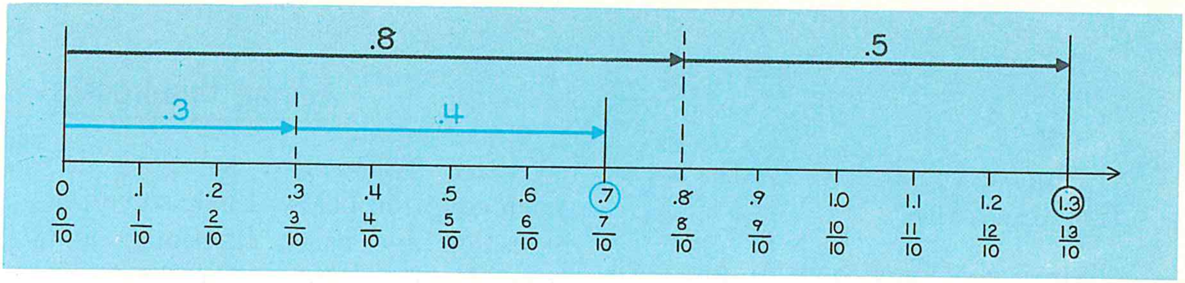
	a	*b*	*c*	*d*
1.	.7	1.3	.07	9.14
2.	.5	3.4	.53	7.01
3.	.3	10.7	.96	21.33
4.	.9	51.6	.44	73.67
5.	8.0	2.22	.13	652.8

Written Write each of the following fractions as decimals.

	a	*b*	*c*	*d*
1.	$\frac{7}{10}$	$\frac{13}{100}$	$5\frac{1}{100}$	$1\frac{11}{100}$
2.	$3\frac{3}{10}$	$\frac{76}{100}$	$36\frac{21}{100}$	$17\frac{49}{100}$
3.	$\frac{31}{10}$	$\frac{99}{100}$	$147\frac{9}{100}$	$80\frac{3}{100}$
4.	$\frac{71}{10}$	$6\frac{6}{10}$	$\frac{121}{100}$	$\frac{826}{100}$

Write each of the following decimal numerals as fractions or as mixed fractions.

	a	*b*	*c*	*d*
5.	.3	1.47	.17	5.41
6.	7.7	15.9	.03	17.09
7.	.1	36.51	1.01	86.73
8.	1.7	726.3	5.78	250.11
9.	.21	99.99	7.60	30.3



Adding Tenths

Ralph read that .3 of an inch of rain fell on Saturday and .4 of an inch fell on Sunday. What was the total rainfall for the two days?

To find the total rainfall, add the measures of each rainfall. The equation would be $.3+.4=n$.

If the decimals were written as fractions, the equation would be as shown below.

$$\frac{3}{10}+\frac{4}{10}=\frac{3+4}{10}=\frac{7}{10}$$

This addition is shown by the blue arrows on the number line.

$$.3+.4=.7$$

The results show that the total rainfall was $\frac{7}{10}$ or .7 of an inch.

Suppose you were to solve the equation $.8+.5=n$. If these decimals were written as fractions, the equation would be as shown below.

$$\frac{8}{10}+\frac{5}{10}=\frac{8+5}{10}=\frac{13}{10}=1\frac{3}{10}$$

This addition is shown by the gray arrows on the number line.

$$.8+.5=1.3$$

Since fractions may be written as decimals, the arithmetic of decimals should be in full agreement with the arithmetic of fractions.

$$\begin{array}{r} \frac{8}{10} \\ +\frac{5}{10} \\ \hline \frac{13}{10}=1\frac{3}{10} \end{array} \qquad \begin{array}{r} 8 \text{ tenths} \\ +5 \text{ tenths} \\ \hline 13 \text{ tenths} \end{array} \qquad \begin{array}{r} .8 \\ +.5 \\ \hline 1.3 \end{array}$$

Oral How would you write each of the following equations by changing the decimals to fractions?

	a	*b*
1.	$.2+.3=n$	$.4+.5=n$
2.	$.7+.6=n$	$.3+.8=n$

Written Write two solutions for each equation. First, solve the equation by using fractions; second, solve it by using decimals.

	a	*b*
1.	$.5+.2=n$	$.6+.3=n$
2.	$.6+.9=n$	$.4+.9=n$
3.	$.9+.7=n$	$.8+.9=n$

MORE PRACTICE PAGE 317

Adding Hundredths

Suppose you are to solve the equation .32+.25=n.

Since you know how to add with fractions, solve the equation first in fractional form.

$$\frac{32}{100}+\frac{25}{100}=\frac{32+25}{100}=\frac{57}{100}$$

This shows that .32+.25=.57.

When showing this addition in column form, write the decimals in such a way that all the decimal points lie on a vertical line. This keeps each digit in its proper place-value position.

$\begin{array}{r} \frac{46}{100} \\ +\frac{72}{100} \\ \hline \frac{118}{100}=1\frac{18}{100} \end{array}$	$\begin{array}{r} 46 \text{ hundredths} \\ +72 \text{ hundredths} \\ \hline 118 \text{ hundredths} \end{array}$	$\begin{array}{r} .46 \\ +.72 \\ \hline 1.18 \end{array}$

You added hundredths when you solved problems involving dollars and cents. One cent or $.01 is another way of writing .01 or $\frac{1}{100}$ of a dollar.

Oral How would you write each of the following additions in fractional form?

	a	*b*
1.	.07+.12=n	.72+.42=n
2.	$\begin{array}{r} .36 \\ +.17 \\ \hline \end{array}$	$\begin{array}{r} .58 \\ +.83 \\ \hline \end{array}$

Written Write two solutions for each equation below. First, solve the equation by using fractions; and second, solve the equation by using decimals.

	a	*b*
1.	.04+.61=n	.75+.05=n
2.	.35+.56=n	.95+.17=n
3.	.42+.37=n	.87+.29=n
4.	.63+.57=n	.53+.48=n
5.	.98+.36=n	.44+.39=n

Copy. Find each sum in decimal form.

	a	*b*	*c*	*d*
6.	$\begin{array}{r} .05 \\ +.08 \\ \hline \end{array}$	$\begin{array}{r} .32 \\ +.15 \\ \hline \end{array}$	$\begin{array}{r} .47 \\ +.28 \\ \hline \end{array}$	$\begin{array}{r} .57 \\ +.63 \\ \hline \end{array}$
7.	$\begin{array}{r} .72 \\ +.67 \\ \hline \end{array}$	$\begin{array}{r} .39 \\ +.65 \\ \hline \end{array}$	$\begin{array}{r} .87 \\ +.67 \\ \hline \end{array}$	$\begin{array}{r} .54 \\ +.29 \\ \hline \end{array}$
8.	$\begin{array}{r} .64 \\ +.36 \\ \hline \end{array}$	$\begin{array}{r} .29 \\ +.76 \\ \hline \end{array}$	$\begin{array}{r} .72 \\ +.28 \\ \hline \end{array}$	$\begin{array}{r} .96 \\ +.12 \\ \hline \end{array}$
9.	$\begin{array}{r} .76 \\ +.06 \\ \hline \end{array}$	$\begin{array}{r} .39 \\ +.48 \\ \hline \end{array}$	$\begin{array}{r} .18 \\ +.78 \\ \hline \end{array}$	$\begin{array}{r} .08 \\ +.59 \\ \hline \end{array}$
10.	$\begin{array}{r} .82 \\ +.75 \\ \hline \end{array}$	$\begin{array}{r} .56 \\ +.65 \\ \hline \end{array}$	$\begin{array}{r} .67 \\ +.33 \\ \hline \end{array}$	$\begin{array}{r} .92 \\ +.08 \\ \hline \end{array}$
11.	$\begin{array}{r} .47 \\ +.64 \\ \hline \end{array}$	$\begin{array}{r} .83 \\ +.99 \\ \hline \end{array}$	$\begin{array}{r} .32 \\ +.19 \\ \hline \end{array}$	$\begin{array}{r} .30 \\ +.76 \\ \hline \end{array}$

Adding Tenths and Hundredths

The equation, $.4+.37=n$, is solved first in fractional form and then in decimal form below.

$$.4+.37=n$$

$$\frac{4}{10}+\frac{37}{100}=\frac{40}{100}+\frac{37}{100}=\frac{77}{100}$$

$$.4+.37=.40+.37=.77$$

The fractional form shows that $\frac{4}{10}$ was written as $\frac{40}{100}$. Then the fractions $\frac{40}{100}$ and $\frac{37}{100}$ had a common denominator and the addition could be completed.

In decimal form, this means that .4 was written as .40. Does $.4=.40$? Does $\frac{4}{10}=\frac{40}{100}$? How do you know that it does?

Does $\frac{7}{10}=\frac{70}{100}$? Why? Does $.7=.70$? Does $.30=.3$?

If the addition of .4 and .37 is written in column form, it is not necessary to write .4 as .40. Be sure to write the numerals so that all of the decimal points lie on the same vertical line.

$$\begin{array}{r}.40\\+.37\\\hline .77\end{array}\quad\text{or}\quad\begin{array}{r}.4\\+.37\\\hline .77\end{array}$$

Oral Answer the following questions.

1. How would you express .2 as hundredths?

2. How would you express .60 as tenths?

3. How would you solve $.29+.6=n$ in fractional form? In decimal form?

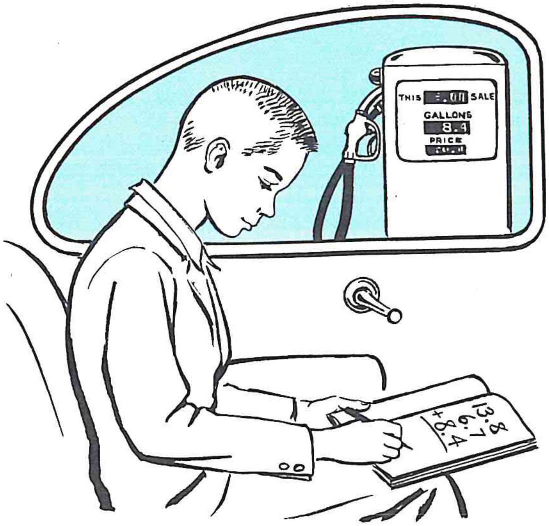
Written Solve each of the following equations by first writing it in fractional form and then writing it in decimal form.

	a	*b*
1.	$.18+.5=n$	$.9+.82=n$
2.	$.07+.6=n$	$.2+.38=n$
3.	$.96+.5=n$	$.7+.48=n$
4.	$.11+.7=n$	$.9+.72=n$

Copy. Find each sum.

	a	*b*	*c*	*d*
5.	$\begin{array}{r}.43\\+.30\\\hline\end{array}$	$\begin{array}{r}.26\\+.8\\\hline\end{array}$	$\begin{array}{r}.43\\+.9\\\hline\end{array}$	$\begin{array}{r}.82\\+.26\\\hline\end{array}$
6.	$\begin{array}{r}.7\\+.27\\\hline\end{array}$	$\begin{array}{r}.6\\+.53\\\hline\end{array}$	$\begin{array}{r}.8\\+.05\\\hline\end{array}$	$\begin{array}{r}.70\\+.55\\\hline\end{array}$
7.	$\begin{array}{r}.92\\+.40\\\hline\end{array}$	$\begin{array}{r}.3\\+.57\\\hline\end{array}$	$\begin{array}{r}.87\\+.56\\\hline\end{array}$	$\begin{array}{r}.6\\+.99\\\hline\end{array}$
8.	$\begin{array}{r}.77\\+.7\\\hline\end{array}$	$\begin{array}{r}.38\\+.8\\\hline\end{array}$	$\begin{array}{r}.80\\+.7\\\hline\end{array}$	$\begin{array}{r}.05\\+.5\\\hline\end{array}$
9.	$\begin{array}{r}.60\\+.07\\\hline\end{array}$	$\begin{array}{r}.97\\+.09\\\hline\end{array}$	$\begin{array}{r}.79\\+.8\\\hline\end{array}$	$\begin{array}{r}.36\\+.98\\\hline\end{array}$

Addition with Decimals



Bob's father asked him to keep a record of the amount of gasoline they bought on their trip. At the end of the first day Bob had listed 13.8 gallons, 6.7 gallons, and 8.4 gallons. How many gallons of gasoline did they buy that day?

$$\begin{array}{r}13.8\\6.7\\+8.4\\\hline 28.9\end{array}$$

The rules for adding whole numbers may be used for fractional numbers named by decimals. See example.

Add for the tenths column first. That is, .8+.7+.4=1.9. Write .9 in the tenths column, and use the 1 as an addend for the ones column. Then add the whole numbers as usual.

They bought 28.9 gallons of gasoline the first day.

Study additions A and B. Notice that the numerals are written so that each digit is in its proper place-value position. Also notice that the addition with decimals is like the addition of whole numbers.

$$\begin{array}{rr}\textbf{A} & \textbf{B}\\ 32.6 & 584.26\\ 26.4 & 473.79\\ +7.7 & +67.83\\ \hline 66.7 & 1125.88\end{array}$$

Write the addends of additions A and B on your paper. Do the additions and compare your answers.

Oral Give the answer to each addition below. Do the work mentally.

	a	*b*	*c*
1.	3.2 +.8	4.71 +.20	20.3 +9.4
2.	7.02 +5.07	50.21 +5.06	72.7 +7.3
3.	9.2 +.37	32.3 +8.05	75.0 +3.12
4.	67.0 +.4	38.01 +.67	15.04 +5.2
5.	59.2 +8.9	38.16 +20.45	74.37 +3.40

Written Copy. Find each sum.

	a	*b*	*c*
1.	89.7 +6.8	321.7 +58.4	231.42 +82.37
2.	3.86 9.42 +1.33	85.27 8.16 +20.05	3.18 41.57 +53.09
3.	7.6 5.37 +.52	49.2 78.76 +9.42	7.09 55.6 +8.87
4.	.27 +8.99	89.75 +34.92	136.2 +82.9
5.	37.45 +16.82	385.76 +72.93	726.98 +97.05
6.	3.97 .50 7.03 +.68	429.51 38.7 357.09 +49.27	597.02 406.9 721.54 +80.08

Solve the following problems. Read each problem carefully before doing the computation.

7. A surveyor measured the lengths of the sides of a city building lot. The sides of the lot measured 82.3 feet, 251.26 feet, 94.7 feet, and 196.78 feet. What was the perimeter of the lot?

8. Randy deposited $3.72 in his saving account during January, $4.24 during February, and $3.93 during March. What was the total amount deposited in the 3 months?

9. What is the combined thickness of three pieces of plastic measuring .17 inch, .29 inch, and .08 inch in thickness?

10. The sides of a triangle had measures of 1.75 inches, 2.5 inches, and 3.38 inches. What was its perimeter?

11. An automobile company tested one of its new cars by taking it on three test drives which were 1729.2 miles, 3281.7 miles, and 2957.3 miles in length. How many miles was the car driven on the three tests?

A quick review Write each of the following as decimals.

1. Two and three tenths
2. Sixty-five hundredths
3. Fourteen and nine tenths
4. Ten and thirteen hundredths
5. Forty-five and one tenth
6. Eighty and six tenths
7. Nine hundredths
8. One and eight hundredths
9. One hundred and one tenth
10. Seventy and seven tenths
11. Thirty and three tenths
12. Twenty-two hundredths

MORE PRACTICE PAGE 317

Subtracting Tenths and Hundredths

Pam ran the 50-yard dash in 9.9 seconds and Jim ran it in 8.7 seconds. How much longer did it take Pam than Jim?

Since you are to find the difference between the two lengths of time, the equation is $9.9-8.7=n$. The subtraction is shown below in fractional form and decimal form.

$$\begin{array}{r} 9\frac{9}{10} \\ -8\frac{7}{10} \\ \hline 1\frac{2}{10} \end{array} \qquad\qquad \begin{array}{r} 9.9 \\ -8.7 \\ \hline 1.2 \end{array}$$

Study the subtraction shown below. Compare the decimal form with the fractional form.

$$\begin{array}{rcr} 7\frac{3}{10} & = & 6\frac{13}{10} \\ -4\frac{6}{10} & = & 4\frac{6}{10} \\ \cline{1-1}\cline{3-3} & & 2\frac{7}{10} \end{array} \qquad\qquad \begin{array}{r} 7.3 \\ -4.6 \\ \hline 2.7 \end{array}$$

It is not possible to subtract .6 from .3. Think of 7.3 as 6 and 13 tenths. Then subtract as you did with whole numbers.

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Study the subtractions below.

$$\begin{array}{rcr} 6\frac{7}{100} & = & 5\frac{107}{100} \\ -5\frac{23}{100} & = & 5\frac{23}{100} \\ \cline{1-1}\cline{3-3} & & \frac{84}{100} \end{array} \qquad\qquad \begin{array}{r} 6.07 \\ -5.23 \\ \hline .84 \end{array}$$

Oral Tell the answer for each subtraction below. Do the computations mentally.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} .9 \\ -.6 \\ \hline \end{array}$	$\begin{array}{r} 7.2 \\ -.5 \\ \hline \end{array}$	$\begin{array}{r} .78 \\ -.45 \\ \hline \end{array}$	$\begin{array}{r} 7.12 \\ -.08 \\ \hline \end{array}$
2.	$\begin{array}{r} .7 \\ -.1 \\ \hline \end{array}$	$\begin{array}{r} 3.4 \\ -.9 \\ \hline \end{array}$	$\begin{array}{r} .36 \\ -.16 \\ \hline \end{array}$	$\begin{array}{r} 3.24 \\ -3.18 \\ \hline \end{array}$
3.	$\begin{array}{r} 8.9 \\ -.6 \\ \hline \end{array}$	$\begin{array}{r} 7.4 \\ -2.4 \\ \hline \end{array}$	$\begin{array}{r} .41 \\ -.38 \\ \hline \end{array}$	$\begin{array}{r} 9.57 \\ -.57 \\ \hline \end{array}$

Written Copy. Find the difference in each subtraction.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 4.2 \\ -2.6 \\ \hline \end{array}$	$\begin{array}{r} 64.8 \\ -42.9 \\ \hline \end{array}$	$\begin{array}{r} 8.27 \\ -5.05 \\ \hline \end{array}$	$\begin{array}{r} 85.27 \\ -36.42 \\ \hline \end{array}$
2.	$\begin{array}{r} 9.2 \\ -6.4 \\ \hline \end{array}$	$\begin{array}{r} 20.5 \\ -19.8 \\ \hline \end{array}$	$\begin{array}{r} 4.02 \\ -3.75 \\ \hline \end{array}$	$\begin{array}{r} 89.75 \\ -14.57 \\ \hline \end{array}$
3.	$\begin{array}{r} 6.3 \\ -3.5 \\ \hline \end{array}$	$\begin{array}{r} 82.3 \\ -34.7 \\ \hline \end{array}$	$\begin{array}{r} 5.15 \\ -4.07 \\ \hline \end{array}$	$\begin{array}{r} 80.01 \\ -19.98 \\ \hline \end{array}$
4.	$\begin{array}{r} 6.5 \\ -.7 \\ \hline \end{array}$	$\begin{array}{r} 32.6 \\ -24.4 \\ \hline \end{array}$	$\begin{array}{r} 9.87 \\ -6.49 \\ \hline \end{array}$	$\begin{array}{r} 46.15 \\ -37.26 \\ \hline \end{array}$

Rewrite the following in column form and find each difference.

	a	*b*
5.	$7.2-6.3=n$	$3.27-1.09=n$
6.	$73.7-56.8=n$	$121.04-98.98=n$

Subtraction with Decimals

The weight of the instruments in the satellite Vanguard II was 20.74 pounds and in Explorer VII it was 91.5 pounds. What is the difference between these weights?

To find the difference between the weights, subtract their measures as shown below.

$$\begin{array}{r} 91.5 \\ -20.74 \\ \hline 70.76 \end{array} \qquad \text{or} \qquad \begin{array}{r} 91.50 \\ -20.74 \\ \hline 70.76 \end{array}$$

Since $.5 = .50$, think of 91.5 as 91.50 and subtract as you would with whole numbers. Write the 0 in the minuend if this makes the subtraction easier for you.

If the minuend or subtrahend is a whole number, it is helpful to express it as a decimal. Since $3 = 3\frac{0}{10}$, you can say that $3 = 3.0$. Likewise, $17 = 17\frac{0}{100}$, so $17 = 17.00$.

Study the following subtractions. Be ready to explain each of them.

$$\begin{array}{r} 27.36 \\ -19.5 \\ \hline 7.86 \end{array} \qquad \begin{array}{r} 72.5 \\ -36. \\ \hline 36.5 \end{array} \qquad \begin{array}{r} 84. \\ -69.73 \\ \hline 14.27 \end{array}$$

Why is it important to write the numerals so that the decimal points lie on the same vertical line? How do you know where to place the decimal point in the answer?

Oral Express each of the following decimals as a whole number and tenths.

	a	*b*	*c*
1.	5	6.70	90.90
2.	26	18.20	576.10

Express each of the following decimals as a whole number and hundredths.

	a	*b*	*c*
3.	7	26.6	367.8
4.	13	87.3	921.4

Written Copy. Find the difference in each subtraction.

	a	*b*	*c*
1.	$\begin{array}{r} 92.3 \\ -41.25 \\ \hline \end{array}$	$\begin{array}{r} 59. \\ -42.8 \\ \hline \end{array}$	$\begin{array}{r} 157.2 \\ -87.49 \\ \hline \end{array}$
2.	$\begin{array}{r} 87. \\ -53.6 \\ \hline \end{array}$	$\begin{array}{r} 67.3 \\ -49. \\ \hline \end{array}$	$\begin{array}{r} 375. \\ -269.3 \\ \hline \end{array}$
3.	$\begin{array}{r} 47.98 \\ -28.7 \\ \hline \end{array}$	$\begin{array}{r} 56.04 \\ -9.3 \\ \hline \end{array}$	$\begin{array}{r} 2004. \\ -876.2 \\ \hline \end{array}$

Rewrite the following in column form and find each difference.

	a	*b*
4.	$15 - 9.6 = n$	$281.2 - 47.54 = n$
5.	$32.21 - 19 = n$	$1205.7 - 32.73 = n$

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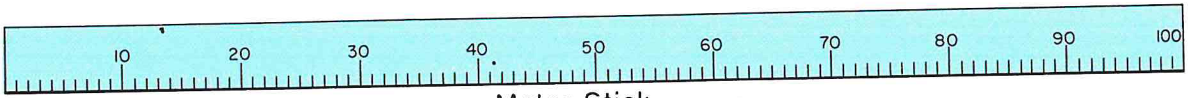
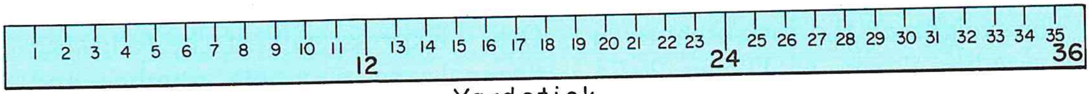


Meter Stick



Yardstick

Solving Problems with Decimals

Read each problem carefully. Decide what it tells, what it asks, and how you will find the answer. Write the equation. Then solve the problem.

1. During August the average speed of the wind in Honolulu is 14 miles per hour. During September it is 11.6 miles per hour. What is the difference between these two average speeds?

2. In an automobile economy run, the winning car averaged 32.07 miles per gallon of gasoline. The second-place car averaged 27.85 miles per gallon. How many more miles per gallon did the winner average than the second-place car?

3. One meter is a measure of about 39.37 inches. How much greater in length is 1 meter than 1 yard?

4. A weather station reported the rainfall for the spring months as follows: April, 12.26 inches; May, 8.5 inches; and June, 6.37 inches. What was the total rainfall for the three months?

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5. At the start of a day's trip the odometer on the Smiths' car read 30118.4 miles. At the end of the day the reading was 30488.7 miles. How many miles did the Smiths travel that day?

6. When Sally was sick, her temperature was 100.7° at noon and 102.3° at 6 p.m. How much higher was her temperature at 6 p.m. than at noon?

7. In April Mr. Sawyer bought 124.6 gallons of gasoline, in May 78.8 gallons, in June 89.9 gallons, and in July 104.3 gallons. How much gasoline did he buy in these months?

8. The average amount of snow and sleet during one year in Juneau, Alaska, is 92.1 inches. In Detroit, Michigan, it is 31.5 inches. How much less snow and sleet can be expected per year in Detroit than in Juneau?

9. During one week Mrs. Jones wrote checks for $36.75, $12.88, $73.55, and $6.03. What was the total amount of the four checks?

Review and Practice

Part 1 Copy. Find each sum, difference, or product by performing the indicated operation.

	a	*b*	*c*
1.	$\begin{array}{r}6276\\+9854\\\hline\end{array}$	$\begin{array}{r}39.47\\-29.52\\\hline\end{array}$	$\begin{array}{r}4826\\\times 7\\\hline\end{array}$
2.	$\begin{array}{r}3007\\-768\\\hline\end{array}$	$\begin{array}{r}3624\\\times 33\\\hline\end{array}$	$\begin{array}{r}3.78\\+5.47\\\hline\end{array}$
3.	$\begin{array}{r}821\\\times 29\\\hline\end{array}$	$\begin{array}{r}9827\\+8995\\\hline\end{array}$	$\begin{array}{r}\$9.76\\-7.98\\\hline\end{array}$
4.	$\begin{array}{r}7.09\\-3.84\\\hline\end{array}$	$\begin{array}{r}3175\\\times 425\\\hline\end{array}$	$\begin{array}{r}7208\\\times 904\\\hline\end{array}$
5.	$\begin{array}{r}516\\3829\\5472\\823\\+714\\\hline\end{array}$	$\begin{array}{r}4621\\347\\3653\\2008\\+541\\\hline\end{array}$	$\begin{array}{r}52.14\\8.71\\32.08\\17.6\\+5.37\\\hline\end{array}$

Part 2 Copy. Find each quotient and remainder.

	a	*b*
1.	$70\overline{)2800}$	$34\overline{)24162}$
2.	$82\overline{)5626}$	$58\overline{)72541}$
3.	$27\overline{)9428}$	$67\overline{)45423}$
4.	$17\overline{)1529}$	$23\overline{)98324}$
5.	$33\overline{)6709}$	$92\overline{)40506}$

Part 3 Solve each of the following equations.

	a	*b*
1.	$\frac{2}{9}+\frac{5}{9}=n$	$(\frac{2}{3}+\frac{5}{6})-\frac{1}{4}=n$
2.	$\frac{4}{6}+\frac{3}{4}=n$	$(\frac{5}{8}-\frac{1}{2})+\frac{3}{4}=n$
3.	$3\frac{7}{9}+\frac{4}{9}=n$	$7\frac{1}{2}+\frac{1}{6}+4\frac{2}{3}=n$
4.	$6.1+7=n$	$9.7+.32+.5=n$
5.	$3.09-.37=n$	$(2.6+7.03)-.8=n$

Part 4 Write whether each of the following is *true* or *false*. Then draw a picture which shows that your answer is correct.

1. Two line segments in the same plane always intersect.

2. If two lines are parallel they never intersect.

3. The shortest distance between two points in a plane is the line segment joining the two points.

4. An angle is a simple closed figure.

5. Any two rays form an angle.

6. If the diameter of a circle is 4 inches long, then the radius is 2 inches long.

7. Every rectangle is a square.

8. Every square is a rectangle.

Read each problem carefully. Decide which operation or operations to use. Write an equation which describes the problem.

Solve the equation and answer the question asked in the problem.

1. Janet used 12 rolls of film last year. She took 16 pictures with each roll of film. How many pictures did she take last year?

2. Susan bought a notebook for $1.98, paper for $.75, and a ball-point pen for $2.49. She gave the clerk a $10.00 bill. How much change did she receive?

3. A basketball team scored 656 points in 16 games. What was the average score for each game?

4. A store advertised paint to sell for $5.60 per 1-gallon can and $19.75 per 5-gallon can. How much would you save on 5 gallons of paint by buying it in the larger container?

5. The scores on a spelling test were as follows: John, 92; Susan, 95; Bill, 84; Mark, 79; Jean, 88; and Rita, 90. How many points above average was Susan's score?

6. Mr. Ellis bought 75 feet of fence to enclose a rectangular pen. The pen was 4 yards wide and 7 yards long. How many feet of fence were left over?

7. How much would 2 bushels of potatoes cost at $.69 a peck?

8. Mr. Johnson's yard is 125 feet long and 60 feet wide. If 1 pound of grass seed covers 250 square feet of ground, how many pounds of seed does he need?

9. Janice worked for $\frac{2}{5}$ hour on her reading assignment and 18 minutes on arithmetic. For how many minutes did she work on these assignments?

10. Robert made a standing broad jump of $6\frac{2}{3}$ feet. David's jump measured 66 inches. How many inches more than David did Robert jump?

11. The sides of a triangle measured 29.7 inches, 18.56 inches, and 41.07 inches. What was its perimeter in inches?

12. The width of a rectangle is 6 feet and its perimeter is 32 feet. How long is the rectangle?

Can you do this? A rectangle and a square have the same perimeter. The rectangle has dimensions of 9 inches and 7 inches. How long is a side of the square?

Checkup Time

The numerals in ( ) tell the pages where you can turn for help.

Important Ideas

1. Fractions can be written as decimals and become part of our decimal system of numeration. (281)

2. If a fraction has a denominator of 10 or 100, the fraction can easily be written as a decimal. (281, 282)

3. The arithmetic of decimals should be in full agreement with the arithmetic of fractions. (283)

4. Knowing how to add and subtract in fractional form tells you how to add and subtract in decimal form. (283, 284, 285, 288)

5. When writing decimals in column form for addition or subtraction, be sure to write them so that all the decimal points lie on the same vertical line. (285, 289)

6. Every whole number can be expressed as a decimal. (289)

Words to Know

1. Decimal numerals (281)

2. Decimals (281)

3. Decimal point (281)

4. Tenths (281)

5. Hundredths (282)

Questions to Discuss

1. How do you read the decimal 3.5? (281) The decimal 6.31? (282)

2. How do you write *seven and nine tenths* as a decimal? (281)

3. How would you write *thirty-one and forty-eight hundredths* as a decimal numeral? (282)

4. How would you write the addition of 21.8, 7.43, and 5.06 in column form? (286)

Written Practice

Copy. Find each sum or difference by performing the indicated operation. Check each result.

	a	*b*	*c*	
1.	.5 +.2	.8 +.7	.3 +.9	(283)
2.	.05 +.23	.26 +.38	.76 +.47	(284)
3.	.36 .04 +.25	36.2 79.59 +8.17	14.56 83.37 +6.9	(287)
4.	7.8 −3.5	12.34 −8.56	40.02 −9.27	(288)
5.	4.73 −3.5	36.7 −7.54	27.0 −5.87	(289)

Self-Evaluation

Part 1 Write each of the following by using only decimal numerals.

1. Thirty-one and two tenths
2. Eighty-seven hundredths
3. 137 and 92 hundredths
4. Five and eight hundredths
5. One thousand and two tenths
6. 4218 and 7 tenths
7. Three hundred seventy
8. Six and sixty-six hundredths
9. 222 and 22 hundredths
10. 1 hundred and 1 hundredth

Part 2 Write each of the following numbers in decimal form.

	a	*b*	*c*
1.	$\frac{8}{10}$	$13\frac{6}{10}$	$351\frac{0}{10}$
2.	$\frac{19}{100}$	$27\frac{31}{100}$	$607\frac{7}{100}$
3.	$\frac{3}{100}$	$74\frac{0}{100}$	$923\frac{23}{100}$

Write each of the following numbers in fractional form.

	a	*b*	*c*
4.	.3	31.9	726.1
5.	.47	51.07	950.63
6.	.09	77.7	682.9

Part 3 Copy. Find each sum or difference by performing the indicated operation.

	a	*b*	*c*
1.	.7 +.5	21.2 +7.6	321.9 +82.6
2.	.62 +.27	9.09 +5.71	26.47 +17.75
3.	.8 −.5	17.9 −8.7	215.3 −65.7
4.	.58 −.37	31.71 −7.35	118.24 −93.68
5.	3.7 .6 8.1 +.4	21.6 3.02 51.59 +6.34	92.34 .67 70.05 +3.93
6.	9.7 −3.21	83. −5.26	107.15 −38.6

Part 4 Solve each of the following problems.

1. The radius of a circle measures 2.38 inches. How long is a diameter of the circle?

2. Mr. Jackson had 375 feet of fence to enclose his yard. The sides of his yard measured 72.5 feet, 136.23 feet, and 157.09 feet. How much extra fence did he have?

18

You can review the many important ideas you have learned this year by reading the statements in the tint blocks.

You can get practice in using these ideas by completing the exercises in this chapter. The practice may be oral or written, whichever your teacher thinks will help you most. Before you do the exercises in each practice lesson, read the statement that tells you about it and then study the example.

Numbers and Numerals

When a number is named in the decimal system, it can be thought of as the sum of ones, tens, hundreds, and so on.

36 = 3 tens + 6 ones = 30 + 6

507 = 5 hundreds + 7 ones = 500 + 7

Use with pages 7 and 8.

Write each of the following as the sum of ones, tens, and hundreds.

	a	*b*	*c*
1.	354	270	803
2.	777	93	357

Numbers can be named by using words or by using numerals.

721 means *seven hundred twenty-one.*

Use with pages 10 and 12.

Write the following in numerals.

1. Nine hundred fifty-three

2. Seven thousand two hundred

3. Four million eighty-three

4. One thousand one hundred two

5. Nine billion two thousand

Express these numbers in words.

	a	*b*
6.	3,012	801,320
7.	7,005	1,006,015

A number can be rounded off to the nearer 10, 100, and so on.

57, to the nearer 10, is 60

348, to the nearer 100, is 300

Use with page 13.

Express each number below to the nearer 10, then to the nearer 100, and finally to the nearer 1000.

	a	*b*	*c*
1.	3,528	4,556	7,192
2.	8,717	2,626	3,377

Addition

You add ones to ones and tens to tens, making changes in form whenever the sum of ones or tens is greater than 9.

$$\begin{array}{r|l} T & O \\ 3 & 7 \\ +4 & 5 \\ \hline 8 & 2 \end{array} \qquad \begin{array}{r|c|l} H & T & O \\ & 8 & 4 \\ + & 5 & 7 \\ \hline 1 & 4 & 1 \end{array}$$

Use with page 21.

	a	*b*	*c*	*d*
1.	36 +4	49 +7	93 +42	88 +36
2.	72 +18	79 +66	93 +19	69 +38
3.	9 +68	17 +37	27 +58	49 +94

Regrouping addends does not change the sum.

$(7+12)+3=19+3=22$

$7+(12+3)=7+15=22$

Use with pages 22 and 23.

	a	*b*	*c*	*d*
1.	54 6 28 +12	72 39 4 +76	7 8 74 +9	40 65 37 +83
2.	19 43 55 +67	76 49 28 +76	29 36 87 +23	90 79 57 +66

If two sets are to be joined, add the numbers of the sets.

A set containing 5 objects joined with a set having 7 objects forms a set having 5+7 or 12 objects.

Use with page 24.

1. In the school election Jane received 24 votes, John 19, and Don 18. How many votes were cast?

2. It took Robert 15 minutes to sweep the garage, 23 minutes to clean the basement, and 19 minutes to burn the trash. How many minutes did the three jobs take?

Parentheses in an equation indicate which operations are to be done first.

$$\begin{aligned} (30+10)+(9+2) &= n \\ 40 \quad + \quad 11 &= n \\ 51 &= n \end{aligned}$$

Use with page 25.

Solve the following equations by performing the operations within the parentheses first.

1. $(70+20)+(4+8)=n$

2. $(90+60)+(5+7)=n$

3. $(100+30)+(7+3)=n$

4. $(40+35)+(12+13)=n$

5. $(9+8)+(120+70)=n$

You add hundreds just as you add tens and ones.

$$\begin{array}{r} 342 \\ +837 \\ \hline 1179 \end{array} \qquad \begin{array}{r} 758 \\ +569 \\ \hline 1327 \end{array}$$

Use with page 26.

	a	*b*	*c*	*d*
1.	404 +657	359 +566	737 +284	862 +39
2.	207 526 84 +713	573 88 392 +777	538 705 413 +171	727 846 919 +285

Thousands are added in the same way as hundreds, tens, and ones.

$$\begin{array}{r} 476 \\ +2352 \\ \hline 2828 \end{array} \qquad \begin{array}{r} 4513 \\ +7638 \\ \hline 12151 \end{array}$$

Use with page 27.

	a	*b*	*c*
1.	3419 +657	7305 +817	6285 +964
2.	2888 +673	3721 +5293	6729 +7508
3.	7327 849 +5277	3875 269 +338	1909 5000 +3492
4.	5974 8000 9866 +5490	4289 7629 9745 +6542	3567 8755 9988 +4879

In column form, the dollar sign is used with the first addend and in the sum; the cents points are written directly underneath each other.

Use with page 28.

	a	*b*	*c*
1.	$6.75 +.98	$18.75 +5.50	$36.20 +7.87
2.	$56.82 49.19 +18.46	$29.80 10.25 +30.44	$35.67 98.66 +58.13
3.	$76.08 59.15 8.47 +3.84	$47.95 90.72 6.49 +87.56	$63.72 29.98 8.05 +83.40

Knowing how to add numbers of 4 digits enables you to add numbers with any number of digits.

$$\begin{array}{r} 83427 \\ +6546 \\ \hline 89973 \end{array} \qquad \begin{array}{r} 65348 \\ +87475 \\ \hline 152823 \end{array}$$

Use with page 30.

	a	*b*	*c*
1.	53421 +4729	33789 +6544	35174 +6798
2.	72157 3746 +39817	69875 64536 +42674	64536 75467 +32154
3.	43205 26327 71812 +53451	65030 42653 17630 +12925	27653 3594 1732 +37934

Subtraction

Subtraction is the inverse of addition, and addition is the inverse of subtraction.

If $7+n=32$ then $32-n=7$.
If $n-5=12$ then $12+5=n$.

Use with pages 33 and 34.

	a	*b*	*c*	*d*
1.	4 +8	12 −8	6 +9	15 −9
2.	4 +6	10 −6	8 +5	13 −5
3.	9 +7	16 −7	7 +4	11 −4
4.	7 +6	13 −6	9 +3	12 −3

In an equation, ( ) means *do this operation first.*

$$\begin{aligned} (59-5)-24&=n\\ 54-24&=n\\ 30&=n \end{aligned}$$

Use with page 35.

	a	*b*
1.	$23-7=n$	$42-8=n$
2.	$13-9=n$	$28-7=n$
3.	$(26-4)-5=n$	$(52-40)-6=n$
4.	$(76-3)-4=n$	$(65-34)-11=n$
5.	$(98-4)-17=n$	$(47-9)-20=n$

When there are more ones in the subtrahend than in the minuend, you change the form of 1 ten in the minuend to 10 ones before you subtract.

$$\begin{array}{r} 93\\ -8\\ \hline 85 \end{array} \qquad 93-8=85$$

Use with page 36.

	a	*b*	*c*	*d*
1.	87 −9	62 −7	40 −4	71 −6
2.	82 −78	36 −27	60 −49	53 −16
3.	93 −38	84 −26	74 −56	91 −36

You subtract tens and hundreds as you subtract ones, making changes in form whenever necessary.

$$\begin{array}{r} 704\\ -89\\ \hline 615 \end{array} \qquad \begin{array}{r} 445\\ -279\\ \hline 166 \end{array}$$

Use with page 37.

	a	*b*	*c*	*d*
1.	456 −97	173 −85	900 −23	901 −11
2.	645 −557	794 −195	500 −269	391 −166
3.	426 −287	345 −288	819 −524	506 −427

You use subtraction to solve a problem that asks you to separate a set or to find the difference between the numbers of two sets.

Ann had 78 cents. She bought some bananas and had 57 cents left. How much did the bananas cost?

$$78-57=21$$

The bananas cost 21 cents.

Use with pages 38 through 41.

Solve these problems.

1. Jack had 74 trading stamps; Wally had 58. How many more stamps did Jack have than Wally?

2. The Hansons drove 903 mi. on their vacation last year and 798 mi. this year. How many miles less did they drive this year?

3. 64 pupils of the Emerson School were waiting for school buses to take them to the museum. The first bus held 36 pupils. How many boarded the second bus?

4. Paul's father weighed 168 pounds. Paul weighed 81 pounds. How many pounds more did Paul's father weigh than did Paul?

5. Mrs. White had 120 tulip bulbs. She planted tulips along the side of the lilac hedge and had 54 bulbs left. How many tulip bulbs did Mrs. White plant?

You subtract thousands as you subtract ones, tens, and hundreds, making changes in form as needed.

$$\begin{array}{r} 5016 \\ -427 \\ \hline 4589 \end{array} \qquad \begin{array}{r} 8117 \\ -4958 \\ \hline 3159 \end{array}$$

Use with page 42.

	a	*b*	*c*
1.	7452 −579	9043 −774	7100 −909
2.	7869 −6970	5437 −2369	6093 −1819
3.	4910 −3819	7012 −4813	3801 −1912

You subtract ten-thousands as you subtract ones, tens, hundreds, and thousands.

$$\begin{array}{r} 41265 \\ -2276 \\ \hline 38989 \end{array} \qquad \begin{array}{r} 30502 \\ -17599 \\ \hline 12903 \end{array}$$

Use with pages 43 and 44.

	a	*b*	*c*
1.	50093 −4994	71812 −3814	28679 −8990
2.	47010 −6911	62412 −7403	50378 −8479
3.	83853 −57906	74797 −26808	69558 −24589
4.	52691 −25909	30604 −13356	42918 −27819

In multiplying, you do not change the product by changing the order of the factors.

$\begin{array}{r} 3 \\ \times 5 \\ \hline 15 \end{array}$ $\quad$ $\begin{array}{r} 5 \\ \times 3 \\ \hline 15 \end{array}$ $\quad$ $5 \times 3 = 3 \times 5$

Use with pages 49 and 50.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 4 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 7 \\ \hline \end{array}$
2.	$\begin{array}{r} 2 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$
3.	$\begin{array}{r} 4 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 9 \\ \hline \end{array}$

Whenever one of the factors in a multiplication is 0, the product is always 0.

$4 \times 0 = 0 \qquad 0 \times 4 = 0$

Use with page 49.

	a	*b*
1.	$3 \times 0 = n$	$n \times 9 = 0$
2.	$7 \times n = 0$	$0 \times 5 = n$

Whenever you multiply a number by 1 or 1 by a number, you do not change the number.

$\begin{array}{r} 8 \\ \times 1 \\ \hline 8 \end{array}$ $\quad$ $\begin{array}{r} 1 \\ \times 8 \\ \hline 8 \end{array}$

Use with page 50.

	a	*b*
1.	$6 \times n = 6$	$1 \times 7 = n$
2.	$n \times 4 = 4$	$5 \times 1 = n$

When you multiply by a one-digit number, first multiply the ones, then the tens, then the hundreds, and so on. Add the partial products.

$\begin{array}{r} 47 \\ \times 5 \\ \hline 35 \\ 200 \\ \hline 235 \end{array}$ $\quad$ $\begin{array}{r} 223 \\ \times 3 \\ \hline 9 \\ 60 \\ 600 \\ \hline 669 \end{array}$ $\quad$ $\begin{array}{r} 2541 \\ \times 2 \\ \hline 2 \\ 80 \\ 1000 \\ 4000 \\ \hline 5082 \end{array}$ $\quad$ $\begin{array}{r} 3451 \\ \times 4 \\ \hline 13804 \end{array}$

Use with page 51.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 51 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 38 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 45 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 87 \\ \times 8 \\ \hline \end{array}$
2.	$\begin{array}{r} 67 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 84 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 95 \\ \times 4 \\ \hline \end{array}$

Use with page 52.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 454 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 349 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 902 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 722 \\ \times 6 \\ \hline \end{array}$
2.	$\begin{array}{r} 8019 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 2736 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 3980 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 4328 \\ \times 7 \\ \hline \end{array}$
3.	$\begin{array}{r} 5943 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 1614 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 7230 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 5881 \\ \times 4 \\ \hline \end{array}$

You multiply by tens as you multiply by ones, changing the form of the partial products when necessary.

$$\begin{array}{r} 48 \\ \times 34 \\ \hline 192 \\ 144 \\ \hline 1632 \end{array} \qquad \begin{array}{r} 486 \\ \times 35 \\ \hline 2430 \\ 1458 \\ \hline 17010 \end{array} \qquad \begin{array}{r} 1759 \\ \times 53 \\ \hline 5277 \\ 8795 \\ \hline 93227 \end{array}$$

Use with pages 57, 58, 59, and 61.

	a	*b*	*c*	*d*
1.	34 ×22	23 ×33	42 ×12	40 ×21
2.	17 ×51	25 ×13	18 ×24	13 ×54
3.	73 ×74	89 ×58	92 ×32	58 ×32
4.	83 ×72	77 ×21	73 ×55	92 ×13
5.	96 ×54	83 ×27	54 ×33	65 ×46

Use with page 62.

	a	*b*	*c*	*d*
1.	846 ×76	134 ×34	500 ×43	301 ×58
2.	549 ×59	190 ×21	507 ×28	825 ×69
3.	572 ×23	200 ×89	814 ×43	156 ×76
4.	409 ×56	760 ×47	638 ×62	317 ×81

Use with page 63.

	a	*b*	*c*	*d*
1.	5000 ×70	9002 ×45	2883 ×55	6054 ×27
2.	6072 ×78	2867 ×53	9175 ×63	8657 ×96
3.	4334 ×71	6492 ×87	4305 ×36	8411 ×29

You multiply by a three-digit number as you multiply by a two-digit number, changing the form of the partial products whenever necessary.

$$\begin{array}{r} 426 \\ \times 344 \\ \hline 1704 \\ 1704 \\ 1278 \\ \hline 146544 \end{array} \qquad \begin{array}{r} 3124 \\ \times 454 \\ \hline 12496 \\ 15620 \\ 12496 \\ \hline 1418296 \end{array}$$

Use with pages 64 and 65.

	a	*b*	*c*	*d*
1.	800 ×453	798 ×642	907 ×809	653 ×302
2.	221 ×192	555 ×413	870 ×376	499 ×510
3.	7000 ×736	8729 ×456	5024 ×412	3452 ×707
4.	6307 ×691	7143 ×700	5211 ×816	9210 ×539
5.	4902 ×503	8169 ×726	9064 ×148	7154 ×683

Division and multiplication are inverse operations.

If $3 \times 7 = 21$ then $21 \div 7 = 3$.

Use with page 74.

	a	*b*
1.	$4 \times 9 = n$	$36 \div 9 = n$
2.	$n \times 7 = 56$	$56 \div 7 = n$

The quotient is 0 when the dividend is 0 and the divisor is not 0.

$0 \div 6 = 0$ $\qquad \begin{array}{r} 0 \\ 6\overline{)0} \end{array}$

Use with page 75.

	a	*b*	*c*	*d*
1.	$8\overline{)0}$	$9\overline{)54}$	$4\overline{)0}$	$9\overline{)36}$
2.	$5\overline{)40}$	$7\overline{)0}$	$9\overline{)81}$	$6\overline{)60}$

A remainder other than 0 is written beside the quotient.

$$\begin{array}{r} 32 \\ 3\overline{)96} \\ \underline{9} \\ 6 \\ \underline{6} \\ 0 \end{array} \qquad \begin{array}{r} 14 \\ 4\overline{)56} \\ \underline{4} \\ 16 \\ \underline{16} \\ 0 \end{array} \qquad \begin{array}{r} 12 \text{ r3} \\ 5\overline{)63}\phantom{\text{ r3}} \\ \underline{5}\phantom{3\text{ r3}} \\ 13\phantom{\text{ r3}} \\ \underline{10}\phantom{\text{ r3}} \\ 3\phantom{\text{ r3}} \end{array}$$

Use with pages 77, 78, and 79.

	a	*b*	*c*	*d*
1.	$6\overline{)66}$	$5\overline{)75}$	$8\overline{)92}$	$4\overline{)83}$
2.	$3\overline{)49}$	$7\overline{)84}$	$6\overline{)95}$	$9\overline{)97}$

Divide hundreds and thousands as you divide tens and ones.

$$\begin{array}{r} 62 \\ 3\overline{)186} \\ \underline{18} \\ 6 \\ \underline{6} \\ 0 \end{array} \qquad \begin{array}{r} 265 \text{ r2} \\ 3\overline{)797}\phantom{\text{ r2}} \\ \underline{6}\phantom{97\text{ r2}} \\ 19\phantom{7\text{ r2}} \\ \underline{18}\phantom{7\text{ r2}} \\ 17\phantom{\text{ r2}} \\ \underline{15}\phantom{\text{ r2}} \\ 2\phantom{\text{ r2}} \end{array} \qquad \begin{array}{r} 1393 \text{ r5} \\ 6\overline{)8363}\phantom{\text{ r5}} \\ \underline{6}\phantom{363\text{ r5}} \\ 23\phantom{63\text{ r5}} \\ \underline{18}\phantom{63\text{ r5}} \\ 56\phantom{3\text{ r5}} \\ \underline{54}\phantom{3\text{ r5}} \\ 23\phantom{\text{ r5}} \\ \underline{18}\phantom{\text{ r5}} \\ 5\phantom{\text{ r5}} \end{array}$$

Use with pages 82, 83, and 87.

	a	*b*	*c*
1.	$6\overline{)565}$	$7\overline{)835}$	$6\overline{)988}$
2.	$5\overline{)376}$	$2\overline{)178}$	$8\overline{)319}$
3.	$2\overline{)195}$	$6\overline{)249}$	$4\overline{)283}$
4.	$5\overline{)523}$	$8\overline{)578}$	$6\overline{)553}$
5.	$7\overline{)639}$	$4\overline{)256}$	$7\overline{)348}$
6.	$8\overline{)367}$	$5\overline{)632}$	$3\overline{)174}$
7.	$4\overline{)360}$	$3\overline{)222}$	$9\overline{)474}$
8.	$9\overline{)570}$	$7\overline{)561}$	$3\overline{)101}$
9.	$9\overline{)9703}$	$3\overline{)9875}$	$7\overline{)8001}$
10.	$5\overline{)7119}$	$4\overline{)5027}$	$3\overline{)7706}$
11.	$3\overline{)3009}$	$5\overline{)6872}$	$7\overline{)6459}$
12.	$9\overline{)5598}$	$8\overline{)7225}$	$4\overline{)9725}$

Division—Two-Digit Divisors

In dividing a two-digit number by a two-digit number, you find the trial quotient by dividing the number of tens in the dividend by the number of tens in the divisor.

$$\begin{array}{r} 4 \\ 20\overline{)80} \\ \underline{80} \\ 0 \end{array} \qquad \begin{array}{r} 2 \text{ r}3 \\ 21\overline{)45} \\ \underline{42} \\ 3 \end{array} \qquad \begin{array}{r} 2 \\ 35\overline{)70} \\ \underline{70} \\ 0 \end{array}$$

Use with pages 94, 95, 98, and 99.

	a	*b*	*c*	*d*
1.	$60\overline{)68}$	$20\overline{)89}$	$30\overline{)90}$	$21\overline{)63}$
2.	$15\overline{)17}$	$11\overline{)58}$	$30\overline{)75}$	$41\overline{)89}$
3.	$18\overline{)82}$	$41\overline{)68}$	$13\overline{)72}$	$24\overline{)73}$
4.	$37\overline{)76}$	$29\overline{)68}$	$18\overline{)72}$	$46\overline{)92}$
5.	$27\overline{)58}$	$15\overline{)35}$	$29\overline{)63}$	$18\overline{)35}$
6.	$48\overline{)88}$	$31\overline{)92}$	$34\overline{)69}$	$13\overline{)82}$
7.	$60\overline{)69}$	$20\overline{)81}$	$40\overline{)89}$	$41\overline{)84}$
8.	$30\overline{)38}$	$50\overline{)62}$	$17\overline{)29}$	$33\overline{)47}$
9.	$65\overline{)84}$	$79\overline{)95}$	$81\overline{)90}$	$19\overline{)50}$
10.	$40\overline{)77}$	$16\overline{)36}$	$14\overline{)57}$	$48\overline{)96}$
11.	$29\overline{)57}$	$58\overline{)89}$	$32\overline{)67}$	$21\overline{)71}$
12.	$23\overline{)92}$	$43\overline{)71}$	$26\overline{)85}$	$18\overline{)72}$

When you divide a three-digit number by a two-digit number, you find the trial quotient by dividing the number of tens in the dividend by the number of tens in the divisor.

$$\begin{array}{r} 4 \text{ r}2 \\ 62\overline{)250} \\ \underline{248} \\ 2 \end{array} \qquad \begin{array}{r} 6 \\ 19\overline{)114} \\ \underline{114} \\ 0 \end{array} \qquad \begin{array}{r} 30 \text{ r}20 \\ 25\overline{)770} \\ \underline{75} \\ 20 \end{array}$$

Use with pages 102, 103, and 105.

	a	*b*	*c*
1.	$60\overline{)180}$	$80\overline{)160}$	$90\overline{)720}$
2.	$57\overline{)228}$	$92\overline{)552}$	$63\overline{)332}$
3.	$23\overline{)210}$	$47\overline{)271}$	$72\overline{)350}$
4.	$50\overline{)450}$	$90\overline{)540}$	$40\overline{)360}$
5.	$82\overline{)416}$	$63\overline{)371}$	$57\overline{)587}$
6.	$37\overline{)259}$	$72\overline{)155}$	$95\overline{)468}$
7.	$19\overline{)181}$	$35\overline{)307}$	$49\overline{)384}$
8.	$61\overline{)396}$	$98\overline{)971}$	$15\overline{)509}$
9.	$23\overline{)675}$	$37\overline{)729}$	$27\overline{)530}$
10.	$40\overline{)500}$	$60\overline{)701}$	$28\overline{)393}$
11.	$56\overline{)974}$	$13\overline{)119}$	$18\overline{)218}$
12.	$34\overline{)102}$	$47\overline{)329}$	$96\overline{)508}$

Measurement

Measurements may be different in form but equal in value. = is used to mean *measures the same as*. See page 332 for tables of measure.

Use with page 114.

1. 15 ft. = ___in.
2. 700 rd. = ___mi.___rd.
3. 2 mi. = ___yd.
4. 1 mi. = ___in.
5. 17 yd. 2 ft. = ___ft.

Use with page 116.

	a	*b*
1.	2 gal. = ___pt.	5 c. = ___fl. oz.
2.	3 qt. = ___fl. oz.	52 pt. = ___qt.
3.	___c. = 2 gal.	___fl. oz. = 215 qt.
4.	___pt. = 112 fl. oz.	200 qt. = ___c.

Use with page 117.

	a	*b*
1.	7 bu. = ___pk.	216 qt. = ___pt.
2.	56 qt. = ___bu.	96 qt. = ___pk.
3.	17 pk. = ___qt.	16 pt. = ___pk.
4.	20 pk. = ___pt.	4 bu. = ___qt.
5.	3 bu. = ___pt.	192 pt. = ___bu.

Use with page 119.

1. 5 T. 2 cwt. = ___lb.
2. 7500 lb. = ___T.___cwt.
3. 272 lb. = ___oz.
4. 117 oz. = ___lb.___oz.
5. 2 T. 73 lb. = ___lb.
6. 352 lb. = ___cwt.___lb.
7. 76 oz. = ___lb.___oz.

Use with pages 122 and 123.

1. 114 mo. = ___yr.___ mo.
2. 17 yr. 5 mo. = ___mo.
3. 220 yr. = ___decades
4. 2300 yr. = ___centuries
5. 47 wk. = ___da.
6. 71 hr. = ___da.___hr.
7. 115 min. = ___hr.___min.

Use with pages 126 and 127.

Use numerals to write each of the times given below.

	a	*b*
1.	Two twenty-two	Seven thirty
2.	One forty-five	Eleven ten
3.	Six fifteen	Eight fourteen

Problem Solving

To solve a story problem, you first put it in equation form. Then you solve the equation. A problem involving the use of more than one operation to solve it may be expressed in a single equation.

Mrs. Jones bought 5 cards of buttons, with 6 buttons on each card. How many buttons did she buy?

$$5 \times 6 = n$$
$$30 = n$$

Mrs. Jones bought 30 buttons.

Paul, Mary, and Jack weighed 192 lb. together. Paul weighed 71 lb., and Jack weighed 62 lb. How much did Mary weigh?

$$192 - (71 + 62) = n$$
$$192 - 133 = n$$
$$59 = n$$

Mary weighed 59 lb.

Use with pages 131 through 133.

1. Phil gave 13 of his marbles to Bob and had 28 marbles left. How many marbles did Phil have before he gave 13 away?

2. Mrs. Olson made 15 cupcakes. After dinner she found that there were 7 left. How many cupcakes had been eaten?

3. Three boys returned 12 books to the library. If each carried the same number, how many books did each boy carry?

4. There were 53 people in the auditorium by 8:15. 68 more came in before the play started. How many people were there altogether?

Use with pages 134 through 136.

1. In one place 12 inches of snow fell in January. By the end of February 43 inches of snow had fallen. How many inches of snow fell in the month of February?

2. Mary's mother bought 3 boxes of greeting cards with 12 cards in each box and 2 boxes with 24 in each box. How many greeting cards in all did Mary's mother buy?

3. Mr. Hood bought 36 tomato plants. He set them out in rows with 9 plants in each row. How many rows of tomato plants did he set out?

4. Joan's score in spelling was 85 on Monday, 91 on Tuesday, 80 on Wednesday, 87 on Thursday, and 97 on Friday. What was her average score for the 5 days?

Use with pages 137 through 139.

1. Terry practiced his horn for 30 minutes every afternoon for 5 days. Jack practiced for 3 hours. How much longer did Jack practice?

2. Mrs. Jackson gathered 137 eggs. She packed them in 11 cartons holding a dozen eggs each. How many eggs were left over?

3. Sally, Alice, and Ann filled nut cups and placed 4 of them on each of 6 card tables for their mother's party. On an average, how many nut cups did each girl fill?

4. Phil sorted the shells in his collection into 13 boxes of 8 shells each and had 6 shells left over. How many shells did he have?

To solve a problem, take these steps: Study the problem, decide what to do with the sets, decide which operation to use, and write an equation. Then solve the equation.

Use with pages 140 and 141.

1. Mary's mother canned 7 quarts of tomatoes and her aunt canned 19 pints. How many pints of tomatoes did they can altogether?

2. Fred had 17 books on one bookshelf and 33 on another. He gave 20 of the books to the Boys' Club. How many books did he have left?

3. Bob picked 46 pumpkins. He put 4 aside to take home and divided the rest into 6 equal piles. How many pumpkins were in each pile?

4. When milk is 96¢ a gallon, what is the cost of 1 pint?

You may estimate answers by rounding off the numbers given to the nearer 10, 100, or 1000.

Philip had 186 pennies in his bank. He took out 28 of them to buy some stamps. How many pennies did Philip have left?

When the numbers are rounded off to the nearer ten, the equation is

$$190 - 30 = n$$

and the estimated answer is

$$160 = n.$$

Philip had about 160 pennies left.

Use with pages 142 and 143.

Estimate an answer to each of the following problems. Write your estimate. Then write an equation for each problem. Solve the equation.

1. There were 311 boys in the junior division of the summer camp and 189 in the senior division. How many boys attended the camp?

2. The class collected 115 pounds of paper in March and 168 pounds in April. How much more did they collect in April than in March?

3. If Andy practiced his accordion for 14 hours every month, how many hours would he practice in a year?

4. Janice said that she had read 36 books one year. On an average, how many had she read each month?

More Two-Digit Divisors—Larger Dividends

You divide four-digit dividends as you divide three-digit dividends.

$$\begin{array}{r} 40 \\ 40\overline{)1600} \\ \underline{160} \\ 0 \end{array} \qquad \begin{array}{r} 60 \\ 61\overline{)3660} \\ \underline{366} \\ 0 \end{array}$$

Use with pages 147 and 148.

	a	*b*	*c*
1.	$80\overline{)3200}$	$78\overline{)4680}$	$50\overline{)3000}$
2.	$57\overline{)2280}$	$96\overline{)3840}$	$30\overline{)2100}$
3.	$49\overline{)3430}$	$60\overline{)5400}$	$27\overline{)1890}$
4.	$67\overline{)2010}$	$90\overline{)1800}$	$87\overline{)4350}$
5.	$36\overline{)1944}$	$63\overline{)6111}$	$27\overline{)1080}$

You must find a trial quotient for each digit of the quotient.

$$\begin{array}{r} 81 \\ 37\overline{)2997} \\ \underline{296} \\ 37 \\ \underline{37} \\ 0 \end{array} \qquad \begin{array}{r} 120 \\ 31\overline{)3720} \\ \underline{31} \\ 62 \\ \underline{62} \\ 0 \end{array} \qquad \begin{array}{r} 25 \text{ r}37 \\ 60\overline{)1537} \\ \underline{120} \\ 337 \\ \underline{300} \\ 37 \end{array}$$

Use with page 149.

	a	*b*	*c*
1.	$44\overline{)2156}$	$35\overline{)3290}$	$32\overline{)3168}$
2.	$56\overline{)2240}$	$70\overline{)2450}$	$73\overline{)2336}$
3.	$29\overline{)1218}$	$20\overline{)1120}$	$68\overline{)6108}$
4.	$50\overline{)2550}$	$91\overline{)2457}$	$37\overline{)1591}$
5.	$70\overline{)1540}$	$29\overline{)2407}$	$38\overline{)3344}$

Use with pages 150 and 151.

	a	*b*	*c*
1.	$36\overline{)1852}$	$16\overline{)1300}$	$57\overline{)4768}$
2.	$78\overline{)3962}$	$23\overline{)1177}$	$42\overline{)2736}$
3.	$15\overline{)1103}$	$39\overline{)5500}$	$84\overline{)1962}$
4.	$16\overline{)1100}$	$75\overline{)2040}$	$39\overline{)3598}$
5.	$17\overline{)1655}$	$47\overline{)3690}$	$83\overline{)9516}$

Use with page 153.

	a	*b*	*c*
1.	$11\overline{)1320}$	$43\overline{)9890}$	$21\overline{)2541}$
2.	$21\overline{)7350}$	$40\overline{)9200}$	$45\overline{)9450}$
3.	$55\overline{)6050}$	$32\overline{)6720}$	$70\overline{)8400}$
4.	$30\overline{)8100}$	$29\overline{)3190}$	$43\overline{)9030}$
5.	$15\overline{)9150}$	$51\overline{)5610}$	$61\overline{)9760}$
6.	$39\overline{)8190}$	$72\overline{)7920}$	$28\overline{)6160}$

Use with page 158.

	a	b	c
1.	$17\overline{)2636}$	$38\overline{)5712}$	$70\overline{)8480}$
2.	$30\overline{)7590}$	$55\overline{)6166}$	$30\overline{)6170}$
3.	$13\overline{)4301}$	$82\overline{)9370}$	$37\overline{)7483}$
4.	$28\overline{)5909}$	$31\overline{)3782}$	$83\overline{)8466}$
5.	$38\overline{)9006}$	$78\overline{)8814}$	$66\overline{)9174}$

When each new dividend is less than the divisor, a 0 is written in the proper place in the quotient.

```
     309 r1            208 r14
14)4327            17)3550
   42                 34
    127                150
    126                136
      1                 14
```

Use with page 159.

	a	b	c
1.	$14\overline{)1526}$	$38\overline{)7801}$	$24\overline{)7320}$
2.	$83\overline{)9137}$	$22\overline{)8910}$	$37\overline{)7549}$
3.	$38\overline{)7829}$	$15\overline{)1539}$	$21\overline{)2185}$
4.	$27\overline{)1629}$	$63\overline{)7625}$	$34\overline{)3517}$
5.	$55\overline{)1706}$	$17\overline{)5204}$	$35\overline{)7034}$
6.	$66\overline{)1330}$	$39\overline{)8001}$	$90\overline{)9809}$
7.	$23\overline{)4720}$	$51\overline{)9303}$	$80\overline{)7096}$

You divide 5-digit dividends as you divide 4-digit dividends.

```
      454             2636 r8
60)27240          32)84360
   240               64
    324              203
    300              192
     240              116
     240               96
       0               200
                       192
                         8
```

Use with pages 167 and 168.

	a	b
1.	$40\overline{)16000}$	$70\overline{)35000}$
2.	$60\overline{)54000}$	$90\overline{)72000}$
3.	$26\overline{)25167}$	$50\overline{)31759}$
4.	$48\overline{)41365}$	$11\overline{)10787}$
5.	$70\overline{)16730}$	$96\overline{)12192}$
6.	$74\overline{)25012}$	$68\overline{)32693}$
7.	$59\overline{)36681}$	$47\overline{)34232}$
8.	$83\overline{)45446}$	$97\overline{)59655}$
9.	$60\overline{)42360}$	$39\overline{)28726}$

Use with page 171.

	a	b
1.	$26\overline{)37549}$	$80\overline{)90000}$
2.	$15\overline{)30645}$	$75\overline{)85150}$

	a	*b*
3.	$20\overline{)70158}$	$81\overline{)99270}$
4.	$26\overline{)78286}$	$41\overline{)64062}$
5.	$13\overline{)56847}$	$23\overline{)32149}$
6.	$90\overline{)91603}$	$87\overline{)98458}$
7.	$69\overline{)83490}$	$56\overline{)89769}$
8.	$39\overline{)89973}$	$44\overline{)79685}$
9.	$71\overline{)85958}$	$26\overline{)80913}$
10.	$31\overline{)99800}$	$66\overline{)83952}$

You divide 6-digit dividends as you divide 5-digit dividends.

$$\begin{array}{r} 2583 \\ 53\overline{)136899} \\ \underline{106} \\ 308 \\ \underline{265} \\ 439 \\ \underline{424} \\ 159 \\ \underline{159} \\ 0 \end{array} \qquad \begin{array}{r} 10059\rlap{\text{ r7}} \\ 47\overline{)472780} \\ \underline{47} \\ 278 \\ \underline{235} \\ 430 \\ \underline{423} \\ 7 \end{array}$$

Use with pages 174 and 175.

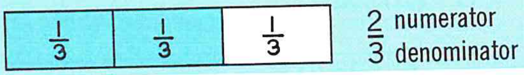
	a	*b*
1.	$30\overline{)716202}$	$60\overline{)841092}$
2.	$72\overline{)371240}$	$51\overline{)110750}$
3.	$55\overline{)400287}$	$76\overline{)510505}$
4.	$37\overline{)204001}$	$90\overline{)810000}$

	a	*b*
5.	$46\overline{)310537}$	$67\overline{)584970}$
6.	$40\overline{)918402}$	$35\overline{)176505}$
7.	$64\overline{)324308}$	$29\overline{)234707}$
8.	$30\overline{)646200}$	$96\overline{)209760}$
9.	$74\overline{)892295}$	$41\overline{)965889}$
10.	$43\overline{)879039}$	$67\overline{)998569}$
11.	$51\overline{)579048}$	$39\overline{)271336}$

Use with pages 176 and 177.

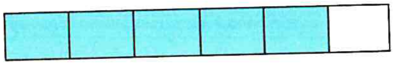
	a	*b*
1.	$18\overline{)170908}$	$21\overline{)291809}$
2.	$63\overline{)633984}$	$49\overline{)485990}$
3.	$29\overline{)234707}$	$69\overline{)640698}$
4.	$85\overline{)819683}$	$35\overline{)345500}$
5.	$65\overline{)716956}$	$48\overline{)596840}$
6.	$38\overline{)987582}$	$41\overline{)897736}$
7.	$78\overline{)964236}$	$67\overline{)478498}$
8.	$43\overline{)556794}$	$36\overline{)719502}$
9.	$91\overline{)983219}$	$65\overline{)905126}$
10.	$74\overline{)896048}$	$67\overline{)619530}$
11.	$49\overline{)476327}$	$53\overline{)565972}$

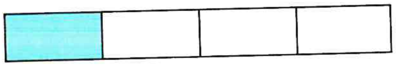
Fractions are numerals which name fractional numbers. A fraction has a numerator and a denominator.

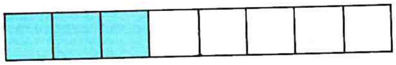


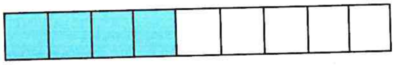
Use with page 183.

After the number of each drawing below, write the fraction that names the fractional part that is blue.

1.


2.


3.


4.


Fractions that are different in form may be equivalent.

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}$$

$$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\frac{8}{24}=\frac{9}{27}$$

Use with pages 184 through 187.

Copy. Replace each n with the fraction that makes the statement true.

	a	*b*	*c*
1.	$\frac{2}{7}=\frac{n}{28}$	$\frac{3}{5}=\frac{9}{n}$	$\frac{1}{12}=\frac{n}{96}$
2.	$\frac{4}{11}=\frac{24}{n}$	$\frac{5}{8}=\frac{n}{48}$	$\frac{5}{6}=\frac{n}{54}$
3.	$\frac{3}{5}=\frac{24}{n}$	$\frac{1}{4}=\frac{n}{12}$	$\frac{2}{5}=\frac{6}{n}$
4.	$\frac{1}{2}=\frac{n}{18}$	$\frac{5}{13}=\frac{n}{39}$	$\frac{11}{16}=\frac{n}{64}$
5.	$\frac{5}{6}=\frac{25}{n}$	$\frac{2}{17}=\frac{n}{68}$	$\frac{7}{15}=\frac{49}{n}$
6.	$\frac{3}{8}=\frac{n}{56}$	$\frac{7}{9}=\frac{42}{n}$	$\frac{5}{12}=\frac{60}{n}$
7.	$\frac{1}{23}=\frac{n}{92}$	$\frac{3}{4}=\frac{93}{n}$	$\frac{5}{14}=\frac{n}{70}$
8.	$\frac{5}{8}=\frac{n}{72}$	$\frac{1}{6}=\frac{n}{114}$	$\frac{2}{9}=\frac{n}{63}$
9.	$\frac{2}{3}=\frac{n}{144}$	$\frac{10}{11}=\frac{n}{132}$	$\frac{5}{7}=\frac{n}{35}$
10.	$\frac{7}{10}=\frac{42}{n}$	$\frac{2}{13}=\frac{n}{78}$	$\frac{1}{8}=\frac{n}{128}$

A fraction is in lowest terms when you can find no whole number, other than 1, by which both the numerator and the denominator are divisible.

$$\frac{20\div5}{35\div5}=\frac{4}{7} \qquad \frac{27\div9}{36\div9}=\frac{3}{4}$$

Use with page 188.

Copy. Reduce to lowest terms.

	a	*b*	*c*	*d*
1.	$\frac{14}{49}$	$\frac{33}{99}$	$\frac{15}{95}$	$\frac{72}{144}$
2.	$\frac{9}{21}$	$\frac{18}{54}$	$\frac{2}{16}$	$\frac{6}{10}$
3.	$\frac{21}{56}$	$\frac{4}{24}$	$\frac{12}{12}$	$\frac{21}{28}$
4.	$\frac{15}{60}$	$\frac{18}{72}$	$\frac{9}{9}$	$\frac{34}{68}$
5.	$\frac{25}{115}$	$\frac{7}{91}$	$\frac{60}{90}$	$\frac{40}{48}$
6.	$\frac{9}{114}$	$\frac{14}{84}$	$\frac{36}{45}$	$\frac{96}{108}$
7.	$\frac{18}{24}$	$\frac{60}{120}$	$\frac{13}{39}$	$\frac{10}{35}$

You find a fractional part of a number by dividing the number by the number named in the denominator and multiplying this quotient by the number named in the numerator. Or multiply by the number named in the numerator and then divide by the number named in the denominator.

$$\frac{3}{7} \text{ of } 84 = (84 \div 7) \times 3 = 12 \times 3 = 36$$

or

$$\frac{3}{7} \text{ of } 84 = (3 \times 84) \div 7 = 252 \div 7 = 36$$

Use with page 189.

Copy. Solve, using either method.

	a	*b*
1.	$\frac{1}{4}$ of $52 = n$	$\frac{1}{2}$ of $74 = n$
2.	$\frac{3}{8}$ of $128 = n$	$\frac{4}{5}$ of $225 = n$
3.	$\frac{5}{6}$ of $930 = n$	$\frac{7}{9}$ of $837 = n$
4.	$\frac{3}{7}$ of $168 = n$	$\frac{3}{10}$ of $120 = n$
5.	$\frac{3}{4}$ of $96 = n$	$\frac{2}{9}$ of $135 = n$
6.	$\frac{5}{8}$ of $192 = n$	$\frac{3}{11}$ of $132 = n$

Use with page 190.

	a	*b*
1.	$\frac{1}{2}$ bu. = ___ qt.	$\frac{3}{4}$ gal. = ___ pt.
2.	$\frac{2}{3}$ hr. = ___ min.	45 min. = ___ hr.
3.	2 oz. = ___ lb.	3 da. = ___ wk.
4.	$\frac{2}{3}$ yd. = ___ in.	1 ft. = ___ yd.
5.	$\frac{2}{5}$ mi. = ___ ft.	$\frac{3}{5}$ mi. = ___ yd.
6.	310 rd. = ___ mi.	3 ft. = ___ in.

Fractions may be changed so they have a common denominator.

$$\frac{2}{3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15} \qquad \frac{3}{5} = \frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$$

Use with pages 194 and 195.

Find the common denominator of each set of fractions below.

	a	*b*
1.	$\frac{1}{6}, \frac{2}{3}$	$\frac{1}{2}, \frac{3}{5}$
2.	$\frac{3}{14}, \frac{2}{7}, \frac{1}{2}$	$\frac{1}{6}, \frac{2}{7}, \frac{3}{21}$
3.	$\frac{5}{6}, \frac{2}{3}, \frac{3}{21}$	$\frac{1}{5}, \frac{3}{15}, \frac{3}{10}$
4.	$\frac{1}{8}, \frac{5}{24}, \frac{1}{48}$	$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

Remainders in division may be written as fractions.

$$\begin{array}{r} 88\frac{5}{14} \\ 14\overline{)1237} \\ \underline{112} \\ 117 \\ \underline{112} \\ 5 \end{array} \qquad \begin{array}{r} 254\frac{21}{23} \\ 23\overline{)5863} \\ \underline{46} \\ 126 \\ \underline{115} \\ 113 \\ \underline{92} \\ 21 \end{array}$$

Use with page 197.

	a	*b*
1.	$32\overline{)711}$	$50\overline{)620}$
2.	$42\overline{)3903}$	$39\overline{)2861}$
3.	$81\overline{)7426}$	$70\overline{)6821}$
4.	$52\overline{)1963}$	$95\overline{)9636}$
5.	$40\overline{)4287}$	$36\overline{)2695}$

Addition of Fractional Numbers

In the addition of fractional numbers, the sum is best expressed in lowest terms.

$\frac{3}{10}+\frac{3}{10}=\frac{3+3}{10}=\frac{6}{10}=\frac{6\div2}{10\div2}=\frac{3}{5}$

$$\begin{array}{r} \frac{3}{10} \\ +\frac{3}{10} \\ \hline \frac{6}{10}=\frac{3}{5} \end{array}$$

Use with pages 201 through 203.

	a	*b*	*c*	*d*	*e*
1.	$\begin{array}{r}\frac{3}{16}\\+\frac{5}{16}\\\hline\end{array}$	$\begin{array}{r}\frac{4}{5}\\+\frac{1}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{18}\\+\frac{5}{18}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{14}\\+\frac{9}{14}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{4}\\+\frac{3}{4}\\\hline\end{array}$
2.	$\begin{array}{r}\frac{1}{8}\\+\frac{5}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{4}{15}\\+\frac{8}{15}\\\hline\end{array}$	$\begin{array}{r}\frac{17}{25}\\+\frac{3}{25}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{50}\\+\frac{3}{50}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{26}\\+\frac{3}{26}\\\hline\end{array}$
3.	$\begin{array}{r}\frac{7}{13}\\+\frac{6}{13}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{22}\\+\frac{13}{22}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{21}\\+\frac{9}{21}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{34}\\+\frac{6}{34}\\\hline\end{array}$	$\begin{array}{r}\frac{6}{49}\\+\frac{8}{49}\\\hline\end{array}$

A sum expressed as an improper fraction may be changed to a mixed fraction.

$\frac{5}{8}+\frac{5}{8}=\frac{5+5}{8}=\frac{10}{8}=1\frac{2}{8}=1\frac{1}{4}$

$$\begin{array}{r} \frac{5}{8} \\ +\frac{5}{8} \\ \hline \frac{10}{8}=1\frac{2}{8}=1\frac{1}{4} \end{array}$$

Use with page 204.

	a	*b*
1.	$\frac{4}{7}+\frac{4}{7}=n$	$\frac{3}{5}+\frac{4}{5}=n$
2.	$\frac{9}{15}+\frac{8}{15}=n$	$\frac{7}{12}+\frac{10}{12}=n$
3.	$\frac{17}{22}+\frac{19}{22}=n$	$\frac{9}{16}+\frac{11}{16}=n$
4.	$\frac{14}{17}+\frac{11}{17}=n$	$\frac{27}{51}+\frac{29}{51}=n$
5.	$\frac{51}{73}+\frac{31}{73}=n$	$\frac{31}{55}+\frac{29}{55}=n$

Use with page 205.

	a	*b*
1.	$\frac{5}{8}+\frac{2}{8}+\frac{7}{8}=n$	$\frac{2}{9}+\frac{7}{9}+\frac{5}{9}=n$
2.	$\frac{12}{15}+\frac{8}{15}+\frac{13}{15}=n$	$\frac{14}{18}+\frac{13}{18}+\frac{7}{18}=n$
3.	$\frac{8}{21}+\frac{9}{21}+\frac{10}{21}=n$	$\frac{2}{10}+\frac{2}{10}+\frac{4}{10}=n$
4.	$\frac{11}{17}+\frac{16}{17}+\frac{8}{17}=n$	$\frac{4}{25}+\frac{19}{25}+\frac{11}{25}=n$
5.	$\frac{40}{43}+\frac{29}{43}+\frac{20}{43}=n$	$\frac{24}{35}+\frac{13}{35}+\frac{8}{35}=n$
6.	$\frac{11}{27}+\frac{17}{27}+\frac{14}{27}=n$	$\frac{7}{11}+\frac{5}{11}+\frac{9}{11}=n$

Fractional numbers to be added must have a common denominator.

$$\begin{array}{r} \frac{3}{5}=\frac{3\times6}{5\times6}=\frac{18}{30} \\ +\frac{5}{6}=\frac{5\times5}{6\times5}=\frac{25}{30} \\ \hline \frac{43}{30}=\frac{30}{30}+\frac{13}{30}=1+\frac{13}{30}=1\frac{13}{30} \end{array}$$

Use with pages 206 and 207.

	a	*b*	*c*	*d*	*e*
1.	$\begin{array}{r}\frac{5}{8}\\+\frac{5}{6}\\\hline\end{array}$	$\begin{array}{r}\frac{10}{15}\\+\frac{3}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{2}{3}\\+\frac{7}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{7}\\+\frac{1}{3}\\\hline\end{array}$	$\begin{array}{r}\frac{15}{16}\\+\frac{5}{6}\\\hline\end{array}$
2.	$\begin{array}{r}\frac{11}{21}\\+\frac{4}{5}\\\hline\end{array}$	$\begin{array}{r}\frac{12}{15}\\+\frac{7}{8}\\\hline\end{array}$	$\begin{array}{r}\frac{8}{9}\\+\frac{10}{11}\\\hline\end{array}$	$\begin{array}{r}\frac{12}{15}\\+\frac{2}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{19}{20}\\+\frac{3}{17}\\\hline\end{array}$
3.	$\begin{array}{r}\frac{5}{9}\\+\frac{7}{15}\\\hline\end{array}$	$\begin{array}{r}\frac{1}{2}\\+\frac{7}{9}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{4}\\+\frac{8}{11}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{26}\\+\frac{5}{13}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{50}\\+\frac{4}{25}\\\hline\end{array}$
4.	$\begin{array}{r}\frac{14}{17}\\+\frac{1}{4}\\\hline\end{array}$	$\begin{array}{r}\frac{7}{18}\\+\frac{9}{26}\\\hline\end{array}$	$\begin{array}{r}\frac{17}{30}\\+\frac{14}{15}\\\hline\end{array}$	$\begin{array}{r}\frac{5}{17}\\+\frac{13}{14}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{5}\\+\frac{7}{10}\\\hline\end{array}$
5.	$\begin{array}{r}\frac{7}{8}\\+\frac{4}{7}\\\hline\end{array}$	$\begin{array}{r}\frac{11}{12}\\+\frac{1}{14}\\\hline\end{array}$	$\begin{array}{r}\frac{3}{23}\\+\frac{7}{46}\\\hline\end{array}$	$\begin{array}{r}\frac{9}{17}\\+\frac{19}{20}\\\hline\end{array}$	$\begin{array}{r}\frac{14}{15}\\+\frac{4}{5}\\\hline\end{array}$

Use with page 208.

	a	*b*
1.	$\frac{1}{2}+\frac{2}{3}+\frac{5}{6}=n$	$\frac{3}{5}+\frac{1}{3}+\frac{7}{10}=n$
2.	$\frac{2}{27}+\frac{7}{9}+\frac{7}{18}=n$	$\frac{12}{15}+\frac{17}{24}+\frac{5}{9}=n$
3.	$\frac{3}{4}+\frac{7}{12}+\frac{15}{16}=n$	$\frac{5}{9}+\frac{1}{2}+\frac{7}{18}=n$
4.	$\frac{3}{5}+\frac{5}{6}+\frac{4}{7}=n$	$\frac{7}{10}+\frac{8}{15}+\frac{17}{25}=n$

If addends are named by mixed fractions, add the fractional parts and then add the whole numbers.

$$\begin{array}{rcr} 4\frac{2}{5} & = & 4\frac{12}{30} \\ 5\frac{7}{10} & = & 5\frac{21}{30} \\ +\frac{4}{15} & = & \frac{8}{30} \\ \hline & & 9\frac{41}{30}=9+1+\frac{11}{30}=10\frac{11}{30} \end{array}$$

Use with pages 212 and 213.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 2\frac{3}{5} \\ +\frac{7}{10} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{1}{6} \\ +\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} 15\frac{8}{9} \\ +\frac{5}{18} \\ \hline \end{array}$	$\begin{array}{r} 26\frac{4}{21} \\ +\frac{5}{7} \\ \hline \end{array}$
2.	$\begin{array}{r} \frac{5}{12} \\ +11\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{9}{14} \\ +42\frac{13}{21} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{6}{18} \\ +\frac{7}{9} \\ \hline \end{array}$	$\begin{array}{r} \frac{10}{11} \\ +47\frac{21}{22} \\ \hline \end{array}$
3.	$\begin{array}{r} \frac{2}{3} \\ 14\frac{5}{7} \\ +\frac{5}{9} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{3}{10} \\ \frac{7}{11} \\ +\frac{4}{23} \\ \hline \end{array}$	$\begin{array}{r} \frac{13}{20} \\ \frac{15}{17} \\ +96\frac{7}{10} \\ \hline \end{array}$	$\begin{array}{r} 101\frac{3}{29} \\ \frac{4}{5} \\ +\frac{1}{2} \\ \hline \end{array}$
4.	$\begin{array}{r} 141\frac{1}{2} \\ \frac{3}{4} \\ +\frac{17}{56} \\ \hline \end{array}$	$\begin{array}{r} \frac{19}{21} \\ 283\frac{4}{7} \\ +\frac{3}{14} \\ \hline \end{array}$	$\begin{array}{r} 314\frac{16}{35} \\ \frac{13}{28} \\ +\frac{53}{63} \\ \hline \end{array}$	$\begin{array}{r} \frac{17}{54} \\ 87\frac{11}{18} \\ +\frac{7}{9} \\ \hline \end{array}$
5.	$\begin{array}{r} 14\frac{11}{12} \\ \frac{7}{16} \\ +\frac{17}{18} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{39} \\ \frac{12}{13} \\ +17\frac{15}{52} \\ \hline \end{array}$	$\begin{array}{r} 910\frac{7}{25} \\ \frac{17}{30} \\ +\frac{9}{10} \\ \hline \end{array}$	$\begin{array}{r} \frac{37}{42} \\ 84\frac{13}{14} \\ +\frac{69}{70} \\ \hline \end{array}$

Use with page 214.

Copy. Find each sum.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 5\frac{6}{7} \\ +8\frac{9}{14} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{11}{12} \\ +8\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 14\frac{2}{3} \\ +19\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{1}{9} \\ +7\frac{19}{36} \\ \hline \end{array}$
2.	$\begin{array}{r} 21\frac{11}{18} \\ +13\frac{11}{18} \\ \hline \end{array}$	$\begin{array}{r} 47\frac{21}{30} \\ +52\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 8\frac{7}{17} \\ +76\frac{15}{34} \\ \hline \end{array}$	$\begin{array}{r} 1\frac{15}{16} \\ +98\frac{5}{8} \\ \hline \end{array}$
3.	$\begin{array}{r} 14\frac{1}{7} \\ 5\frac{5}{14} \\ +\frac{9}{28} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{1}{12} \\ 13\frac{5}{6} \\ +21\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{21}{22} \\ 11 \\ +17\frac{3}{11} \\ \hline \end{array}$	$\begin{array}{r} 26\frac{13}{14} \\ 2\frac{3}{7} \\ +\frac{6}{35} \\ \hline \end{array}$
4.	$\begin{array}{r} 57\frac{19}{20} \\ 63\frac{4}{5} \\ +19\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 64\frac{7}{8} \\ 81\frac{31}{40} \\ +56\frac{19}{20} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{2}{5} \\ 13\frac{2}{7} \\ +15\frac{13}{35} \\ \hline \end{array}$	$\begin{array}{r} 16\frac{14}{15} \\ 5\frac{3}{4} \\ +11\frac{11}{15} \\ \hline \end{array}$
5.	$\begin{array}{r} \frac{19}{20} \\ 7\frac{4}{5} \\ +11\frac{9}{10} \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 17\frac{17}{19} \\ +28\frac{4}{5} \\ \hline \end{array}$	$\begin{array}{r} 14\frac{4}{15} \\ 13\frac{4}{5} \\ +\frac{17}{20} \\ \hline \end{array}$	$\begin{array}{r} 21\frac{11}{14} \\ 6\frac{5}{8} \\ +11 \\ \hline \end{array}$
6.	$\begin{array}{r} 15\frac{1}{3} \\ \frac{7}{9} \\ +21\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 5\frac{1}{5} \\ 18\frac{3}{10} \\ +1\frac{7}{15} \\ \hline \end{array}$	$\begin{array}{r} 21 \\ 3\frac{1}{2} \\ +\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{3}{10} \\ 11\frac{2}{5} \\ +\frac{9}{10} \\ \hline \end{array}$

Copy. Solve for n.

	a	*b*
7.	$5\frac{6}{7}+3\frac{1}{2}=n$	$4\frac{1}{2}+3\frac{1}{4}+5\frac{1}{3}=n$
8.	$3\frac{7}{8}+1\frac{5}{9}=n$	$\frac{18}{19}+11\frac{1}{2}+3\frac{21}{38}=n$
9.	$\frac{15}{16}+21\frac{5}{8}=n$	$7+\frac{10}{11}+84\frac{2}{3}=n$
10.	$4\frac{7}{9}+8\frac{7}{8}=n$	$3\frac{1}{10}+18\frac{11}{12}+3\frac{5}{14}=n$
11.	$\frac{15}{16}+96\frac{5}{8}=n$	$15+23\frac{7}{9}+9\frac{4}{5}=n$

Subtraction of Fractional Numbers

To subtract fractional numbers with the same denominator, write the difference of the numerators over the denominator.

$\frac{11}{13}-\frac{5}{13}=\frac{6}{13}$

$$\begin{array}{r} \frac{11}{13} \\ -\frac{5}{13} \\ \hline \frac{6}{13} \end{array}$$

Use with page 220.

	a	b
1.	$\frac{17}{19}-\frac{11}{19}=n$	$\frac{9}{27}-\frac{7}{27}=n$
2.	$\frac{26}{31}-\frac{16}{31}=n$	$\frac{41}{50}-\frac{18}{50}=n$
3.	$\frac{4}{11}-\frac{3}{11}=n$	$\frac{71}{72}-\frac{69}{72}=n$
4.	$\frac{35}{42}-\frac{16}{42}=n$	$\frac{20}{63}-\frac{16}{63}=n$

To subtract a fractional number from a whole number, you first rename 1 as a fraction.

$$\begin{array}{rr} 1 & =\frac{5}{5} \\ -\frac{2}{5} & =\frac{2}{5} \\ \hline & \frac{3}{5} \end{array}$$

$$\begin{array}{rl} 2\frac{1}{8} & =1\frac{9}{8} \\ -\frac{5}{8} & =\frac{5}{8} \\ \hline & 1\frac{4}{8}=1\frac{1}{2} \end{array}$$

Use with pages 221 and 222.

	a	b	c	d
1.	$\begin{array}{r} 13 \\ -\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 117 \\ -\frac{13}{16} \\ \hline \end{array}$	$\begin{array}{r} 1 \\ -\frac{17}{49} \\ \hline \end{array}$	$\begin{array}{r} 226 \\ -\frac{51}{100} \\ \hline \end{array}$
2.	$\begin{array}{r} 118 \\ -\frac{19}{27} \\ \hline \end{array}$	$\begin{array}{r} 42 \\ -\frac{9}{19} \\ \hline \end{array}$	$\begin{array}{r} 3 \\ -\frac{97}{108} \\ \hline \end{array}$	$\begin{array}{r} 21 \\ -\frac{14}{23} \\ \hline \end{array}$
3.	$\begin{array}{r} 1 \\ -\frac{83}{91} \\ \hline \end{array}$	$\begin{array}{r} 14 \\ -\frac{7}{63} \\ \hline \end{array}$	$\begin{array}{r} 92 \\ -\frac{17}{18} \\ \hline \end{array}$	$\begin{array}{r} 810 \\ -\frac{9}{15} \\ \hline \end{array}$

To subtract with mixed fractions, subtract the fractional parts and then subtract the whole numbers.

$$\begin{array}{r} 17\frac{5}{11} \\ -7 \\ \hline 10\frac{5}{11} \end{array} \qquad \begin{array}{r} 5\frac{5}{9} \\ -2\frac{4}{9} \\ \hline 3\frac{1}{9} \end{array} \qquad \begin{array}{rl} 5 & =4\frac{8}{8} \\ -3\frac{1}{8} & =3\frac{1}{8} \\ \hline & 1\frac{7}{8} \end{array}$$

Use with pages 224 and 225.

	a	b	c	d
1.	$\begin{array}{r} 31 \\ -8\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 26 \\ -19\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 112\frac{7}{12} \\ -83\frac{5}{12} \\ \hline \end{array}$	$\begin{array}{r} 120\frac{14}{15} \\ -47\frac{3}{15} \\ \hline \end{array}$
2.	$\begin{array}{r} 21\frac{7}{9} \\ -3\frac{5}{9} \\ \hline \end{array}$	$\begin{array}{r} 60\frac{2}{9} \\ -43\frac{5}{9} \\ \hline \end{array}$	$\begin{array}{r} 108\frac{13}{15} \\ -79\frac{8}{15} \\ \hline \end{array}$	$\begin{array}{r} 901\frac{4}{21} \\ -98\frac{19}{21} \\ \hline \end{array}$
3.	$\begin{array}{r} 26\frac{5}{6} \\ -9 \\ \hline \end{array}$	$\begin{array}{r} 57\frac{5}{6} \\ -29 \\ \hline \end{array}$	$\begin{array}{r} 200\frac{13}{25} \\ -67\frac{9}{25} \\ \hline \end{array}$	$\begin{array}{r} 108\frac{3}{16} \\ -78\frac{9}{16} \\ \hline \end{array}$
4.	$\begin{array}{r} 61\frac{4}{7} \\ -8\frac{1}{7} \\ \hline \end{array}$	$\begin{array}{r} 48\frac{7}{8} \\ -39\frac{3}{8} \\ \hline \end{array}$	$\begin{array}{r} 187\frac{19}{29} \\ -93\frac{3}{29} \\ \hline \end{array}$	$\begin{array}{r} 404\frac{2}{39} \\ -99\frac{13}{39} \\ \hline \end{array}$
5.	$\begin{array}{r} 26\frac{1}{6} \\ -9\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 89\frac{3}{8} \\ -39\frac{5}{8} \\ \hline \end{array}$	$\begin{array}{r} 191\frac{1}{51} \\ -87\frac{3}{51} \\ \hline \end{array}$	$\begin{array}{r} 183\frac{3}{16} \\ -6\frac{9}{16} \\ \hline \end{array}$

Copy. Solve for n.

	a	b
6.	$18\frac{1}{6}-13\frac{1}{6}=n$	$27\frac{9}{10}-17\frac{7}{10}=n$
7.	$34\frac{3}{7}-25\frac{2}{7}=n$	$38\frac{11}{15}-29\frac{8}{15}=n$
8.	$109\frac{3}{4}-99\frac{3}{4}=n$	$78-69\frac{1}{9}=n$
9.	$91\frac{5}{9}-72=n$	$48\frac{1}{8}-36\frac{3}{8}=n$
10.	$100\frac{1}{5}-75\frac{3}{5}=n$	$132\frac{7}{12}-96\frac{11}{12}=n$

To subtract fractional numbers with unequal denominators, you first rewrite the fractions so that they have a common denominator.

$$\begin{array}{r} \frac{7}{8}=\frac{7}{8} \\ -\frac{3}{4}=\frac{6}{8} \\ \hline \frac{1}{8} \end{array} \qquad \begin{array}{r} 8\frac{1}{6}=7\frac{7}{6} \\ -\frac{1}{3}=\frac{2}{6} \\ \hline 7\frac{5}{6} \end{array} \qquad \begin{array}{r} 9\frac{1}{2}=8\frac{9}{6} \\ -3\frac{2}{3}=3\frac{4}{6} \\ \hline 5\frac{5}{6} \end{array}$$

Use with page 226.

	a	*b*	*c*	*d*	*e*
1.	$\begin{array}{r} \frac{5}{8} \\ -\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{10} \\ -\frac{1}{4} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{6} \\ -\frac{1}{10} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{7} \\ -\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{5} \\ \hline \end{array}$
2.	$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{18} \\ -\frac{2}{27} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{22} \\ -\frac{2}{33} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{9} \\ -\frac{5}{36} \\ \hline \end{array}$	$\begin{array}{r} \frac{3}{5} \\ -\frac{4}{15} \\ \hline \end{array}$
3.	$\begin{array}{r} \frac{3}{4} \\ -\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} \frac{7}{9} \\ -\frac{2}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{17}{25} \\ -\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} \frac{1}{2} \\ -\frac{8}{17} \\ \hline \end{array}$	$\begin{array}{r} \frac{5}{6} \\ -\frac{3}{5} \\ \hline \end{array}$

Use with page 227.

Copy. Find each difference.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 17\frac{2}{7} \\ -\frac{5}{8} \\ \hline \end{array}$	$\begin{array}{r} 13\frac{5}{9} \\ -\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 17\frac{11}{12} \\ -\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 63\frac{3}{8} \\ -\frac{7}{10} \\ \hline \end{array}$
2.	$\begin{array}{r} 14\frac{2}{7} \\ -\frac{1}{14} \\ \hline \end{array}$	$\begin{array}{r} 37\frac{1}{11} \\ -\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 21\frac{7}{12} \\ -\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 51\frac{2}{17} \\ -\frac{5}{9} \\ \hline \end{array}$
3.	$\begin{array}{r} 19\frac{1}{4} \\ -\frac{5}{7} \\ \hline \end{array}$	$\begin{array}{r} 40\frac{5}{9} \\ -\frac{5}{7} \\ \hline \end{array}$	$\begin{array}{r} 1\frac{1}{3} \\ -\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 2\frac{1}{12} \\ -\frac{1}{15} \\ \hline \end{array}$
4.	$\begin{array}{r} 2\frac{4}{15} \\ -\frac{15}{16} \\ \hline \end{array}$	$\begin{array}{r} 4\frac{1}{10} \\ -\frac{3}{15} \\ \hline \end{array}$	$\begin{array}{r} 20\frac{1}{14} \\ -\frac{2}{7} \\ \hline \end{array}$	$\begin{array}{r} 3\frac{7}{11} \\ -\frac{7}{10} \\ \hline \end{array}$
5.	$\begin{array}{r} 18\frac{1}{5} \\ -\frac{3}{6} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{3}{4} \\ -\frac{6}{7} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{5}{18} \\ -\frac{14}{15} \\ \hline \end{array}$	$\begin{array}{r} 91\frac{1}{20} \\ -\frac{3}{5} \\ \hline \end{array}$

Copy. Solve for n.

	a	*b*
1.	$6\frac{1}{9}-\frac{2}{3}=n$	$4\frac{3}{8}-\frac{3}{10}=n$
2.	$7\frac{5}{18}-6\frac{1}{3}=n$	$17\frac{1}{6}-\frac{1}{2}=n$
3.	$4\frac{3}{8}-\frac{3}{7}=n$	$1\frac{1}{6}-\frac{1}{18}=n$
4.	$30\frac{3}{8}-\frac{3}{4}=n$	$1\frac{1}{2}-\frac{5}{8}=n$

Use with page 228.

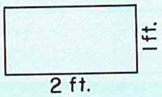
Copy. Solve for n.

	a	*b*
1.	$6\frac{1}{3}-4\frac{1}{2}=n$	$5\frac{1}{5}-2\frac{1}{4}=n$
2.	$16\frac{2}{7}-15\frac{3}{5}=n$	$11\frac{2}{9}-5\frac{5}{6}=n$
3.	$24\frac{1}{12}-18\frac{3}{4}=n$	$42\frac{7}{15}-37\frac{7}{8}=n$
4.	$51\frac{3}{8}-15\frac{3}{5}=n$	$96\frac{2}{3}-17\frac{2}{3}=n$

Copy. Find each difference.

	a	*b*	*c*	*d*
1.	$\begin{array}{r} 54\frac{5}{9} \\ -17\frac{1}{6} \\ \hline \end{array}$	$\begin{array}{r} 28\frac{3}{7} \\ -11\frac{5}{14} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{4}{15} \\ -5\frac{7}{12} \\ \hline \end{array}$	$\begin{array}{r} 36\frac{3}{8} \\ -18\frac{1}{2} \\ \hline \end{array}$
2.	$\begin{array}{r} 15\frac{1}{7} \\ -9\frac{3}{14} \\ \hline \end{array}$	$\begin{array}{r} 9\frac{5}{18} \\ -7\frac{7}{9} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{3}{7} \\ -9\frac{5}{6} \\ \hline \end{array}$	$\begin{array}{r} 59\frac{1}{4} \\ -16\frac{11}{12} \\ \hline \end{array}$
3.	$\begin{array}{r} 49\frac{1}{14} \\ -14\frac{2}{7} \\ \hline \end{array}$	$\begin{array}{r} 24\frac{1}{15} \\ -13\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 63\frac{7}{9} \\ -56\frac{11}{18} \\ \hline \end{array}$	$\begin{array}{r} 108\frac{4}{11} \\ -96\frac{10}{11} \\ \hline \end{array}$
4.	$\begin{array}{r} 21\frac{3}{20} \\ -16\frac{4}{15} \\ \hline \end{array}$	$\begin{array}{r} 70\frac{1}{4} \\ -59\frac{1}{5} \\ \hline \end{array}$	$\begin{array}{r} 29\frac{4}{13} \\ -9\frac{8}{9} \\ \hline \end{array}$	$\begin{array}{r} 111\frac{1}{6} \\ -107\frac{5}{9} \\ \hline \end{array}$
5.	$\begin{array}{r} 17\frac{1}{3} \\ -9\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 25\frac{3}{10} \\ -11\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 36\frac{2}{15} \\ -10\frac{3}{5} \\ \hline \end{array}$	$\begin{array}{r} 14\frac{1}{7} \\ -6\frac{17}{21} \\ \hline \end{array}$

You find the perimeter of a rectangle by multiplying the sum of its length and its width by 2.



$p=2\times(1+2)=2\times3=6$

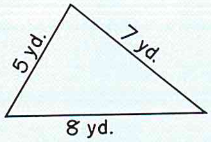
The perimeter of the rectangle is 6 feet.

Use with pages 248 and 249.

Find the perimeter of a rectangle with the following dimensions.

1. 3 yd. and 14 ft.
2. 69 ft. and 142 ft.
3. 3 ft. and 18 in.

To find the perimeter of a triangle, add the measures of its sides.



$p=5+7+8=20$

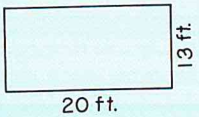
The perimeter of the triangle is 20 yards.

Use with page 250.

Find the perimeter of a triangle with the following dimensions.

1. 18 ft., 18 ft., 7 yd.
2. 115 ft., 97 ft., 79 ft.
3. $10\frac{1}{2}$ yd., $13\frac{1}{2}$ yd., $15\frac{1}{4}$ yd.

The area of a rectangle is the product of its length and its width. It is expressed in square measure.



$13\times20=260$

The area of the rectangle is 260 square feet.

Use with pages 254 through 256.

Find the area of a rectangle with the following dimensions.

1. 17 ft. and 49 ft.
2. 108 yd. and 317 yd.
3. 117 ft. and 288 ft.
4. 15 yd. and 67 ft.

The area of land is usually measured in acres.

43,560 sq. ft. = 1 acre
160 sq. rd. = 1 acre
640 acres = 1 sq. mi.

Use with page 260.

Copy. Replace each blank so that the statement becomes true.

1. 12 sq. mi. = ___ acres
2. 13 acres = ___ sq. ft.
3. $\frac{3}{4}$ acre = ___ sq. rd.
4. ___ sq. ft. = $\frac{6}{10}$ acre

Adding with Decimal Numerals

If a fraction has a denominator of 10 or 100, the fraction can easily be written as a decimal.

$\frac{3}{10}$ is written .3
$\frac{9}{100}$ is written .09

Use with pages 281 and 282.

Write the following as decimals.

	a	*b*	*c*	*d*
1.	$\frac{7}{10}$	$1\frac{11}{100}$	$\frac{8}{10}$	$20\frac{4}{10}$
2.	$15\frac{7}{100}$	$\frac{75}{100}$	$27\frac{9}{10}$	$1\frac{97}{100}$

Write the following as fractions or as mixed fractions.

	a	*b*	*c*	*d*
3.	105.1	80.51	.08	.9
4.	315.62	4.06	37.6	.63

Add with decimals as you add fractional and whole numbers.

$$\begin{array}{r}\frac{7}{10}\\+\frac{4}{10}\\\hline\frac{11}{10}=1\frac{1}{10}\end{array}\qquad\begin{array}{r}7\text{ tenths}\\+4\text{ tenths}\\\hline 11\text{ tenths}\end{array}\qquad\begin{array}{r}.7\\+.4\\\hline 1.1\end{array}$$

Use with page 283.

Copy. Solve by using the decimal form.

	a	*b*
1.	.5+.6=n	.1+.8=n
2.	.8+.3=n	.4+.2=n
3.	.9+.7=n	.6+.9=n
4.	.7+.5=n	.1+.9=n
5.	.9+.1=n	.7+.3=n

When decimals are given in column form, the decimal points should be written on a vertical line.

$$\begin{array}{r}.17\\+.85\\\hline 1.02\end{array}\qquad\begin{array}{r}17\text{ hundredths}\\+85\text{ hundredths}\\\hline 102\text{ hundredths}\end{array}\qquad\begin{array}{r}\$\ .17\\+.85\\\hline \$1.02\end{array}$$

Use with pages 284 through 287.

	a	*b*	*c*	*d*
1.	$\begin{array}{r}.98\\+.04\\\hline\end{array}$	$\begin{array}{r}.06\\+.34\\\hline\end{array}$	$\begin{array}{r}.35\\+.87\\\hline\end{array}$	$\begin{array}{r}\$.76\\+.24\\\hline\end{array}$
2.	$\begin{array}{r}.65\\+.53\\\hline\end{array}$	$\begin{array}{r}.89\\+.27\\\hline\end{array}$	$\begin{array}{r}.09\\+.85\\\hline\end{array}$	$\begin{array}{r}\$.50\\+.62\\\hline\end{array}$
3.	$\begin{array}{r}.8\\+.45\\\hline\end{array}$	$\begin{array}{r}.87\\+.6\\\hline\end{array}$	$\begin{array}{r}.3\\+.98\\\hline\end{array}$	$\begin{array}{r}\$.05\\+.90\\\hline\end{array}$

Use with pages 286 and 287.

	a	*b*	*c*
1.	$\begin{array}{r}57.4\\+6.81\\\hline\end{array}$	$\begin{array}{r}1.96\\+27.3\\\hline\end{array}$	$\begin{array}{r}102.31\\+43.52\\\hline\end{array}$
2.	$\begin{array}{r}.76\\27.43\\+5.01\\\hline\end{array}$	$\begin{array}{r}218.6\\50.14\\+.93\\\hline\end{array}$	$\begin{array}{r}371.06\\320.83\\+96.15\\\hline\end{array}$
3.	$\begin{array}{r}8.7\\15.84\\207.16\\+100.42\\\hline\end{array}$	$\begin{array}{r}595.06\\14.29\\326.4\\+.84\\\hline\end{array}$	$\begin{array}{r}72.94\\180.80\\673.5\\+213.62\\\hline\end{array}$

Subtracting with Decimal Numerals

You subtract with decimals as you subtract whole numbers.

$$\begin{array}{r} 8\frac{2}{10} = 7\frac{12}{10} \\ -6\frac{7}{10} = 6\frac{7}{10} \\ \hline 1\frac{5}{10} \end{array} \qquad \begin{array}{r} 8.2 \\ -6.7 \\ \hline 1.5 \end{array}$$

Use with page 288.

Copy the following in column form. Find each difference.

	a	*b*
1.	$10.1 - 7.3 = n$	$4.09 - 1.89 = n$
2.	$28.7 - 9.9 = n$	$79.22 - 78.29 = n$
3.	$26.0 - 18.1 = n$	$53.2 - 16.16 = n$

When numbers are subtracted in column form, the decimal points should lie on a vertical line.

$$\begin{array}{r} 41.27 \\ -17.3 \\ \hline 23.97 \end{array} \qquad \begin{array}{r} 34.62 \\ -17. \\ \hline 17.62 \end{array} \qquad \begin{array}{r} 65. \\ -38.91 \\ \hline 26.09 \end{array}$$

Use with page 289.

	a	*b*	*c*
1.	$\begin{array}{r} 80.29 \\ -48.39 \\ \hline \end{array}$	$\begin{array}{r} 63.01 \\ -28.89 \\ \hline \end{array}$	$\begin{array}{r} 208.3 \\ -79.5 \\ \hline \end{array}$
2.	$\begin{array}{r} 64. \\ -24.75 \\ \hline \end{array}$	$\begin{array}{r} 59.1 \\ -29. \\ \hline \end{array}$	$\begin{array}{r} 1600.19 \\ -403.8 \\ \hline \end{array}$
3.	$\begin{array}{r} 92.7 \\ -69.05 \\ \hline \end{array}$	$\begin{array}{r} 356.4 \\ -166.5 \\ \hline \end{array}$	$\begin{array}{r} 4716.3 \\ -716.4 \\ \hline \end{array}$
4.	$\begin{array}{r} 91.01 \\ -42.8 \\ \hline \end{array}$	$\begin{array}{r} 129.3 \\ -89.29 \\ \hline \end{array}$	$\begin{array}{r} 3796.3 \\ -357.47 \\ \hline \end{array}$

To solve a problem with decimals, you decide what it tells, what it asks, and how you will find the answer. Then you write the equation and solve the problem.

Jerry is 56.25 inches tall and Al is 53.75 inches tall. How much shorter is Al than Jerry?

$$56.25 - 53.75 = n \qquad \begin{array}{r} 56.25 \\ -53.75 \\ \hline 2.50 \end{array}$$

$$2.50 = n$$

Al is 2.50 (or 2.5) inches shorter than Jerry.

Use with page 290.

Solve the following problems.

1. Paul wants to buy a bicycle for $49.95. He has saved $28.98. How much more does he need?

2. Mr. Black stopped for gas when the odometer on his car read 44018.2 miles. When he stopped for gas the next time it read 44203.5 miles. How many miles had he driven?

3. At 8 a.m. the thermometer read 78.3°. At noon the temperature was 93.2°. How much had the temperature risen since 8 o'clock?

4. Mary's mother bought a coat on sale for $59.95. It had been reduced from $74.50. How much did she save by buying the coat on sale?

Diagnostic Self-Tests

The tests on pages 319 through 322 of your book will help you and your teacher see how much you have learned in arithmetic. Do each test carefully, and check your work before you give your paper to your teacher.

Self-Test 1—The Meaning of Numbers

Copy these numerals, using commas. After each numeral write the words you would use to read it.

	a	*b*
1.	29364	500037
2.	12109002056	100961240120

Round off each number to the nearer ten, hundred, and thousand.

	a	*b*	*c*
3.	9,548	6,853	11,961

Express the numbers named by the number words below, first in Hindu-Arabic numerals and then in Roman numerals.

4. Two hundred fourteen

5. One thousand, ninety-two

Self-Test 2—Addition

Copy. Replace each blank with the word or sign from the following list that makes the statement true: sum, addends, 0, 10, quotient, number.

1. The identity number of addition is —.

2. Regrouping the — does not change the sum.

3. Adding is simpler if you look for combinations of —.

Copy. Find each sum. Check by adding in another order.

	a	*b*	*c*	*d*
4.	808	571	900	710
	26	88	846	423
	142	151	552	118
	+235	+77	+763	+237

Copy. Find each sum.

	a	*b*	*c*
5.	5240	8692	$4.98
	378	3917	5.06
	+2136	+9008	+1.03
6.	10183	24678	$197.61
	6432	43902	843.02
	56305	80613	90.89
	+10567	+54302	+716.53

Copy in column form. Solve. When necessary, reduce to lowest terms.

	a	*b*
7.	$\frac{6}{12}+\frac{9}{12}+\frac{1}{12}=n$	$1\frac{3}{4}+4\frac{3}{4}+\frac{1}{4}=n$
8.	$\frac{1}{7}+\frac{2}{3}+\frac{1}{6}=n$	$15+4\frac{5}{8}+11\frac{3}{4}=n$

(Turn the page)

Self-Test 2 (*continued*)

	a	*b*
9.	$.6 + .54 = n$	$\$.98 + \$.76 = n$
10.	$1.89 + 8.06 = n$	$\$60.50 + \$8.59 = n$

Copy. Find each sum.

	a	*b*	*c*
11.	$\begin{array}{r} 24\frac{3}{7} \\ 15\frac{3}{4} \\ +6\frac{1}{2} \\ \hline \end{array}$	$\begin{array}{r} 42\frac{5}{9} \\ 1\frac{5}{6} \\ +62\frac{17}{18} \\ \hline \end{array}$	$\begin{array}{r} 10\frac{5}{9} \\ 16\frac{7}{10} \\ +\frac{3}{15} \\ \hline \end{array}$
12.	$\begin{array}{r} 9\frac{6}{25} \\ 70\frac{3}{50} \\ +11\frac{1}{3} \\ \hline \end{array}$	$\begin{array}{r} 18\frac{1}{2} \\ 24\frac{4}{5} \\ +4\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 50\frac{7}{8} \\ 31\frac{5}{7} \\ +6\frac{5}{28} \\ \hline \end{array}$
13.	$\begin{array}{r} 102.6 \\ 569.02 \\ +84.88 \\ \hline \end{array}$	$\begin{array}{r} 29.69 \\ 803.8 \\ +562.91 \\ \hline \end{array}$	$\begin{array}{r} \$462.32 \\ 291.75 \\ +49.95 \\ \hline \end{array}$
14.	$\begin{array}{r} 606.99 \\ 412.12 \\ 386.47 \\ +203.56 \\ \hline \end{array}$	$\begin{array}{r} 804.08 \\ 933.0 \\ 486.51 \\ +390.87 \\ \hline \end{array}$	$\begin{array}{r} \$\ 89.60 \\ 100.59 \\ 468.21 \\ +505.70 \\ \hline \end{array}$

Self-Test 3—Subtraction

Copy each word and sign below that makes you think of subtraction.

	a	*b*
1.	addend	minuend
2.	difference	sum
3.	+	×
4.	subtrahend	÷
5.	quotient	−

Copy. Find each difference. Check by adding.

	a	*b*	*c*	*d*
6.	$\begin{array}{r} 90 \\ -70 \\ \hline \end{array}$	$\begin{array}{r} 65 \\ -15 \\ \hline \end{array}$	$\begin{array}{r} 802 \\ -108 \\ \hline \end{array}$	$\begin{array}{r} 201 \\ -109 \\ \hline \end{array}$
7.	$\begin{array}{r} \frac{18}{19} \\ -\frac{7}{19} \\ \hline \end{array}$	$\begin{array}{r} \frac{35}{49} \\ -\frac{4}{7} \\ \hline \end{array}$	$\begin{array}{r} 8 \\ -\frac{2}{3} \\ \hline \end{array}$	$\begin{array}{r} 11\frac{13}{25} \\ -\frac{17}{25} \\ \hline \end{array}$
8.	$\begin{array}{r} 42\frac{1}{4} \\ -32\frac{3}{4} \\ \hline \end{array}$	$\begin{array}{r} 53\frac{2}{5} \\ -39\frac{7}{8} \\ \hline \end{array}$	$\begin{array}{r} 76\frac{9}{10} \\ -58 \\ \hline \end{array}$	$\begin{array}{r} 84 \\ -29\frac{15}{17} \\ \hline \end{array}$
9.	$\begin{array}{r} 18.2 \\ -9.5 \\ \hline \end{array}$	$\begin{array}{r} 83. \\ -66.7 \\ \hline \end{array}$	$\begin{array}{r} 7.07 \\ -5.9 \\ \hline \end{array}$	$\begin{array}{r} \$7.61 \\ -.98 \\ \hline \end{array}$
10.	$\begin{array}{r} 9.27 \\ -6.49 \\ \hline \end{array}$	$\begin{array}{r} 4.02 \\ -1.08 \\ \hline \end{array}$	$\begin{array}{r} 6.3 \\ -4.28 \\ \hline \end{array}$	$\begin{array}{r} \$20.01 \\ -14.89 \\ \hline \end{array}$

Copy. Find each difference. Check.

	a	*b*	*c*
11.	$\begin{array}{r} 3864 \\ -907 \\ \hline \end{array}$	$\begin{array}{r} 4700 \\ -900 \\ \hline \end{array}$	$\begin{array}{r} 5020 \\ -827 \\ \hline \end{array}$
12.	$\begin{array}{r} 9436 \\ -5949 \\ \hline \end{array}$	$\begin{array}{r} 83001 \\ -4961 \\ \hline \end{array}$	$\begin{array}{r} 51010 \\ -38650 \\ \hline \end{array}$
13.	$\begin{array}{r} 529.1 \\ -403.2 \\ \hline \end{array}$	$\begin{array}{r} 60.01 \\ -49.8 \\ \hline \end{array}$	$\begin{array}{r} \$189.26 \\ -79.42 \\ \hline \end{array}$
14.	$\begin{array}{r} 43.3 \\ -27.08 \\ \hline \end{array}$	$\begin{array}{r} 387.1 \\ -288.29 \\ \hline \end{array}$	$\begin{array}{r} \$1412.90 \\ -1024.89 \\ \hline \end{array}$

Copy in column form. Solve.

	a	*b*
15.	$24\frac{1}{9} - 15\frac{2}{5} = n$	$80 - 39\frac{3}{7} = n$
16.	$54.2 - 5.9 = n$	$58.26 - 2.58 = n$

Self-Test 4—Multiplication

Copy. Replace each blank with either = or ≠ to make each statement true.

	a	*b*
1.	41×0 __ 41	0×28 __ 0
2.	91×1 __ 91	1×87 __ 0
3.	5×18 __ 18×5	23×4 __ 96

Copy. Find each product. Then reverse the factors and multiply again.

	a	*b*	*c*	*d*
4.	$\begin{array}{r}89\\ \times8\\ \hline\end{array}$	$\begin{array}{r}508\\ \times7\\ \hline\end{array}$	$\begin{array}{r}987\\ \times9\\ \hline\end{array}$	$\begin{array}{r}621\\ \times5\\ \hline\end{array}$
5.	$\begin{array}{r}80\\ \times60\\ \hline\end{array}$	$\begin{array}{r}300\\ \times40\\ \hline\end{array}$	$\begin{array}{r}18\\ \times47\\ \hline\end{array}$	$\begin{array}{r}82\\ \times48\\ \hline\end{array}$
6.	$\begin{array}{r}859\\ \times21\\ \hline\end{array}$	$\begin{array}{r}425\\ \times30\\ \hline\end{array}$	$\begin{array}{r}907\\ \times871\\ \hline\end{array}$	$\begin{array}{r}468\\ \times315\\ \hline\end{array}$
7.	$\begin{array}{r}\$.65\\ \times23\\ \hline\end{array}$	$\begin{array}{r}\$.84\\ \times19\\ \hline\end{array}$	$\begin{array}{r}\$.48\\ \times30\\ \hline\end{array}$	$\begin{array}{r}\$.60\\ \times75\\ \hline\end{array}$
8.	$\begin{array}{r}\$.95\\ \times45\\ \hline\end{array}$	$\begin{array}{r}\$.42\\ \times19\\ \hline\end{array}$	$\begin{array}{r}\$.64\\ \times20\\ \hline\end{array}$	$\begin{array}{r}\$.70\\ \times57\\ \hline\end{array}$
9.	$\begin{array}{r}\$.14\\ \times43\\ \hline\end{array}$	$\begin{array}{r}\$.09\\ \times84\\ \hline\end{array}$	$\begin{array}{r}\$.89\\ \times31\\ \hline\end{array}$	$\begin{array}{r}\$.67\\ \times55\\ \hline\end{array}$

Copy. Find each product. Check by multiplying again.

	a	*b*	*c*
10.	$\begin{array}{r}2004\\ \times2\\ \hline\end{array}$	$\begin{array}{r}6193\\ \times6\\ \hline\end{array}$	$\begin{array}{r}3500\\ \times47\\ \hline\end{array}$
11.	$\begin{array}{r}5280\\ \times25\\ \hline\end{array}$	$\begin{array}{r}2765\\ \times75\\ \hline\end{array}$	$\begin{array}{r}5300\\ \times340\\ \hline\end{array}$
12.	$\begin{array}{r}9652\\ \times405\\ \hline\end{array}$	$\begin{array}{r}6117\\ \times519\\ \hline\end{array}$	$\begin{array}{r}4995\\ \times374\\ \hline\end{array}$

Self-Test 5—Division

Write the equation that expresses the inverse of each of the following. Solve each equation.

	a	*b*
1.	$8\overline{)32}$	$6\overline{)48}$
2.	$8\times11=n$	$16\div4=n$

Copy. Find each quotient. Check by multiplying.

	a	*b*	*c*
3.	$4\overline{)76}$	$6\overline{)834}$	$9\overline{)2790}$
4.	$37\overline{)82}$	$46\overline{)92}$	$90\overline{)630}$
5.	$57\overline{)2850}$	$45\overline{)2925}$	$52\overline{)2715}$
6.	$36\overline{)3960}$	$38\overline{)4674}$	$27\overline{)8545}$

Copy. Find each quotient. Check.

	a	*b*
7.	$35\overline{)14525}$	$70\overline{)95970}$
8.	$92\overline{)815304}$	$53\overline{)772369}$
9.	$46\overline{)484023}$	$68\overline{)618940}$

Self-Test 6—Measures

Write the number of each statement below. Then write T or F after each number to show whether the statement is true or false.

1. You would measure the time in hours between noon and 6 p.m. by direct measurement.

2. All measurements are approximate—never exact.

3. Units of measure have been standardized by law.

Write each statement below, supplying the numeral or numerals in each one that makes the statement true.

	a	*b*
4.	$1\frac{1}{3}$ mi. = ___ft.	5 sq. mi. = ___acres
5.	25 bu. = ___qt.	$2\frac{1}{4}$ gal. = ___pt.
6.	$1\frac{1}{4}$ yr. = ___mo.	$5\frac{1}{2}$ cwt. = ___lb.

Find the perimeter of figures with dimensions as follows:

7. Rectangle, 1 yd. 2 in. by 4 yd.

8. Square, each side 7 ft. 1 in.

9. Triangle, with sides 10 in., $1\frac{1}{2}$ ft., and 8 in.

Self-Test 7—Problem Solving

Solve each problem below.

1. In September there were 387 pupils in the Wadsworth School. By December, 46 new pupils enrolled. What was the average size of each of the 8 grades in the school?

2. At the P.T.A. carnival Alice spent 20¢ for a sandwich, 10¢ for a soft drink, 15¢ for a pony ride, and 7¢ for candy. How much change did she have left from $1.25?

3. Fred left for school at 8:45 a.m. He returned home at 3:20 p.m. How long had he been gone?

4. Tom weighed $52\frac{1}{2}$ lb., Jack weighed $57\frac{1}{4}$ lb., and Phil weighed $59\frac{3}{8}$ lb. How much did the three boys weigh altogether?

5. What is the perimeter of a rectangular field measuring 155 ft. by 84 ft.? What is its area in square feet? In square yards?

6. Mr. Barton bought 50.3 gal. of gas in October, 63.8 gal. in November, 47.7 gal. in December, and 43.8 gal. in January. How many gallons of gas did he buy altogether?

7. Mrs. Judson paid a heating bill of $77.87 in February. In January she paid $81.37. How much more did she pay in January than in February?

8. Mrs. Ames had $637.89 in her checking account. She made out checks for $110.00, $14.37, $29.61, and $68.54. She then deposited $250.00. What was her balance?

HANDBOOK

Many of your questions about arithmetic can be answered by using this handbook. Form the habit of turning to it when you need to review an idea you have studied.

The handbook is easy to use. Its seven sections are arranged in alphabetical order. Each section is divided into parts. The numerals in ( ) tell the pages in the book that give more information on the topic.

Addition

The Main Ideas

1. Meanings, words, and signs When you are to find the sum of two numbers, you should add. (19, 27, 88)

$7+9=16$

$$\begin{array}{rl} 7 & \text{addend} \\ +9 & \text{addend} \\ \hline 16 & \text{sum} \end{array}$$

2. Commutative property The order of addition of two numbers does not change the sum. (19, 88)

$8+6=14$ $6+8=14$

3. Checking addition Add in a different order. (26, 88)

$5+6+7=18$ $7+6+5=18$

4. Associative property Regrouping addends does not change the sum. (22-23)

$8+9+5=n$

$$\begin{cases} (8+9)+5=n \\ 8+(9+5)=n \end{cases}$$

5. Zero is the identity number of addition A number remains the same, whether you add 0 to the number or add the number to 0. (19)

$7+0=7$ $0+7=7$

How to Add

1. Learn to use the basic addition combinations Then you will be able to add the digits in any place-value position. (20, 21, 26, 27, 30)

$$\begin{array}{r} 58 \\ +41 \\ \hline 99 \end{array} \qquad \begin{array}{r} 234 \\ +763 \\ \hline 997 \end{array} \qquad \begin{array}{r} 42316 \\ +41502 \\ \hline 83818 \end{array}$$

2. Make changes in form Change the form of the sum in any place-value position whenever it is greater than 9. (21, 26, 27, 30)

$$\begin{array}{r} 135 \\ +97 \\ \hline 232 \end{array} \qquad \begin{array}{r} 3412 \\ +6589 \\ \hline 10001 \end{array} \qquad \begin{array}{r} 56738 \\ +47275 \\ \hline 104013 \end{array}$$

Decimals

The Main Ideas

1. Meanings, words, and signs Fractions and mixed fractions with denominators of 10 or 100 may be written as decimal numerals. (281, 282)

$\frac{5}{10}$ is written .5
$\frac{3}{100}$ is written .03
$4\frac{17}{100}$ is written 4.17

2. The decimal point separates numerals naming a whole number from those naming a number less than one The numerals to the left of the decimal point name a whole number. The numerals to the right of the decimal point name a number less than one. (281)

Hundreds	Tens	Ones	Tenths	Hundredths
2	6	3.	4	5

How to Add and Subtract Decimal Numbers

1. You add and subtract with decimals in the same way as fractional and whole numbers (283, 288)

$\frac{8}{10}+\frac{9}{10}=\frac{17}{10}=1\frac{7}{10}$ $\qquad .8+.9=1.7$

$6\frac{3}{10}-1\frac{2}{10}=5\frac{1}{10}$ $\qquad 6.3-1.2=5.1$

2. In column form, write the numerals so that the decimal points lie on a vertical line (284, 289)

$$\begin{array}{r} .44 \\ +.59 \\ \hline 1.03 \end{array} \qquad \begin{array}{r} 66.5 \\ -17.7 \\ \hline 48.8 \end{array}$$

3. You add or subtract hundredths when you add or subtract numbers that stand for money (284, 285)

$$\begin{array}{r} \$.05 \\ +.20 \\ \hline \$.25 \end{array} \qquad \begin{array}{r} \$7.18 \\ -5.39 \\ \hline \$1.79 \end{array}$$

Division

The Main Ideas

1. Meanings, words, and signs When a set or group of things is to be separated into sets with an equal number you use the operation of division with the numbers. (73, 74, 76, 89)

$$546 \div 6 = 91 \qquad \begin{array}{rrl} & 91 & \text{quotient} \\ \text{divisor} & 6\overline{)546} & \text{dividend} \end{array}$$

2. Zero in division When you divide 0 by a number the quotient is 0. Dividing by 0 is meaningless. (75)

$$\begin{array}{r} 0 \\ 2\overline{)0} \end{array} \qquad \begin{array}{r} 0 \\ 58\overline{)0} \end{array}$$

3. Zero and non-zero remainders The part of the dividend left over after the final subtraction is called the remainder. Write a remainder, other than zero, beside the quotient. (78, 79)

$$\begin{array}{rcc} & & 3 \\ \cline{2-3} 6) & 1 & 8 \\ & 1 & 8 \\ \cline{2-3} & & 0 \end{array} \qquad \begin{array}{rccl} & & 9 & \text{r}3 \\ \cline{2-4} 5) & 4 & 8 & \\ & 4 & 5 & \\ \cline{2-3} & & 3 & \end{array}$$

4. **Checking division** Multiply the divisor by the quotient; add the remainder. The result should be the dividend. (77, 78, 89)

$$\begin{array}{rcc} & 3 & 2 \\ \cline{2-3} 3) & 9 & 6 \\ & 9 & \\ \cline{2-2} & & 6 \\ & & 6 \\ \cline{3-3} & & 0 \end{array} \qquad \begin{array}{r} 3 \\ \times 32 \\ \hline 6 \\ 90 \\ \hline 96 \end{array} \qquad \begin{array}{rccl} & 1 & 3 & \text{r}3 \\ \cline{2-4} 4) & 5 & 5 & \\ & 4 & & \\ \cline{2-2} & 1 & 5 & \\ & 1 & 2 & \\ \cline{2-3} & & 3 & \end{array} \qquad \begin{array}{r} 13 \\ \times 4 \\ \hline 52 \\ +3 \\ \hline 55 \end{array}$$

5. **Finding averages** You divide the sum by the number of addends. The result is the average. (84)

$$\begin{array}{r} 86 \\ 45 \\ +31 \\ \hline 162 \end{array} \qquad \begin{array}{rccc} & & 5 & 4 \\ \cline{2-4} 3) & 1 & 6 & 2 \\ & 1 & 5 & \\ \cline{2-3} & & 1 & 2 \\ & & 1 & 2 \\ \cline{3-4} & & & 0 \end{array}$$

6. **Finding trial quotients** Divide the first digit of the dividend by the first digit of the divisor, changing the form of the dividend or rounding off numbers as necessary. (77, 98, 99, 102, 103, 105, 148-150)

96÷4=n Think: 9 T÷4 0=2 T

960÷32=n Think: 9 H÷3 T=3 T

160÷32=n Think: 16 T÷3 T=5 0

5850÷39=n Think: 5 Th÷4 T=1 H

How to Divide—One-Digit Divisor

1. **Learn to use the basic division combinations** Then you can take these steps in division: Divide, multiply, and subtract. (77, 82, 87)

$$\begin{array}{rc} & 2 \\ \cline{2-2} 4) & 8 \\ & 8 \\ \cline{2-2} & 0 \end{array} \qquad \begin{array}{rcc} & 2 & 1 \\ \cline{2-3} 4) & 8 & 4 \\ & 8 & \\ \cline{2-2} & & 4 \\ & & 4 \\ \cline{3-3} & & 0 \end{array} \qquad \begin{array}{rccc} & & 7 & 0 \\ \cline{2-4} 5) & 3 & 5 & 0 \\ & 3 & 5 & \\ \cline{2-3} & & & 0 \end{array} \qquad \begin{array}{rcccc} & & 5 & 3 & 2 \\ \cline{2-5} 3) & 1 & 5 & 9 & 6 \\ & 1 & 5 & & \\ \cline{2-3} & & & 9 & \\ & & & 9 & \\ \cline{4-4} & & & & 6 \\ & & & & 6 \\ \cline{5-5} & & & & 0 \end{array}$$

2. **Write each digit in its correct place-value position** The digit resulting from any subtraction step names a number according to its place-value position and must be so used. (82, 83, 87)

$$\begin{array}{rccl} & & 9 & \text{r}1 \\ \cline{2-4} 4) & 3 & 7 & \\ & 3 & 6 & \\ \cline{2-3} & & 1 & \end{array} \qquad \begin{array}{rcccl} & 1 & 4 & 5 & \text{r}5 \\ \cline{2-5} 6) & 8 & 7 & 5 & \\ & 6 & & & \\ \cline{2-2} & 2 & 7 & & \\ & 2 & 4 & & \\ \cline{2-3} & & 3 & 5 & \\ & & 3 & 0 & \\ \cline{3-4} & & & 5 & \end{array} \qquad \begin{array}{rccccl} & 1 & 8 & 6 & 4 & \text{r}1 \\ \cline{2-6} 5) & 9 & 3 & 2 & 1 & \\ & 5 & & & & \\ \cline{2-2} & 4 & 3 & & & \\ & 4 & 0 & & & \\ \cline{2-3} & & 3 & 2 & & \\ & & 3 & 0 & & \\ \cline{3-4} & & & 2 & 1 & \\ & & & 2 & 0 & \\ \cline{4-5} & & & & 1 & \end{array}$$

How to Divide a Two-Digit Number by a Two-Digit Divisor

1. **Divide the tens digit of the dividend by the tens digit of the divisor** Then you will know a trial quotient. (98, 99)

69÷23=n Think: 6 T÷2 T=3

2. Multiply the divisor by the trial quotient You should obtain a number equal to or less than the dividend. (98, 99)

$$3 \times 23 = 69$$

3. Subtract Then write the quotient in ones place. (98, 99)

$$69 \div 23 = n \qquad \begin{array}{r} 3 \\ 23\overline{)69} \\ \underline{69} \\ 0 \end{array}$$

How to Divide a Three-Digit Number by a Two-Digit Divisor, Resulting in a One-Digit Quotient

1. Change the form of the hundreds to tens Then divide tens by tens to find a trial quotient. (102, 103)

$$160 \div 40 = n \qquad \text{Think: } 16\text{ T} \div 4\text{ T} = 4$$

2. Multiply mentally Then you should know if the trial quotient is right. (102, 103)

$$4 \times 40 = 160$$

3. Subtract Then write the quotient in ones place. (102, 103)

$$160 \div 40 = n \qquad \begin{array}{r} 4 \\ 40\overline{)160} \\ \underline{160} \\ 0 \end{array}$$

How to Divide a Three-Digit Number by a Two-Digit Number, Resulting in a Two-Digit Quotient

1. Divide the hundreds digit of the dividend by the tens digit of the divisor Then you will know a trial quotient. (105)

$$820 \div 41 = n \qquad \text{Think: } 8\text{ H} \div 4\text{ T} = 2$$

2. Multiply mentally Then you should know if the trial quotient is right. The product should be equal to or less than the dividend. (105)

$$2 \times 41 = 82$$
$$20 \times 41 = 820$$

3. Subtract Then write the trial quotient in tens place and determine the digit for ones place. (105)

$$820 \div 41 = n \qquad \begin{array}{r} 20 \\ 41\overline{)820} \\ \underline{82} \\ 0 \end{array}$$

How to Divide Larger Numbers by a Two-Digit Divisor

You divide larger numbers as you divide two-digit or three-digit numbers Simply repeat the three steps more often: Divide, multiply, and subtract. (148-150, 153, 158, 167, 168, 171, 174-177)

$$\begin{array}{r} 45 \\ 64\overline{)2880} \\ \underline{256} \\ 320 \\ \underline{320} \\ 0 \end{array} \qquad \begin{array}{r} 11423 \\ 47\overline{)536881} \\ \underline{47} \\ 66 \\ \underline{47} \\ 198 \\ \underline{188} \\ 108 \\ \underline{94} \\ 141 \\ \underline{141} \\ 0 \end{array}$$

Fractions

The Main Ideas

1. Meanings, words, and signs A fraction tells the number of parts of equal size into which the whole number is divided (the denominator), and the number of these being considered (the numerator). (183)

$$\begin{array}{ll} \underline{3} & \text{numerator} \\ 5 & \text{denominator} \end{array}$$

2. Finding equivalent fractions Multiplying or dividing both of the numbers named in a fraction by the same number—other than 1 or 0—produces another name for the same fractional number. (185-188, 210)

$$\frac{2\times4}{3\times4}=\frac{8}{12} \qquad \frac{2}{3}=\frac{8}{12}$$

$$\frac{18\div6}{24\div6}=\frac{3}{4} \qquad \frac{18}{24}=\frac{3}{4}$$

$$3\frac{5}{6}=3+\frac{5}{6}=\frac{18}{6}+\frac{5}{6}=\frac{18+5}{6}=\frac{23}{6}$$

3. Writing remainders as fractions In division, a remainder other than 0 may be written as a fraction. (197)

$$\begin{array}{r} 8\frac{1}{4} \\ 4\overline{)33} \\ \underline{32} \\ 1 \end{array} \qquad \begin{array}{r} 4\frac{11}{26} \\ 26\overline{)115} \\ \underline{104} \\ 11 \end{array}$$

Comparing Fractional Numbers

1. Fractions with the same numerator As the denominators increase, the numbers decrease. (192)

$\frac{1}{3}$ is smaller than $\frac{1}{2}$

$\frac{5}{7}$ is smaller than $\frac{5}{6}$

2. Fractions with a common denominator As the numerators increase, the numbers increase. (193)

$\frac{5}{8}$ is larger than $\frac{4}{8}$

$\frac{5}{9}$ is larger than $\frac{4}{9}$

3. Rename fractions, if necessary, so that they have a common denominator Compare the numerators. (194)

$$\frac{7}{9}=\frac{7\times2}{9\times2}=\frac{14}{18} \qquad \frac{5}{6}=\frac{5\times3}{6\times3}=\frac{15}{18}$$

How to Add Fractional Numbers

1. Adding fractional numbers with a common denominator Write the sum of the numerators over the common denominator. (202-205)

$$\frac{3}{9}+\frac{2}{9}+\frac{7}{9}=\frac{3+2+7}{9}=\frac{12}{9}$$

$$\frac{12}{9}=\frac{9+3}{9}=\frac{9}{9}+\frac{3}{9}=1\frac{3}{9}=1\frac{1}{3}$$

2. Adding fractional numbers with unequal denominators Rename both numbers so that they have a common denominator. Then add. (206-208)

$$\frac{9}{10}+\frac{14}{15}=\frac{27}{30}+\frac{28}{30}=\frac{27+28}{30}=\frac{55}{30}$$

$$\frac{55}{30}=\frac{30}{30}+\frac{25}{30}=1+\frac{25}{30}=1\frac{5}{6}$$

3. Using mixed fractions Add the fractional parts first. Then add the whole numbers. (212-214)

$$\begin{array}{rl} 5\frac{2}{3}= & 5\frac{28}{42} \\ 2\frac{5}{6}= & 2\frac{35}{42} \\ \underline{+4\frac{5}{7}}= & \underline{4\frac{30}{42}} \\ & 11\frac{93}{42}=11+2+\frac{9}{42} \\ & \quad =13+\frac{9}{42}=13\frac{3}{14} \end{array}$$

How to Subtract Fractional Numbers

1. Subtracting fractional numbers with a common denominator Subtract the numbers named by the numerators as you would subtract whole numbers. Write this difference over their common denominator. (220)

$$\frac{5}{7}-\frac{3}{7}=\frac{5-3}{7}=\frac{2}{7} \qquad \begin{array}{r} \frac{5}{7} \\ -\frac{3}{7} \\ \hline \frac{2}{7} \end{array}$$

2. Subtracting fractional numbers with unequal denominators Rewrite the fractions so that they have a common denominator. Then subtract as you did when there was a common denominator. (226)

$$\frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{12-10}{15}=\frac{2}{15}=n$$

$$\frac{3}{8}-\frac{3}{12}=\frac{9}{24}-\frac{6}{24}=\frac{9-6}{24}=\frac{3}{24}=\frac{1}{8}=n$$

3. Subtracting a fractional number from a whole number Rename the whole number as a sum which has 1 as one of its addends. Rename the 1 as a fraction with the same denominator as the subtrahend. Subtract the numbers and write the whole number. Make changes in form as necessary. (221)

$$\begin{array}{r} 3-\frac{7}{8}=n \\ (2+1)-\frac{7}{8}=n \\ 2+\left(\frac{8}{8}-\frac{7}{8}\right)=n \\ 2\frac{1}{8}=n \end{array} \qquad \begin{array}{rcr} 3 & = & 2\frac{8}{8} \\ -\frac{7}{8} & = & \frac{7}{8} \\ \hline & & 2\frac{1}{8} \end{array}$$

4. Subtracting with mixed fractions Subtract the fractional numbers. Subtract the whole numbers. Add these results. (222, 224, 225)

a.
$$\begin{array}{rcrcr} 5\frac{1}{8} & = & 4\frac{9}{8} & & \\ -2\frac{3}{8} & = & 2\frac{3}{8} & & \\ \hline & & 2\frac{6}{8} & = & 2\frac{3}{4} \end{array}$$

b.
$$\begin{array}{rcrcrcr} 8\frac{1}{3} & = & 8\frac{2}{6} & = & 7\frac{8}{6} & & \\ -3\frac{5}{6} & = & 3\frac{5}{6} & = & 3\frac{5}{6} & & \\ \hline & & & & 4\frac{3}{6} & = & 4\frac{1}{2} \end{array}$$

Multiplication

The Main Ideas

1. Meanings, words, and signs When a number is to be added a specified number of times, you multiply. You may express your multiplication in a statement containing numerals and signs, using either the column or the equation form. (49)

$$\begin{array}{rl} 9 & \rbrace \text{ factors} \\ \times 7 & \\ \hline 63 & \text{product} \end{array} \qquad 7\times 9=63$$

2. Commutative and associative properties The order or the grouping of the factors does not affect the product. (50, 66, 88)

$$7\times 8=56 \qquad 8\times 7=56$$

$$(3\times 7)\times 8=168 \qquad 3\times(7\times 8)=168$$

3. One is the identity number of multiplication Multiplication by 1 leaves a number unchanged. (50)

$$6\times 1=6 \qquad 1\times 6=6$$

4. Using 0 in multiplication Any number times 0, and 0 times any number, equals 0. (49, 50)

$8 \times 0 = 0 \qquad 0 \times 8 = 0$

How to Multiply by a One-Digit Number

1. Learn to use the basic multiplication combinations Then you will be able to multiply the digits in any place-value position. (50-53)

$$\begin{array}{lr} & 2143\\ & \times 2\\ \cline{2-2} 2\times3=6 & 6\\ 2\times40=80 & 80\\ 2\times100=200 & 200\\ 2\times2000=4000 & 4000\\ \cline{2-2} \text{Add} & 4286 \end{array}$$

2. Make changes in form Whenever the partial product in any multiplication is greater than 9, you make changes in form. (51)

$$\begin{array}{r} 51\\ \times4\\ \hline 4\\ 200\\ \hline 204 \end{array} \qquad \begin{array}{r} 15\\ \times4\\ \hline 20\\ 40\\ \hline 60 \end{array} \qquad \begin{array}{r} 55\\ \times4\\ \hline 20\\ 200\\ \hline 220 \end{array}$$

How to Multiply by a Two-Digit Number

1. Multiply by tens as you multiply by ones Tens times thousands is equal to ten-thousands. (56)

1 ten× 1= 10
1 ten× 10= 100
1 ten× 100= 1000
1 ten×1000=10000

2. Multiply by the ones and then by the tens Make changes in form as needed. Add the partial products. Their sum is the product. (57-63)

$$\begin{array}{lr} & 1357\\ & \times46\\ \cline{2-2} 6\ \text{O}\times7\ \text{O}=42\ \text{O}= & 42\\ 6\ \text{O}\times5\ \text{T}=30\ \text{T}= & 300\\ 6\ \text{O}\times3\ \text{H}=18\ \text{H}= & 1800\\ 6\ \text{O}\times1\ \text{Th}=6\ \text{Th}= & 6000\\ 4\ \text{T}\times7\ \text{O}=28\ \text{T}= & 280\\ 4\ \text{T}\times5\ \text{T}=20\ \text{H}= & 2000\\ 4\ \text{T}\times3\ \text{H}=12\ \text{Th}= & 12000\\ 4\ \text{T}\times1\ \text{Th}=4\ \text{TTh}= & 40000\\ \cline{2-2} \text{Add} & 62422 \end{array}$$

How to Multiply by a Three-Digit Number

1. Multiply by hundreds as you multiply by tens and by ones Hundreds times thousands is equal to hundred-thousands. (64, 65)

1 hundred× 1= 100
1 hundred× 10= 1000
1 hundred× 100= 10000
1 hundred×1000=100000

2. Multiply first by the ones, then by the tens, and then by the hundreds Make changes in form whenever you find it is necessary. Add the partial products. Their sum is the product. (64, 65)

$$\begin{array}{rl} \left.\begin{array}{r} 3142\\ \times564 \end{array}\right\} & \text{factors}\\ \cline{1-1} \left.\begin{array}{r} 12568\\ 18852\\ 15710 \end{array}\right\} & \text{partial products}\\ \cline{1-1} 1772088 & \text{product} \end{array}$$

Problem Solving

The Main Ideas

1. Story problems Most problems can be written as arithmetic sentences, or equations. (131-133)

Seven more books than I read last year is twenty-five.

$$n+7=25$$

2. Solving story problems Write the equation. Solve it. Then write the answer. (133, 157, 170)

A club had 12 members. Then other boys joined it, and there were 19 members. How many boys joined?

$12+n=19$ $\quad n=19-12$ $\quad n=7$

7 other boys joined the club.

3. Using more than one operation Write one equation for problems involving more than one operation. (Remember: The ( ) mean *do this operation first.*) (134-141, 173)

On 5 days, Mary practiced her piano lesson for 50 minutes, 55 minutes, 60 minutes, 35 minutes, and 40 minutes. On an average, how many minutes did she practice each day?

$$(50+55+60+35+40)\div 5=n$$
$$240 \div 5=n$$
$$48=n$$

Mary practiced an average of 48 minutes a day.

How to Solve Story Problems

1. Addition To join sets with unequal numbers, you add their numbers. (25, 29, 54, 68, 134-136, 209, 290)

Paul bought a sweater for \$5.98, a pair of gloves for \$2.50, and a tie for \$1.49. How much did he spend in all?

$5.98+2.50+1.49=n$

$5.98+2.50+1.49=9.97$

$$\begin{array}{r} \$5.98 \\ 2.50 \\ +1.49 \\ \hline \$9.97 \end{array}$$

Paul spent \$9.97.

2. Subtraction To separate a set, or to compare sets to find by how much they differ in number, you subtract their numbers. (38, 39, 40, 54, 86, 134, 137, 138, 223, 290)

Frank's and Terry's total weight was 125 pounds. Frank weighed 66 pounds. How much did Terry weigh?

$125-66=n$ $\quad 125-66=59$

Terry weighed 59 pounds.

Sally had 157 pennies in her collection, and Carol had 98 pennies in hers. How many more pennies did Sally have than Carol?

$157-98=n$ $\quad 157-98=59$

Sally had 59 more pennies than Carol had.

3. Multiplication To find how many in all of several sets with an equal number, you multiply. (54, 55, 60, 68, 69, 86, 135, 137, 139)

The baker delivered 235 loaves of rye bread to his customers each day. How many loaves did he deliver in 15 days?

$15\times235=n \qquad 15\times235=3525$

He delivered 3525 loaves.

4. Division To find how many sets with an equal number, or to find how many in each set with an equal number, divide. (76, 80, 86, 96, 101, 136, 138, 139, 152, 178)

Mrs. Lang invited 24 people to a party, seating 4 people at each table. How many tables did she have?

$24\div4=n \qquad 24\div4=6$

She had 6 tables.

The 30 pupils were divided into 5 groups with an equal number. How many were in each group?

$30\div5=n \qquad 30\div5=6$

6 pupils were in each group.

Subtraction

The Main Ideas

1. Meanings, words, and signs Subtraction is the inverse of addition and is expressed in numerals and signs. (33)

$$\begin{array}{r} 8 \\ +9 \\ \hline 17 \end{array} \xleftarrow{\quad} \text{inverse} \xrightarrow{\quad} \begin{array}{rl} 17 & \text{minuend} \\ -9 & \text{subtrahend} \\ \cline{1-1} 8 & \text{difference} \end{array}$$

2. Zero in subtraction Subtracting 0 from a number does not change the number, and subtracting a number from itself leaves 0. (34)

$7-0=7 \qquad 7-7=0$

3. Checking You use addition to check subtraction. (88)

$34-18=16 \qquad 16+18=34$

How to Subtract

1. Learn to use the basic subtraction combinations Then you will know how to subtract numbers in any place-value position. (34, 35)

$$\begin{array}{r} 58 \\ -37 \\ \hline 21 \end{array} \qquad \begin{array}{r} 5943 \\ -1932 \\ \hline 4011 \end{array} \qquad \begin{array}{r} 86445 \\ -34402 \\ \hline 52043 \end{array}$$

2. Make changes in form In any subtraction, whenever any digit in the minuend is less than the corresponding digit in the subtrahend, you change the form as needed. (36, 37, 42-44)

$$\begin{array}{r} 83 \\ -69 \\ \hline 14 \end{array} \qquad \begin{array}{r} 693 \\ -497 \\ \hline 196 \end{array} \qquad \begin{array}{r} 4697 \\ -2699 \\ \hline 1998 \end{array} \qquad \begin{array}{r} 56784 \\ -38876 \\ \hline 17908 \end{array}$$

Tables of Measure

In the tables of measure the sign = means *measures the same as* or *is equivalent to.*

Linear Measure

12 inches (in.) = 1 foot (ft.)
36 in. = 1 yard (yd.)
3 ft. = 1 yd.
$16\frac{1}{2}$ ft. = 1 rod (rd.)
$5\frac{1}{2}$ yd. = 1 rd.
5280 ft. = 1 mile (mi.)
1760 yd. = 1 mi.
320 rd. = 1 mi.

Square Measure

144 square inches (sq. in.) = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
43,560 sq. ft. = 1 acre
160 sq. rd. = 1 acre
640 acres = 1 sq. mi.

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)

Liquid Measure

16 fluid ounces (fl. oz.) = 1 pint (pt.)
2 cups (c.) = 1 pt.
2 pt. = 1 quart (qt.)
4 qt. = 1 gallon (gal.)

Time

60 seconds (sec.) = 1 minute (min.)
60 min. = 1 hour (hr.)
24 hr. = 1 day (da.)
7 da. = 1 week (wk.)
365 da. = 1 yr.
366 da. = 1 leap year
12 months (mo.) = 1 yr.
10 yr. = 1 decade
100 yr. = 1 century

Weight

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 ton (T.)

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